- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

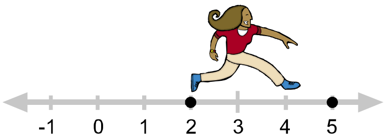
- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

**Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

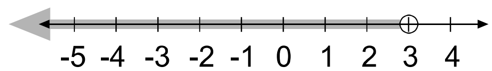
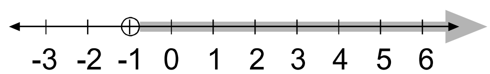
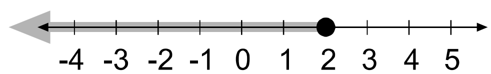
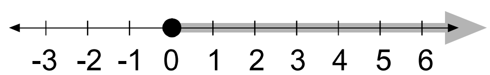
## Graphing Inequalities on a Number Line

In this unit we will graph **inequalities** on a **number line**. A **graph of a number** is the point on a *number line* paired with the number. Graphing solutions on a number line will help you visualize solutions.



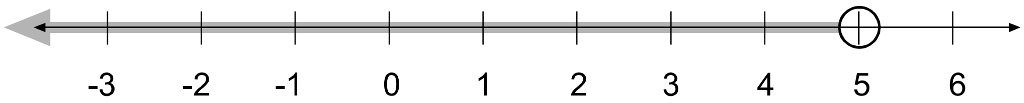
Here are some examples of *inequalities*, their verbal meanings, and their graphs.

**Inequalities**

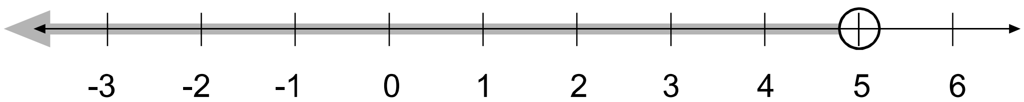
| Inequality | Meaning | Graph |
|---|---|---|
| a. $x < 3$ | All real numbers less than 3. |  The open circle means that 3 is *not* a solution. Shade to left. |
| b. $x > -1$ | All real numbers greater than -1. |  The open circle means that -1 is *not* a solution. Shade to right. |
| c. $x \leq 2$ | All real numbers less than or equal to 2. |  The solid circle means that 2 is a solution. Shade to left. |
| d. $x \geq 0$ | All real numbers greater than or equal to 0. |  The solid circle means that 0 is a solution. Shade to right. |

For each example, the inequality is written with the variable on the left. Inequalities can also be written with the variable on the right. However, graphing is easier if the variable is on the left.

Consider $x < 5$, which means the same as $5 > x$. Note that the graph of $x < 5$ is all real numbers less than 5.




The graph of $5 > x$ is all real number that 5 is greater than.




To write an inequality that is *equivalent* to (or the same as) $x < 5$, move the number and variable to the opposite side of the inequality, and then reverse the inequality.

$x < 5$

$5 > x$

$x < 5$ means the same as

$5 > x$

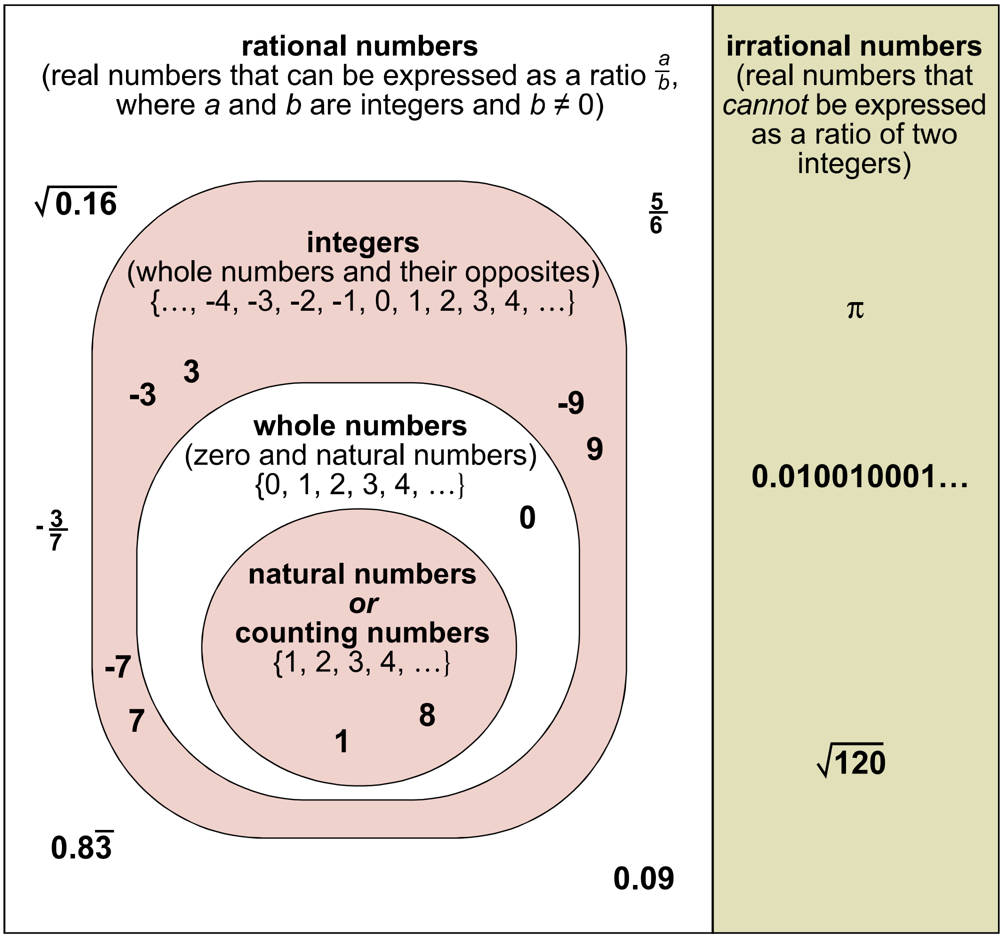
The inequality $y \geq -2$ is equivalent to $-2 \leq y$. Both inequalities can be written as the *set of all* **real numbers** that are *greater than or equal to* -2.

The inequality $0 \leq x$ is equivalent to $x \geq 0$. Each can be written as the *set of all real numbers* that are *greater than or equal to* zero.

**Remember:** *Real numbers* are all **rational numbers** and all **irrational numbers**.

The Venn diagram below is a graphic organizer that aids in visualizing what real numbers are.

**The Set of Real Numbers**




Rational numbers can be expressed as a **ratio** $\frac{a}{b}$, where $a$ and $b$ are *integers* and $b \neq 0$.

| **rational numbers** | 4 | $-3\frac{3}{4}$ | 0.250 | 0 | $0.\overline{3}$ |
|---|---|---|---|---|---|
| **expressed as ratio of two integers** | $\frac{4}{1}$ | $-\frac{15}{4}$ | $\frac{1}{4}$ | $\frac{0}{1}$ | $\frac{1}{3}$ |

**Note:** All integers are rational numbers.

A *ratio* is the comparison of two quantities. For example, a ratio of 8 and 11 is 8:11 or $\frac{8}{11}$.

## Solving Inequalities

We have been solving *equations* since Unit 1. When we solve inequalities, the procedures are the same except for one important difference.

> **When we multiply or divide both sides of an inequality by the same *negative number*, we *reverse* the direction of the inequality symbol.**

**Example:** Solve by *dividing* by a *negative number* and *reversing* the inequality sign.

$$-3x < 6$$

$$\frac{-3x}{-3} > \frac{6}{-3} \quad \longleftarrow \text{divide each side by -3 and reverse the inequality symbol}$$

$$x > -2$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$-3x < 6$$
$$-3(-1) < 6$$
$$3 < 6 \quad \textit{It checks!}$$

substitute 3

$$-3x < 6$$
$$-3(3) < 6$$
$$-9 < 6 \quad \textit{It checks!}$$

substitute 0

$$-3x < 6$$
$$-3(0) < 6$$
$$0 < 6 \quad \textit{It checks!}$$

substitute 3,000

$$-3x < 6$$
$$-3(3{,}000) < 6$$
$$-9{,}000 < 6 \quad \textit{It checks!}$$

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.

Study the following examples.

**Example:** Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$$-\frac{1}{3}y \geq 4$$

$$(-3)\ -\frac{1}{3}y \leq 4(-3)$$ ← multiply each side by -3 and reverse the inequality symbol

$$y \leq -12$$

**Example:** Solve by first adding, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$$-3a - 4 > 2$$

$$-3a - 4 + 4 > 2 + 4$$ ← add 4 to each side

$$-3a > 6$$

$$\frac{-3a}{-3} < \frac{6}{-3}$$ ← divide each side by -3 and reverse the inequality symbol

$$a < -2$$

**Example:** Solve by first subtracting, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$$\frac{y}{-2} + 5 \leq 0$$

$$\frac{y}{-2} + 5 - 5 \leq 0 - 5$$ ← subtract 5 from each side

$$\frac{y}{-2} \leq -5$$

$$\frac{(-2)y}{-2} \geq (-5)(-2)$$ ← multiply each side by -2 and reverse the inequality symbol

$$y \geq 10$$

**Example:** Solve by first subtracting, then *multiplying* by a *positive number* and **not** *reversing* the inequality sign.

$$\frac{n}{2} + 5 \leq 2$$

$$\frac{n}{2} + 5 - 5 \leq 2 - 5 \quad \longleftarrow \text{ subtract 5 from each side}$$

$$\frac{n}{2} \leq -3$$

$$\frac{(2)n}{2} \leq -3(2) \quad \longleftarrow$$

$$n \leq -6$$

multiply each side by 2, *but* do *not* reverse the inequality symbol because we multiplied by a positive number

When multiplying or dividing both sides of an inequality by the same *positive number*, do *not* reverse the inequality symbol—leave it alone.

**Example:** Solve by first adding, then *dividing* by a positive number, and **not** *reversing* the inequality sign.

$$7x - 3 > -24$$

$$7x - 3 + 3 > -24 + 3 \quad \longleftarrow \text{ add 3 to each side}$$

$$7x > -21$$

$$\frac{7x}{7} > \frac{-21}{7} \quad \longleftarrow$$

$$x > -3$$

divide each side by 7 do *not* reverse the inequality symbol because we divided by a positive number