- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

The Distributive Property

Consider 4(2 + 6). The rules for **order of operations** would have us add inside the parentheses first.

$$\begin{aligned} 4(2+6) &= \\ 4(8) &= \\ 32 & \end{aligned}$$

Remember: Rules for the order of operations

Always start on the *left* and move *to* the *right*.

1. Do operations inside *grouping symbols* first.
2. Then do all *powers* (exponents) **or** *roots*.
3. Next do *multiplication* **or** *division*—as they occur from left to right.
4. Finally, do *addition* **or** *subtraction*—as they occur from left to right.

However, there is a second way to do the problem.

$$\begin{aligned} &4(2 + 6) = \\ &4(2) + 4(6) = \\ &8 + 24 = \\ &32 \end{aligned}$$

In the second way, the 4 is *distributed* over the addition. This second way of doing the problem illustrates the **distributive property**.

The Distributive Property

For any numbers a, b, and c,
$a(b + c) = ab + ac$

Also, it works for subtraction:
$a(b - c) = ab - ac$

This property is most useful in simplifying expressions that contain variables, such as $2(x + 4)$.

To **simplify an expression** we must perform as many of the indicated operations as possible. However, in the expression $2(x + 4)$, we can't add first, unless we know what number x represents. The *distributive property* allows us to rewrite the equation:

$$\begin{aligned} &2(x + 4) = \\ &2x + 2(4) = \\ &2x + 8 \end{aligned}$$

The distributive property allows you to multiply each term *inside* a set of parentheses by a factor *outside* the parentheses. We say multiplication is *distributive over* addition and subtraction.

$$\begin{aligned} 5(3 + 1) &= (5 \bullet 3) + (5 \bullet 1) \\ 5(4) &= 15 + 5 \\ 20 &= 20 \end{aligned}$$

$$\begin{aligned} 5(3 - 1) &= (5 \bullet 3) - (5 \bullet 1) \\ 5(2) &= 15 - 5 \\ 10 &= 10 \end{aligned}$$

Not all operations are distributive. You cannot distribute division over addition.

$$\begin{aligned} 14 \div (5 + 2) &\neq 14 \div 5 + 14 \div 2 \\ 14 \div 7 &\neq 2.8 + 7 \\ 2 &\neq 9.8 \end{aligned}$$

Study the chart below.

Properties

Addition		**Multiplication**	
Commutative:	$a + b = b + a$	Commutative:	$ab = ba$
Associative:	$(a + b) + c = a + (b + c)$	Associative:	$(ab)c = a(bc)$
Identity:	0 is the identity. $a + 0 = a$ and $0 + a = a$	Identity:	1 is the identity. $a \cdot 1 = a$ and $1 \cdot a = a$
Addition		**Subtraction**	
Distributive:	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	Distributive:	$a(b - c) = ab - ac$ and $(b - c)a = ba - ca$

These properties deal with the following:

order—**commutative property** of addition and commutative property of multiplication

grouping—**associative property** of addition and associative property of multiplication

identity—**additive identity** property and **multiplicative identity** property

zero—**multiplicative property of zero**

distributive—**distributive property** of multiplication over addition and over subtraction

Notice in the distributive property that it does *not* matter whether $\boldsymbol{a}$ is placed on the *right* or the *left* of the expression in parentheses.

$$\boldsymbol{a}(b + c) = (b + c)\boldsymbol{a} \text{ or } \boldsymbol{a}(b - c) = (b - c)\boldsymbol{a}$$

The **symmetric property of equality** (if $a = b$, then $b = a$) says that if one quantity equals a second quantity, then the second quantity also equals the first quantity. We use the **substitution property of equality** when replacing a variable with a number or when two quantities are equal and one quantity can be replaced by the other. Study the chart and examples below that describe properties of equality.

Properties of Equality

Reflexive:	$a = a$
Symmetric:	If $a = b$, then $b = a$.
Transitive:	If $a = b$ and $b = c$, then $a = c$.
Substitution:	If $a = b$, then a may be replaced by b.

Examples of Properties of Equality	
Reflexive:	$8 - e = 8 - e$
Symmetric:	If 5 + 2 = 7, then 7 = 5 + 2.
Transitive:	If 9 – 2 = 4 + 3 and 4 + 3 = 7, then 9 – 2 = 7.
Substitution:	If $x = 8$, then $x \div 4 = 8 \div 4$. x is replaced by 8.
	or
	If 9 + 3 = 12, then 9 + 3 may be replaced by 12.

Study the following examples of how to simplify expressions. Refer to the charts above and on the previous page as needed.

$$5(6x + 3) + 8$$

$5(6x + 3) + 8 =$ ← use the distributive property to distribute 5 over 6x and 3

$5(6x) + 5(3) + 8 =$

$30x + 15 + 8 =$ ← use the associative property to associate 15 and 8

$30x + 23$

and

$$6 + 2(4x - 3)$$

$6 + 2(4x - 3) =$ ← use order of operations to multiply before adding, then

$6 + 2(4x) + 2(-3) =$ ← distribute 2 over 4x and -3

$6 + 8x + -6 =$ ← use the associative property to associate 6 and -6

$8x + 0 =$ ← use the identity property of addition

$8x$

Simplifying Expressions

Here's how to use the distributive property and the definition of subtraction to simplify the following expressions.

Example 1:

Simplify

$-7a - 3a$

$$\begin{aligned} -7a - 3a &= -7a + -3a \\ &= (-7 + -3)a \quad \longleftarrow \text{ use the distributive property} \\ &= -10a \end{aligned}$$

Example 2:

Simplify

$10c - c$

$$\begin{aligned} 10c - c &= 10c - 1c \\ &= 10c + -1c \\ &= (10 + -1)c \quad \longleftarrow \text{ use the distributive property} \\ &= 9c \end{aligned}$$

The expressions $-7a - 3a$ and $-10a$ are called **equivalent** *expressions*. The expressions $10c - c$ and $9c$ are also called *equivalent* expressions. Equivalent expressions express the same number. An expression is in simplest form when it is replaced by an equivalent expression having no **like terms** and no parentheses.

Study these examples.

$$\begin{aligned} -5x + 4x &= (-5 + 4)x \\ &= -x \end{aligned}$$

$$\begin{aligned} 5y - 5y &= 5y + -5y \\ &= (5 + -5)y \\ &= 0y \quad \longleftarrow \text{ multiplicative property of zero} \\ &= 0 \end{aligned}$$

> The multiplicative property of 0 says for any number a,
> $a \cdot 0 = 0 \cdot a = 0$.

The following shortcut is frequently used to simplify expressions.

First

- rewrite each subtraction as adding the opposite
- then combine *like terms* (terms that have the same variable) by adding.

Simplify

$2a + 3 - 6a$

like terms

$2a + 3 - 6a = 2a + 3 + -6a$ ← rewrite – 6*a* as + -6*a*

$= -4a + 3$ ← combine like terms by adding

Simplify

$8b + 7 - b - 6$

like terms

$8b + 7 - b - 6 = 8b + 7 + -1b + -6$ ← rewrite – *b* as + -1*b* and – 6 as + -6

$= 7b + 1$ ← combine like terms by adding

like terms

Simplify

$7x + 5 + 3x = 10x + 5$ ← combine like terms

like terms

Equations with Like Terms

Consider the following equation.

$$2x + 3x + 4 = 19$$

Look at both sides of the equation and see if either side can be simplified.

> Always simplify first
> by combining like terms.

$$\begin{aligned} 2x + 3x + 4 &= 19 \\ 5x + 4 &= 19 && \longleftarrow \text{add like terms} \\ 5x + 4 - 4 &= 19 - 4 && \longleftarrow \text{subtract 4 from each side} \\ 5x &= 15 \\ \frac{5x}{5} &= \frac{15}{5} && \longleftarrow \text{divide each side by 5} \\ x &= 3 \end{aligned}$$

Always mentally check your answer by *substituting* the solution for the variable in the original equation.

Substitute 3 for x in the equation.

$$\begin{aligned} 2x + 3x + 4 &= 19 \\ 2(3) + 3(3) + 4 &= 19 \\ 6 + 9 + 4 &= 19 \\ 19 &= 19 && \textit{It checks!} \end{aligned}$$

Consider this example.

The product of x and 7 plus the product of x and 3 is 45.

Remember: To work a problem like this one, we need to remember two things. The word *product* means *multiply* and the word *is* always translates to =.

The product of x and 7 plus the product of x and 3 is 45.

$$\begin{aligned} 7x + 3x &= 45 \\ 10x &= 45 && \longleftarrow \text{add like terms} \\ \frac{10x}{10} &= \frac{45}{10} && \longleftarrow \text{divide both sides by 10} \\ x &= 4.5 \end{aligned}$$

Check by substituting 4.5 for x in the original equation.

$$\begin{aligned} 7x + 3x &= 45 \\ 7(4.5) + 3(4.5) &= 45 \\ 31.5 + 13.5 &= 45 \\ 45 &= 45 && \textit{It checks!} \end{aligned}$$

Here is another example which appears to be more challenging.

$$\begin{aligned} 3x - 2 - x + 10 &= -12 \\ 3x - 2 - 1x + 10 &= -12 && \longleftarrow \text{remember: } 1 \bullet x = x \\ 3x - 1x - 2 + 10 &= -12 && \longleftarrow \text{add like terms} \\ 2x + 8 &= -12 \\ 2x + 8 - 8 &= -12 - 8 && \longleftarrow \text{subtract 8 from both sides} \\ 2x &= -20 \\ \frac{2x}{2} &= \frac{-20}{2} && \longleftarrow \text{divide both sides by 2} \\ x &= -10 \end{aligned}$$

Check by substituting -10 into the original equation.

$$\begin{aligned} 3x - 2 - x + 10 &= -12 \\ 3(-10) - 2 - (-10) + 10 &= -12 \\ -30 - 2 + 10 + 10 &= -12 \\ -32 + 20 &= -12 \\ -12 &= -12 && \textit{It checks!} \end{aligned}$$

Putting It All Together

Guidelines for Solving Equations

1. Use the distributive property to clear parentheses.
2. Combine like terms. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by substituting the solution in the original equation.

SAM = **S**implify (steps 1 and 2) then
Add (or subtract)
Multiply (or divide)

Example 1

Solve:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6y + 4y + 8 &= 88 && \longleftarrow \text{use distributive property} \\ 10y + 8 - 8 &= 88 - 8 && \longleftarrow \text{combine like terms and undo addition by subtracting 8 from each side} \\ \frac{10y}{10} &= \frac{80}{10} && \longleftarrow \text{undo multiplication by dividing by 10} \\ y &= 8 \end{aligned}$$

Check solution in the original equation:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6(8) + 4(8 + 2) &= 88 \\ 48 + 4(10) &= 88 \\ 48 + 40 &= 88 \\ 88 &= 88 && \textit{It checks!} \end{aligned}$$

Example 2

Solve:

$$-\tfrac{1}{2}(x + 8) = 10$$

$$-\tfrac{1}{2}x - 4 = 10 \quad \longleftarrow \text{ use distributive property}$$

$$-\tfrac{1}{2}x - 4 + 4 = 10 + 4 \quad \longleftarrow \text{ undo subtraction by adding 4 to both sides}$$

$$-\tfrac{1}{2}x = 14$$

$$(-2)-\tfrac{1}{2}x = 14(-2) \quad \longleftarrow \text{ isolate the variable by multiplying}$$

$$x = -28 \quad \longleftarrow \text{ each side by the reciprocal of } -\tfrac{1}{2}$$

Check solution in the original equation:

$$-\tfrac{1}{2}(x + 8) = 10$$

$$-\tfrac{1}{2}(-28 + 8) = 10$$

$$-\tfrac{1}{2}(-20) = 10$$

$$10 = 10 \quad \textit{It checks!}$$

Example 3

Solve:

$$26 = \tfrac{2}{3}(9x - 6)$$

$$26 = \tfrac{2}{3}(9x) - \tfrac{2}{3}(6) \quad \longleftarrow \text{ use distributive property}$$

$$26 = 6x - 4$$

$$26 + 4 = 6x - 4 + 4 \quad \longleftarrow \text{ undo subtraction by adding 4 to each side}$$

$$\frac{30}{6} = \frac{6x}{6} \quad \longleftarrow \text{ undo multiplication by dividing each side by 6}$$

$$5 = x$$

Check solution in the original equation:

$$26 = \tfrac{2}{3}(9x - 6)$$

$$26 = \tfrac{2}{3}(9 \bullet 5 - 6)$$

$$26 = \tfrac{2}{3}(39)$$

$$26 = 26 \quad \textit{It checks!}$$

Example 4

Solve:

$$x - (2x + 3) = 4$$
$$x - 1(2x + 3) = 4 \quad \leftarrow \text{use the multiplicative property of -1}$$
$$x - 2x - 3 = 4 \quad \leftarrow \text{use the multiplicative identity of 1 and use the distributive property}$$
$$-1x - 3 = 4 \quad \leftarrow \text{combine like terms}$$
$$-1x - 3 + 3 = 4 + 3 \quad \leftarrow \text{undo subtraction}$$
$$\frac{-1x}{-1} = \frac{7}{-1} \quad \leftarrow \text{undo multiplication}$$
$$x = -7$$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

line 1: $x - (2x + 3) = 4$
line 2: $x \mathbf{- 1}(2x + 3) = 4$

Also notice the use of *multiplicative identity* on line three.

line 3: $\mathbf{1}x - 2x - 3 = 4$

The simple variable x was multiplied by 1 ($1 \bullet x$) to equal $1x$. The $1x$ helped to clarify the number of variables when combining like terms on line four.

Check solution in the original equation:

$$x - (2x + 3) = 4$$
$$-7 - (2 \bullet -7 + 3) = 4$$
$$-7 - (-11) = 4$$
$$4 = 4$$

It checks!