- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Addition and Subtraction of Rational Expressions

In order to add and subtract rational expressions in fraction form, it is necessary for the fractions to have a **common denominator** (the same bottom number). We find those *common denominators* in the same way we did with simple fractions. The process requires careful attention.

- When we add $\frac{3}{7} + \frac{5}{8}$, we find a common denominator by multiplying 7 and 8.

- Then we change each fraction to an equivalent fraction whose denominator is 56.

  $\frac{3 \bullet 8}{7 \bullet 8} = \frac{24}{56}$ and $\frac{5 \bullet 7}{8 \bullet 7} = \frac{35}{56}$

- Next we add $\frac{24}{56} + \frac{35}{56} = \frac{59}{56}$.

Finding the Least Common Multiple (LCM)

By multiplying the denominators of the terms we intend to add or subtract, we can always find a common denominator. However, it is often to our advantage to find the **least common denominator (LCD)**, which is also the **least common multiple (LCM)**. The *LCD* or *LCM* is the smallest of the **common multiples** of two or more numbers. This makes simplifying the result easier. Look at the example on the following page.

Let's look at finding the LCM of 36, 27, and 15.

1. Factor each of the denominators and examine the results.

$36 = 2 \bullet 2 \bullet 3 \bullet 3$	←	The new denominator must contain at least **two 2s** and **two 3s**.
$27 = 3 \bullet 3 \bullet 3$	←	The new denominator must contain at least **three 3s**.
$45 = 3 \bullet 3 \bullet 5$	←	The new denominator must contain at least **two 3s** and **one 5**.

2. Find the **minimum** combination of factors that is described by the combination of all the statements above—two 2s, three 3s, and one 5.

$$\text{LCM} = \underbrace{2 \bullet 2}_{\text{two 2s}} \bullet \underbrace{3 \bullet 3 \bullet 3}_{\text{three 3s}} \bullet \underbrace{5}_{\text{one 5}} = 540$$

3. Convert the *terms* to equivalent fractions using the new common denominator and then proceed to add or subtract.

$$\frac{5}{36} = \frac{75}{540}; \frac{8}{27} = \frac{160}{540}; \frac{4}{15} = \frac{144}{540} \longrightarrow \frac{75}{540} + \frac{160}{540} - \frac{144}{540} = \frac{91}{540}$$

Now, let's look at an algebraic example.

$$\frac{y}{y^2 - 9} - \frac{1}{y^2 - 4y - 21} =$$

1. Factor each denominator and examine the results.

$y^2 - 9 = (y + 3)(y - 3)$ ← The new denominator must contain (y + 3) and (y – 3).

$y^2 - 4y - 21 = (y - 7)(y + 3)$ ← The new denominator must contain (y – 7) and (y + 3).

2. Find the *minimum* combination of factors.

$\text{LCM} = (y + 3)(y - 3)(y - 7)$

3. Convert each fraction to an equivalent fraction using the new common denominator and proceed to subtract.

$$\frac{y(y-7)}{(y+3)(y-3)(y-7)} - \frac{1(y-3)}{(y+3)(y-3)(y-7)} =$$

$$\frac{y^2 - 7y - y + 3}{(y+3)(y-3)(y-7)} =$$

$$\frac{y^2 - 8y + 3}{(y+3)(y-3)(y-7)}$$

notice how the minus sign between the fractions *distributes* to make -*y* + 3 in the numerator (**distributive property**)

Hint: Always check to see if the numerator can be factored and then reduce, if possible. Do this to be sure the answer is in the lowest terms.