- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Solving Equations

Recall that an **equation** is a mathematical sentence stating the two expressions have the same value. The equality symbol or equal sign (=) shows that two quantities are equal. An *equation* equates one expression to another.

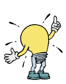
$3x - 7 = 8$ is an example of an equation.

You may be able to solve this problem mentally, without using paper and pencil.

$$3x - 7 = 8$$

The problem reads—3 times what number minus 7 equals 8?

Think: $3 \bullet 4 = 12$
$12 - 7 = 5$ ← too small



Think: $3 \bullet 5 = 15$
$15 - 7 = 8$ ← That's it!

$$3x - 7 = 8$$
$$3(5) - 7 = 8$$

Step-by-Step Process for Solving Equations

A problem like $\frac{x + 12}{5} = \text{-}2(x - 10)$ is a bit more challenging. You could use a guess and check process, but that would take more time, especially when answers involve **decimals** or fractions.

So, as problems become more difficult, you can see that it is important to have a process in mind and to write down the steps as you go.

Unfortunately, there is *no* exact process for solving equations. Every rule has an exception. That is why creative thinking, reasoning, and practice are necessary and keeping a written record of the steps you have used is extremely helpful.

Example 1

Let's look at a step-by-step process for solving the problem above.

$$\frac{x + 12}{5} = \text{-}2(x - 10)$$

← Step 1: Copy the problem **carefully**!

$$\frac{x + 12}{5} = \text{-}2x + 20$$

← Step 2: Simplify each side of the equation as needed by *distributing* the 2.

$$\left(\frac{x + 12}{\cancel{5}}\right) \bullet \cancel{5} = (\text{-}2x + 20) \bullet 5$$

← Step 3: Multiply both sides of the equation by 5 to "undo" the division by 5, which eliminates the fraction.

$$x + 12 = \text{-}10x + 100$$

← Step 4: Simplify by distributing the 5.

$$\boxed{+\ 10x} + 1x + 12 = \text{-}10x \boxed{+\ 10x} + 100$$
$$11x + 12 = 100$$

← Step 5: Add 10x to both sides.

$$11x + 12 \boxed{-\ 12} = 100 \boxed{-\ 12}$$
$$11x = 88$$

← Step 6: Subtract 12 from both sides.

$$11x \boxed{\div\ 11} = 88 \boxed{\div\ 11}$$
$$x = 8$$

← Step 7: Divide both sides by 11.

$$\frac{x + 12}{5} = \text{-}2(x - 10)$$
$$\frac{8 + 12}{5} = \text{-}2(8) + 20$$
$$4 = \text{-}16 + 20$$
$$4 = 4$$

← Step 8: Check by replacing the variable in the original problem.

It checks!

Example 2

What if the original problem had been $5x + 12 = -2(x - 10)$? The process would have been different. Watch for differences.

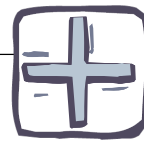
$5x + 12 = -2(x - 10)$	← Step 1:	Copy the problem **carefully**!
$5x + 12 = -2x + 20$	← Step 2:	Simplify each side of the equation as needed by *distributing* the 2.
$5x + 12 - 12 = -2x + 20 - 12$ $5x = -2x + 8$	← Step 3:	Subtract 12 from both sides of the equation.
$5x + 2x = -2x + 2x + 8$ $7x = 8$	← Step 4:	Add $2x$ to both sides of the equation.
$7x \div 7 = 8 \div 7$ $x = \frac{8}{7}$	← Step 5:	Divide both sides by 7.
$5x + 12 = -2(x - 10)$ $5(\frac{8}{7}) + 12 = -2(\frac{8}{7}) + 20$ $\frac{40}{7} + 12 = \frac{-16}{7} + 20$ $5\frac{5}{7} + 12 = -2\frac{2}{7} + 20$	← Step 6:	Check by replacing the variable in the original problem.
$17\frac{5}{7} = 17\frac{5}{7}$	←	*It checks!*

Did you notice that the steps were *not* always the same? The rules for solving equations change to fit the individual needs of each problem. You can see why it is a good idea to check your answers each time. You may need to do some steps in a different order than you originally thought.

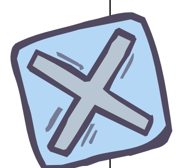
Generally speaking the processes for solving equations are as follows.

- Simplify both sides of the equation as needed.
- "Undo" additions and subtractions.
- "Undo" multiplications and divisions.

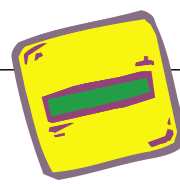
You might notice that this seems to be the *opposite* of the **order of operations**. Typically, we "undo" in the *reverse* order from the original process.

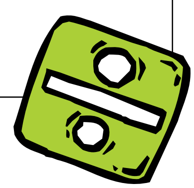


Guidelines for Solving Equations



1. Use the **distributive property** to clear parentheses.
2. Combine **like terms**. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by **substituting** the **solution** in the original equation.





SAM = **S**implify (steps 1 and 2) then
Add (or subtract)
Multiply (or divide)

Here are some additional examples.

Example 3

Solve:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6y + 4y + 8 &= 88 && \leftarrow \text{use \textit{distributive property}} \\ 10y + 8 - 8 &= 88 - 8 && \leftarrow \text{combine \textit{like terms} and undo addition by subtracting 8 from each side} \\ \frac{10y}{10} &= \frac{80}{10} && \leftarrow \text{undo multiplication by dividing by 10} \\ y &= 8 \end{aligned}$$

Check *solution* in the original equation:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6(8) + 4(8 + 2) &= 88 \\ 48 + 4(10) &= 88 \\ 48 + 40 &= 88 \\ 88 &= 88 && \leftarrow \textit{It checks!} \end{aligned}$$

Example 4

Solve:

$$\begin{aligned} -\tfrac{1}{2}(x + 8) &= 10 \\ -\tfrac{1}{2}x - 4 &= 10 && \leftarrow \text{use distributive property} \\ -\tfrac{1}{2}x - 4 + 4 &= 10 + 4 && \leftarrow \text{undo subtraction by adding 4 to both sides} \\ -\tfrac{1}{2}x &= 14 \\ (-2)-\tfrac{1}{2}x &= 14(-2) && \leftarrow \text{isolate the variable by multiplying} \\ x &= -28 && \text{each side by the \textbf{reciprocal} of } -\tfrac{1}{2} \end{aligned}$$

Check solution in the original equation:

$$\begin{aligned} -\tfrac{1}{2}(x + 8) &= 10 \\ -\tfrac{1}{2}(-28 + 8) &= 10 \\ -\tfrac{1}{2}(-20) &= 10 \\ 10 &= 10 && \leftarrow \textit{It checks!} \end{aligned}$$

Example 5

Solve:

$$
\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9x) - \tfrac{2}{3}(6) && \leftarrow \text{use distributive property} \\
26 &= 6x - 4 \\
26 + 4 &= 6x - 4 + 4 && \leftarrow \text{undo subtraction by adding 4 to each side} \\
\tfrac{30}{6} &= \tfrac{6x}{6} && \leftarrow \text{undo multiplication by dividing each side by 6} \\
5 &= x
\end{aligned}
$$

Check solution in the original equation:

$$
\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9 \bullet 5 - 6) \\
26 &= \tfrac{2}{3}(39) \\
26 &= 26 && \leftarrow \textit{It checks!}
\end{aligned}
$$

Example 6

Solve:

$$
\begin{aligned}
x - (2x + 3) &= 4 \\
x - 1(2x + 3) &= 4 && \leftarrow \text{use the } \textbf{multiplicative property of -1} \\
x - 2x - 3 &= 4 && \leftarrow \text{use the } \textbf{multiplicative identity of 1} \\
& && \quad\ \text{and use the distributive property} \\
-1x - 3 &= 4 && \leftarrow \text{combine like terms} \\
-1x - 3 + 3 &= 4 + 3 && \leftarrow \text{undo subtraction} \\
\frac{-1x}{-1} &= \frac{7}{-1} && \leftarrow \text{undo multiplication} \\
x &= -7
\end{aligned}
$$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

line 1: $x - (2x + 3) = 4$
line 2: $x \mathbf{- 1}(2x + 3) = 4$

Also notice the use of *multiplicative identity* on line three.

line 3: $\mathbf{1}x - 2x - 3 = 4$

The simple variable x was multiplied by 1 ($1 \bullet x$) to equal $1x$. The $1x$ helped to clarify the number of variables when combining like terms on line four.

Check solution in the original equation:

$$
\begin{aligned}
x - (2x + 3) &= 4 \\
-7 - (2 \bullet -7 + 3) &= 4 \\
-7 - (-11) &= 4 \\
4 &= 4 && \leftarrow \textit{It checks!}
\end{aligned}
$$