Simplifying Rational Expressions

An **expression** is a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes **variables**. A **fraction**, or any part of a whole, is an *expression* that represents a **quotient**—the result of dividing two numbers. The same *fraction* may be expressed in many different ways.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10}$$

If the **numerator** (top number) and the **denominator** (bottom number) are both **polynomials**, then we call the fraction a **rational expression**. A *rational expression* is a fraction whose *numerator* and/or *denominator* are *polynomials*. The fractions below are all rational expressions.

$$\frac{x}{x+y} \qquad \frac{a^2 - 2a + 1}{a} \qquad \frac{1}{y^2 + 4} \qquad \frac{a}{b-3}$$

When the *variables* or any symbols which could represent numbers (usually letters) are replaced, the result is a numerator and a denominator that are **real numbers**. In this case, we say the entire *expression* is a *real number*. Real numbers are all **rational numbers** and **irrational numbers**. *Rational numbers* are numbers that can be expressed as a **ratio** $\frac{a}{b}$, where a and b are **integers** and $b \neq 0$. *Irrational numbers* are real numbers that *cannot* be expressed as a *ratio* of two *integers*. Of course, there is an exception: when a denominator is equal to 0, we say the fraction is *undefined*.

Note: In this unit, we will agree that *no* denominator equals 0.

Fractions have some interesting properties. Let's examine them.

- If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. $\frac{4}{8} = \frac{6}{12}$ therefore $4 \bullet 12 = 8 \bullet 6$

  $$\frac{a}{b} = \frac{c}{d}$$
  $$a \bullet d = b \bullet c$$
  $$ad = bc$$

  In other words, if two fractions are equal, then the **products** are equal when you **cross multiply**.

  $$\frac{4}{8} = \frac{6}{12}$$
  $$4 \bullet 12 = 8 \bullet 6$$
  $$48 = 48$$

- $\frac{a}{b} = \frac{ac}{bc}$ $\frac{4}{7} = \frac{4 \bullet 3}{7 \bullet 3}$ therefore $\frac{4}{7} = \frac{12}{21}$

  Simply stated, if you *multiply* both the numerator and the denominator by the *same* number, the new fraction will be **equivalent** to the original fraction.

- $\frac{ac}{bc} = \frac{a}{b}$ $\frac{9}{21} = \frac{9 \div 3}{21 \div 3}$ therefore $\frac{9}{21} = \frac{3}{7}$

  In other words, if you *divide* both the numerator and the denominator by the *same* number, the new fraction will be *equivalent* to the original fraction. The same rules are true for **simplifying** rational expressions by performing as many indicated operations as possible. Many times, however, it is necessary to **factor** and find numbers or expressions that divide the numerator or the denominator, or both, so that the **common factors** become easier to see. Look at the following example:

  $$\frac{3x + 3y}{3} = \frac{{}^{1}\cancel{3}(x + y)}{\cancel{3}_{1}} = x + y$$

  Notice that, by **factoring** a 3 out of the numerator, we can divide (or **cancel**) the 3s, leaving $x + y$ as the final result.

Before we move on, do the practice on the following pages.

Additional Factoring

Look carefully at numbers 2-5 in the previous practice. What do you notice about them?

Alert! You cannot cancel individual **terms** (numbers, variables, products, or quotients in an expression)—you can only cancel *factors* (numbers or expressions that exactly divide another number)!

$$\frac{2x+4}{4} \neq \frac{2x}{4} \qquad \frac{3x+6}{3} \neq \frac{x+6}{3} \qquad \frac{9x^2+3}{6x+3} \neq \frac{9x^2x}{6x}$$

Look at how simplifying these expressions was taken a step further. Notice that additional factoring was necessary.

Example

$$\frac{x^2+5x+6}{x+3} = \frac{^{1}\cancel{(x+3)}(x+2)}{_{1}\cancel{x+3}} = (x+2) = x+2$$

Look at the denominator above. It is one of the factors of the numerator. Often, you can use the problem for hints as you begin to factor.