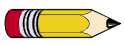
Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Solving Inequalities

Inequalities are mathematical sentences that state two expressions are not equal. Instead of using the equal symbol (=), we use the following with *inequalities*.

- greater than $>$
- less than $<$
- greater than or equal to $\geq$
- less than or equal to $\leq$
- not equal to $\neq$



Remember: The "*is greater than*" (>) or "*is less than*" (<) symbols always *point to the lesser number*.

For example:

$5 > 3$
$3 < 5$

We have been solving *equations* in this unit. When we solve inequalities, the procedures are the same except for one important difference.

When we multiply or divide both sides of an inequality by the same *negative number*, we *reverse* the direction of the inequality symbol.

Example

Solve by *dividing* by a **negative number** and *reversing* the inequality sign.

$$-3x < 6$$
$$\frac{-3x}{-3} > \frac{6}{-3}$$ ← divide each side by -3 and reverse the inequality symbol
$$x > -2$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$-3x < 6$$
$$-3(-1) < 6$$
$$3 < 6$$ ← *It checks!*

substitute 3

$$-3x < 6$$
$$-3(3) < 6$$
$$-9 < 6$$ ← *It checks!*

substitute 0

$$-3x < 6$$
$$-3(0) < 6$$
$$0 < 6$$ ← *It checks!*

substitute 3,000

$$-3x < 6$$
$$-3(3{,}000) < 6$$
$$-9{,}000 < 6$$ ← *It checks!*

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.

Study the following examples.

Example 1

Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$$-\frac{1}{3}y \geq 4$$
$$(-3)-\frac{1}{3}y \leq 4(-3)$$ ← multiply each side by -3 and reverse the inequality symbol
$$y \leq -12$$

Example 2

Solve by first adding, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$$-3a - 4 > 2$$
$$-3a - 4 + 4 > 2 + 4$$ ← add 4 to each side
$$-3a > 6$$
$$\frac{-3a}{-3} < \frac{6}{-3}$$ ← divide each side by -3 and reverse the inequality symbol
$$a < -2$$

Example 3

Solve by first subtracting, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$$\frac{y}{-2} + 5 \leq 0$$
$$\frac{y}{-2} + 5 - 5 \leq 0 - 5$$ ← subtract 5 from each side
$$\frac{y}{-2} \leq -5$$
$$\frac{(-2)y}{-2} \geq (-5)(-2)$$ ← multiply each side by -2 and reverse the inequality symbol
$$y \geq 10$$

Example 4

Solve by first subtracting, then *multiplying* by a **positive number**. Do **not** *reverse* the inequality sign.

$$\frac{n}{2} + 5 \leq 2$$

$$\frac{n}{2} + 5 - 5 \leq 2 - 5 \quad \leftarrow \text{subtract 5 from each side}$$

$$\frac{n}{2} \leq -3$$

$$\frac{(2)n}{2} \leq -3(2) \quad \leftarrow \text{multiply each side by 2, } \textit{but}$$

$$n \leq -6$$

do *not* reverse the inequality symbol because we multiplied by a *positive number*

When multiplying or dividing both sides of an inequality by the same positive number, do not reverse the inequality symbol—leave it alone.

Example 5

Solve by first adding, then *dividing* by a positive number. Do **not** *reverse* the inequality sign.

$$7x - 3 > -24$$

$$7x - 3 + 3 > -24 + 3 \quad \leftarrow \text{add 3 to each side}$$

$$7x > -21 \quad \leftarrow \text{divide each side by 7, but}$$

$$\frac{7x}{7} > \frac{-21}{7}$$

$$x > -3$$

do *not* reverse the inequality symbol because we divided by a positive number