Ratios and Proportions

Ratio is another word for a **fraction**. It is the comparison of two quantities: the **numerator** (top number of a *fraction*) and the **denominator** (bottom number of a fraction). For instance, if a classroom has 32 students and 20 of them are girls, we can say that the *ratio* of the number of girls to the number of students in the class is $\frac{20}{32} = \frac{5}{8}$ or 5:8. There are several other comparisons we can make using the information. We could compare the number of boys to the number of students, $\frac{12}{32} = \frac{3}{8}$.

What about the number of boys to the number of girls? $\frac{12}{20} = \frac{3}{5}$

Or, the number of girls to the number of boys? $\frac{20}{12} = \frac{5}{3}$

When two ratios are equal to each other, we have formed a **proportion**. A *proportion* is a mathematical sentence stating that two ratios are equal.

$$\frac{6}{9} = \frac{2}{3}$$

There are several properties of proportions that will be useful as we continue through this unit.

- We could *switch* the 6 with the 3 and still have a *true* proportion (Example 1).
- We could *switch* the 2 with the 9 and still have a *true* proportion (Example 2).
- We could even *flip* both fractions over and still have a *true* proportion (Example 3).

Example 1	Example 2	Example 3
$\frac{⑥}{9} = \frac{2}{③}$	$\frac{6}{⑨} = \frac{②}{3}$	$\frac{6}{9} = \frac{2}{3}$
$\frac{③}{9} = \frac{2}{⑥}$	$\frac{6}{②} = \frac{⑨}{3}$	$\frac{9}{6} = \frac{3}{2}$

Proportions are also very handy to use for problem solving. We use a process that involves **cross multiplying**, then **solve** the resulting **equation**. Look at the example below as we *solve* the *equation* and find the **value of the variable**.

$$\frac{3}{5} = \frac{x}{x+6}$$

$$\frac{3}{5} \times \frac{x}{x+6}$$ ← cross multiply

$$3(x+6) = 5x$$
$$3x + 18 = 5x$$ ← distribute (**distributive property**)

$$3x - 3x + 18 = 5x - 3x$$
$$18 = 2x$$ ← subtract 3*x* from each side

$$\frac{18}{2} = \frac{2x}{2}$$
$$9 = x$$ ← divide each side by 2

Check your answer. Does $\frac{9}{9+6} = \frac{3}{5}$? Yes, $\frac{9}{15} = \frac{3}{5}$, so 9 is the correct *value* for x.

Try the following practice.

Using Proportions Algebraically

We can use proportions in word problems as well. Here's an example.

In Coach Coffey's physical education class, the ratio of boys to girls is 3 to 4. If there are 12 boys in the class, how many girls are there?

When setting up proportions, you must have a plan and be consistent when you write the ratios. If you set up one ratio as $\frac{\text{boys}}{\text{girls}}$, the you must set up the other ratio in the same order, as $\frac{\text{boys}}{\text{girls}}$.

$\frac{3}{4} = \frac{12}{x}$ ← notice that both fractions indicate $\frac{\text{boys}}{\text{girls}}$

$\frac{3}{4} \times \frac{12}{x}$ ← cross multiply

$3x = 4 \times 12$ ← simplify

$3x = 48$ ← divide each side by 3

$\frac{3x}{3} = \frac{48}{3}$

$x = 16$

Check your answer. Does $\frac{12}{16} = \frac{3}{4}$? Yes, so 16 is the correct answer.

Now it is your turn to practice on the following page.