Addition and Subtraction of Polynomials

Polynomials with *exactly* the same variable combinations can be added or subtracted. For example, $7xy$ and $3xy$ have the same variable combination. We call these **like terms**.

$$7xy + 3xy = 10xy \quad \text{and} \quad 7xy - 3xy = 4xy$$

A polynomial is in *simplest form* if it contains no **grouping symbols** (except a *fraction* bar) and all *like terms* have been combined.

Polynomials can be arranged in any order. In **standard form**, polynomials are arranged from *left to right*, from *greatest to least* degree of *power*. For example:

$$x^7 - x^2 + 8x$$

Polynomials can be added or subtracted in vertical ($\updownarrow$) or horizontal ($\leftrightarrow$) form.

Addition

vertical form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Align like terms in columns and add.

$$
\begin{array}{r}
3y^2 + 2y + 3 \quad \leftarrow \text{write degrees of powers} \\
\text{left to right from greatest} \\
\text{to least} \\
(+) \underline{\; y^2 \qquad + 1} \quad \leftarrow \text{align like terms} \\
4y^2 + 2y + 4 \quad \leftarrow \text{add like terms}
\end{array}
$$

horizontal form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Regroup and add like terms.

$$(3y^2 + y^2) + (2y) + (3 + 1) = \quad \leftarrow \text{group like terms}$$
$$4y^2 \quad + 2y \quad + \quad 4 \qquad \leftarrow \text{add like terms}$$
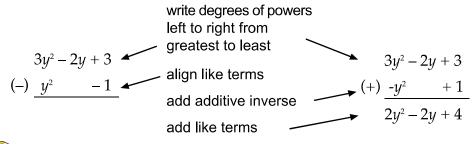
Subtraction

You subtract a polynomial by adding its **additive inverse** or **opposite**. To do this, multiply each term in the *subtracted* polynomial by -1 and add.

polynomial	additive inverse
$-8y + 4x$	$8y - 4x$
$3q^2 - 6r + 11$	$-3q^2 + 6r - 11$
$2a + 7b - 3$	$-2a - 7b + 3$

vertical form

$(3y^2 - 2y + 3) - (y^2 - 1)$

Align like terms in columns and subtract by adding the additive inverse.

write degrees of powers left to right from greatest to least

align like terms

add additive inverse

add like terms

$$3y^2 - 2y + 3$$
$$(-)\ \underline{y^2\qquad - 1}$$

$$3y^2 - 2y + 3$$
$$(+)\ \underline{-y^2\qquad + 1}$$
$$2y^2 - 2y + 4$$

Remember: $-y^2 = -1y^2$

horizontal form

$(3y^2 - 2y + 3) - (y^2 - 1)$

Subtract by adding *additive inverse* and group like terms.

$[3y^2 + (-2y) + 3] + [(-y^2) + 1] =$ ⟵ add additive inverse of $2y$, which is $-2y$, and $y^2 - 1$, which is $-y^2 + 1$

$[3y^2 + (-y^2)] + (-2y) + (3 + 1) =$ ⟵ group like terms

$2y^2 \qquad + -2y + \quad 4 \quad =$ ⟵ add like terms

vertical form

Subtract $2t^2 - 3t + 4$ from the **sum** of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$

write degrees of powers
left to right from
greatest to least

$$
\begin{array}{r}
t^2 + t - 6 \\
3t^2 + 2t - 1 \\
(-)\ \underline{2t^2 - 3t + 4}
\end{array}
$$

align like terms

add additive inverse

$$
\begin{array}{r}
t^2 + t - 6 \\
3t^2 + 2t - 1 \\
(+)\ \underline{-2t^2 + 3t - 4} \\
2t^2 + 6t - 11
\end{array}
$$

horizontal form

Subtract $2t^2 - 3t + 4$ from the *sum* of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$

$t^2 + t - 6 + 3t^2 + 2t - 1 - 2t^2 + 3t - 4 =$

add additive inverse of
$-(2t^2 - 3t + 4)$, which is
$-2t^2 + 3t - 4$

$(t^2 + 3t^2 - 2t^2) + (t + 2t + 3t) + (-6 - 1 - 4) =$

group like terms

$2t^2 + 6t - 11$

combine like terms