- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

Factoring Polynomials

If we look at the product *abc*, we know *a*, *b*, and *c* are *factors* of this product. In the same way, 2 and 3 are *factors* of 6. Other factors of 6 are 6 and 1.

Remember: A factor is a number or expression that divides evenly into another number.

factors of $abc = a, b,$ and c

factors of 6 = 1, 2, 3, and 6

Some numbers, like 5, have no factors other than the number itself and the number 1. These numbers are called **prime numbers**. A *prime number* is any **whole number** {0, 1, 2, 3, 4, …} with only two factors, 1 and itself. The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19.

prime numbers $< 20 =$ 2, 3, 5, 7, 11, 13, 17, and 19

Natural numbers greater than 1 that are *not* prime are called **composite numbers**. A *composite number* is a whole number with more than two factors. For example, 16 has five factors, 1, 2, 4, 8, and 16. Therefore, 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are the composite numbers less than 20.

composite numbers $< 20 =$ 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18

Every composite number can be written as a *product* of prime numbers. We can find this **prime factorization** by factoring the factors and repeating this process until all factors are primes.

For example, find the *prime factorization* of 24 and express it in completely **factored form**.

Factoring a Positive Number—
numbers greater than zero

Method One			Method Two
Use a factor tree.	24 / \\ 6 • 4 / \\ / \\ 3 • 2 • 2 • 2 24 = 3 • 2 • 2 • 2	*or*	24 6 • 4 (3 • 2)(2 • 2) 3 • 2 • 2 • 2

Factoring a Negative Number—
numbers less than zero

Method One		Method Two
-24 / \\ -6 • 4 / \\ / \\ -3 • 2 • 2 • 2 -24 = -3 • 2 • 2 • 2	*or*	-24 -6 • 4 = (-3 • 2)(2 • 2) = -3 • 2 • 2 • 2

An Alternate Method for Factoring a Positive Number

Here is an alternate method for factoring a positive number called *upside-down dividing*. Divide by prime numbers starting with the number 2.

2)24
2)12
2) 6
3) 3
1

The prime factorization is down the left side.

$2 \bullet 2 \bullet 2 \bullet 3$ or $2^3 \bullet 3$

To factor polynomials, you must look carefully at each term and decide if there is a factor that is *common* to each term. If there is, we basically "undistribute" or *factor out* that **greatest common factor (GCF)**. Look at the example below.

$6x^3 - 12x^2 + 3x =$ ⟵ Notice that each term can be divided by 3 and *x*. So, $3x$ is the *greatest factor* these terms have in *common*. Therefore, $3x$ is the *GCF* of $6x^3 - 12x^2 + 3x$.

$3x(2x^2 - 4x + 1)$ ⟵ *undistribute* the $3x$

All of the terms and symbols must be written to make sure that your new expression is *exactly* equal to the original one. You can check your work by distributing the $3x$ to everything within the parentheses to see if it matches the original expression.

$$3x(2x^2 - 4x + 1) = 6x^3 - 12x^2 + 3x$$

Remember: $(a + b) = (b + a)$

The **commutative property** of addition—numbers can be added in any order and the sum will be the same.

Alert!

$(a - b) \neq (b - a)$

The same is *not* true for $a - b$. The *commutative property* does *not* work with subtraction.

$a - b$ does *not* equal $b - a$

$(a - b) = -1(b - a)$

$a - b$ is understood as $a - {}^{+}b$, therefore,

$a - b$ equals $-1(b - a)$