Unit 5: How Radical Are You?

This unit focuses on simplifying radical expressions and performing operations involving radicals.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1 The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2 The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5 The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1 Simplify radical expressions.
- MA.912.A.6.2 Add, subtract, multiply and divide radical expressions (square roots and higher).

Unit 5: How Radical Are You?

Introduction

We will see that radical expressions can be rewritten to conform to the mathematical definitions of simplest terms. We will then be able to perform the operations of addition, subtraction, multiplication and division on these reformatted expressions. We will also explore the effects of multiplying a radical expression by its conjugate.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1 The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2 The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5 The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1 Simplify radical expressions.

Simplifying Radical Expressions

A **radical expression** is any mathematical **expression** that contains a **square root** symbol. Look at the following examples:

$$\sqrt{5} \qquad \frac{\sqrt{6}}{3} \qquad \frac{3}{\sqrt{6}} \qquad \frac{7}{5+2\sqrt{\ }} \qquad \sqrt{36}$$

Certain numbers can be reformatted to make them easier to work with. To do so, mathematicians have rules that make working with numbers uniform. If we all play by the same rules, we should all have the same outcome.

With this in mind, here are the two basic rules for working with *square roots*.

1. Never leave a **perfect square factor** under a **radical sign** ($\sqrt{\ }$).
   Why? Because if you do, the radical expression is *not* simplified.

2. Never leave a *radical sign* in a **denominator**.
   Why? Because if you do, the radical expression is *not* simplified.

Important! Do **not** use your calculator with the *square roots*. It will change the numbers to **decimal** approximations. We are looking for exact answers.

Let's explore each of the rules...one at a time.

Rule One

First, let's review the idea of *perfect squares*. Perfect squares happen whenever you multiply a number times itself. In the following examples,

$3 \times 3 = 9 \qquad 7 \times 7 = 49 \qquad 9 \times 9 = 81$

9, 49, and 81 are all perfect squares.

It will be helpful to learn the chart below. You will be asked to use these numbers many times in this unit and in real-world applications. The chart shows the perfect squares underneath the radical sign, then gives the square root of each perfect square.

Perfect Squares: Square Root = Whole Number

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$
$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$
$\sqrt{400} = 20$

Any time you see a perfect square under a square root symbol, simplify it by writing it as the square root.

Sometimes, perfect squares are hidden in an *expression* and we have to search for them. At first glance, $\sqrt{45}$ looks as if it is in **simplest radical form**. However, when we realize that 45 has a *factor* that is a perfect square, we can rewrite it as

$$\sqrt{45} = \sqrt{9} \cdot \sqrt{5}.$$

From the information in the chart, we know that 9 is a perfect square and that

$\sqrt{9} = 3$. Therefore

$\sqrt{45} = 3 \cdot \sqrt{5}$ or $3\sqrt{5}$.

Let's look at some examples.

$$\begin{aligned}\sqrt{18} &= \sqrt{9} \cdot \sqrt{2} \\ &= 3 \cdot \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \cdot \sqrt{5} \\ &= 2 \cdot \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$

Now you try some in the following practices. Study the chart of perfect squares on page 367 before you start the practices.

Practice

Simplify *each* **radical expression**.

Remember: Never leave a perfect square factor under a radical sign.

1. $\sqrt{50}$ 6. 32 $\sqrt{\ }$

2. $\sqrt{27}$ 7. 12 $\sqrt{\ }$

3. $\sqrt{125}$ **369**

8. $\sqrt{45}$

4. $\sqrt{64}$

9. $\sqrt{300}$

5. $\sqrt{13}$

10. $\sqrt{8}$

Practice

Simplify *each* **radical expression**.

1. $4\sqrt{8}$

2. $2\sqrt{40}$

3. $\sqrt{60}$

4. $\sqrt{242}$

5. $\sqrt{28}$

6. $-\sqrt{200}$

7. $-\sqrt{250}$

8. $\sqrt{108}$

9. $\sqrt{405}$

10. $5\sqrt{90}$

Rule Two

Now it's time to work on that second rule: never leave a square root in the *denominator*. Because if a square root is left in the denominator of a radical expression, the radical expression is *not* simplified.

If a **fraction** has a denominator that is a perfect square root, just rewrite the *fraction* using that square root. Let's look at examples.

$$\frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3} \qquad \frac{4}{\sqrt{81}} = \frac{4}{9}$$

Many times, however, that denominator will *not* be a perfect square root. In those cases, we have to reformat the denominator so that it is a perfect square root. This is called **rationalizing the denominator** or the bottom number of the fraction. To do this, we make it into a **rational number** by using a method to eliminate **radicals** from the denominator of a fraction. Remember, we aren't concerned about what may happen to the format of the **numerator**, just the denominator.

To reformat an **irrational** denominator (one with a square root in it), we find a number to multiply it by that will produce a perfect square root.

Follow the explanation of this example carefully.

$\frac{2}{\sqrt{7}}$ ⟵ Yikes! This denominator is irrational! I need to *rationalize* it.

$\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ ⟵ Look what happens I multiply the denominator by itself. (Since, $\frac{\sqrt{7}}{\sqrt{7}}$ is 1, I have *not* changed the value of the original fraction.)

$\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}}$ ⟵ Because I remember the perfect square roots from the chart on page 240, I see that $\sqrt{49}$ is a perfect square root ... and therefore rational!

$\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$ ⟵ This may not look like a simpler expression than I started with, but it *does conform to the second rule*.

Follow along with this example.

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = \frac{2\sqrt{3}}{1} = 2\sqrt{3}$$

In the above example, notice that we reduced the “real 6” and the “real 3,” but not with the square root of 3. Do *not* mix a rational number with an irrational number, sometimes referred to as a *non-rational* number when you are reducing…they are *not* **like terms**!

It’s time for you to practice.

Practice

Simplify *each* **radical expression**.

Remember: Never leave a square root in the denominator.

Example: $\frac{}{\sqrt{65}} = \frac{}{\sqrt{65}} \cdot \frac{\sqrt{}}{\sqrt{55}} = \frac{6\sqrt{}}{\sqrt{255}} = 6\frac{\sqrt{5}}{5}$

Show all your steps.

1. $\frac{7}{\sqrt{2}}$

2. $\frac{5}{\sqrt{6}}$

3. $\frac{1}{\sqrt{3}}$

4. $\frac{3}{\sqrt{5}}$

5. $\frac{5}{\sqrt{18}}$

6. $\frac{4}{\sqrt{3}}$

7. $\frac{7}{\sqrt{10}}$

8. $\frac{3}{\sqrt{7}}$

9. $\frac{4}{\sqrt{11}}$

10. $\frac{\sqrt{2}}{\sqrt{15}}$

Practice

Simplify *each* **radical expression**.

Example: $\frac{10}{\sqrt{6}} = \frac{10}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{6}}{\sqrt{36}} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$

1. $\frac{9}{\sqrt{6}}$

2. $\frac{-2}{\sqrt{8}}$

3. $\frac{-5\sqrt{27}}{8\sqrt{20}}$

4. $\frac{5}{\sqrt{5}}$

5. $\sqrt{\frac{2}{3}}$

6. $\frac{\sqrt{3}}{\sqrt{18}}$

7. $\sqrt{\frac{1}{3}}$

$\sqrt{}$

9. $\frac{3\sqrt{5}}{6\sqrt{}}$

10. $\frac{7\sqrt{3}}{5\sqrt{}}$

Practice

Match each definition with the correct term. Write the letter on the line provided.

________ 1. a number whose square root is a whole number

________ 2. an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction

________ 3. the symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*

________ 4. terms that have the same variables and the same corresponding exponents

________ 5. a real number that cannot be expressed as a ratio of two integers

________ 6. a number or expression that divides evenly into another number

________ 7. a numerical expression containing a radical sign

________ 8. a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$

________ 9. a positive real number that can be multiplied by itself to produce a given number

A. factor

B. irrational number

C. like terms

D. perfect square

E. radical expression

F. radical sign

G. rational number

H. simplest radical form

I. square root

Lesson Two Purpose

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Add and Subtract Radical Expressions

We can add or subtract radical expressions only when those radical expressions match. For instance,

$$5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}.$$

Notice that we did not change the $\sqrt{2}$'s. We simply added the **coefficients** because they had matching radical parts.

Remember: *Coefficients* are any factor in a **term**. Usually, but not always, a coefficient is a number instead of a **variable** or a *radical*.

The same is true when we subtract radical expressions.

$$5\sqrt{7} - 3\sqrt{7} = 2\sqrt{7}$$

At first glance, it may sometimes appear that there are no matching numbers under the radical sign. But, if we **simplify** the expressions, we often find radical expressions that we can add or subtract.

Look at this example.

$$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32}$$

Notice that $\sqrt{8}$ and $\sqrt{32}$ each have perfect square factors and can be simplified. Follow the simplification process step by step and see what happens.

$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32} =$	
$3\sqrt{4}\sqrt{2} + 5\sqrt{2} - 4\sqrt{16}\sqrt{2} =$	We *found* the perfect square factors of $\sqrt{8}$ and $\sqrt{32}$ and rewrote the problem.
$3 \cdot 2\sqrt{2} + 5\sqrt{2} - 4 \cdot 4\sqrt{2} =$	Next, we *simplified* the perfect square roots.
$6\sqrt{2} + 5\sqrt{2} - 16\sqrt{2} =$	We *multiplied* the new factors for each coefficient.
$-5\sqrt{2}$	Finally, we *added* and *subtracted* matching radical expressions, in order, from *left to right*.

When Radical Expressions Don't Match or Are Not in Radical Form

What happens when radical expressions don't match, or there is a number that is not in radical form? Just follow the steps on the previous pages and leave your answer, with appropriate terms in descending order. Watch this!

$$\sqrt{75} + \sqrt{27} - \sqrt{16} + \sqrt{80} =$$
$$\sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{3} - 4 + \sqrt{16}\sqrt{5} =$$
$$5\sqrt{3} + 3\sqrt{3} - 4 + 4\sqrt{5} =$$
$$8\sqrt{3} - 4 + 4\sqrt{5} =$$
$$8\sqrt{3} + 4\sqrt{5} - 4 \longleftarrow \text{rewritten in descending order}$$

Practice

Simplify *each of the following. Refer to pages 376-378 as needed.*

1. $4\sqrt{7} + 10\sqrt{7}$

2. $-5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2}$

3. $3\sqrt{7} + 5 - \sqrt{7}$

4. $2\sqrt{27} - 4\sqrt{12}$

5. $\sqrt{2} + \sqrt{18} - \sqrt{16}$

6. $\sqrt{3} + 5\sqrt{3} - \sqrt{27}$

7. $\sqrt{50} + \sqrt{18}$

8. $\sqrt{27} + \sqrt{12} - \sqrt{48}$

Practice

Simplify *each of the following. Refer to pages 376-378 as needed.*

1. $-3\sqrt{5} + 4\sqrt{2} - \sqrt{5} + \sqrt{8}$

2. $\sqrt{81} + \sqrt{24} - \sqrt{9} + \sqrt{54}$

3. $\sqrt{50} - \sqrt{45} + \sqrt{32} - \sqrt{80}$

4. $5\sqrt{7} + 2\sqrt{3} - 4\sqrt{7} + \sqrt{27}$ $\sqrt[3]{81}$

5. $\sqrt{200} - \sqrt{8} + 3\sqrt{72} - 6$

6. $12 - 3\sqrt{5} + 2\sqrt{144} - \sqrt{20}$

7. $\sqrt{18} + \sqrt{48} - \sqrt{32} - \sqrt{27}$

Lesson Three Purpose

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Multiply and Divide Radical Expressions

Radical expressions *don't* have to match when we multiply or divide them. The following examples show that we simply multiply or divide the **digits** under the radical signs and then simplify our results, if possible.

Example 1

$\sqrt{5} \text{ x } \sqrt{6} = \sqrt{30}$

Example 2

$\sqrt{8} \text{ x } \sqrt{3} = \sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

Example 3

$\sqrt{18} \text{ x } \sqrt{2} = \sqrt{36} = 6$

Example 4

$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2$

Example 5

$\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$

Example 6

$\frac{\sqrt{8}}{\sqrt{24}} = \sqrt{\frac{1}{3}}$ (we must simplify this) $\rightarrow \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$

After studying the examples above, try the following practice.

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous page as needed.*

1. $\sqrt{5} \cdot \sqrt{10}$

2. $\sqrt{2} \cdot \sqrt{50}$

3. $\sqrt{75} \cdot \sqrt{3}$

4. $\sqrt{6} \cdot \sqrt{10}$

5. $\dfrac{\sqrt{30}}{\sqrt{2}}$

6. $\frac{\sqrt{8}}{\sqrt{32}}$

7. $\frac{\sqrt{6}}{\sqrt{10}}$

8. $\frac{\sqrt{75}}{\sqrt{3}}$

9. $\frac{\sqrt{72}}{\sqrt{18}}$

10. $\frac{\sqrt{5}}{\sqrt{10}}$

Working with a Coefficient for the Radical

What happens when there is a coefficient for the *radical*? It is important to multiply or divide the radical numbers together separately from the coefficients. Then simplify each answer. Look at the following examples.

Example 1

multiply coefficients

$3 \cdot 5 = 15$

$$3\sqrt{7} \cdot 5\sqrt{2} = 15\sqrt{14}$$

multiply radicands

$\sqrt{7} \cdot \sqrt{2} = \sqrt{14}$

Example 2

$$6\sqrt{3} \cdot \sqrt{3} = 6\sqrt{9} = 6 \cdot 3 = 18$$

Remember: If there is *no* written coefficient, then it is understood to be a 1.

Example 3

$$\frac{2\sqrt{14}}{6\sqrt{7}} = \frac{1}{3}\sqrt{2} = \frac{\sqrt{2}}{3}$$

Example 4

$$\frac{2\sqrt{6}}{\sqrt{12}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Example 5

$$\frac{\sqrt{6} - \sqrt{12}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}} - \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{2} - \sqrt{4} = \sqrt{2} - 2$$

Now it's time to practice on the following page.

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous page as needed.*

1. $5\sqrt{3} \cdot 6\sqrt{5}$

2. $2\sqrt{5} \cdot 4\sqrt{2}$

3. $8\sqrt{2} \cdot 5\sqrt{3}$

4. $2\sqrt{7} \cdot \sqrt{7}$

5. $\dfrac{3\sqrt{10}}{5\sqrt{6}}$

6. $\dfrac{4\sqrt{6}}{12\sqrt{2}}$

7. $\dfrac{9\sqrt{5}}{10\sqrt{3}}$

8. $2\sqrt{7} \cdot 5\sqrt{7}$

9. $5\sqrt{6} \cdot 4\sqrt{2}$

Practice

Simplify *each of the following. Refer to the* **examples** *on page 386 as needed.*

1. $\frac{15-\sqrt{20}}{\sqrt{5}}$

2. $\frac{8-\sqrt{12}}{\sqrt{2}}$

3. $\frac{30-\sqrt{50}}{\sqrt{10}}$

4. $\frac{3\sqrt{18}}{\sqrt{3}}$

5. $\frac{4\sqrt{6}}{6\sqrt{2}}$

6. $\frac{10 \quad 8\sqrt{12}}{12\sqrt{}}$

7. $\frac{\sqrt{75} - \sqrt{50}}{25\sqrt{}}$

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Multiple Terms and Conjugates

Sometimes it is necessary to multiply or divide radical expressions with more than one *term*. To multiply radicals with multiple terms by a single term, we use the old reliable **distributive property**. See how the *distributive property* works for these examples.

Example 1

$$6(\sqrt{5} + \sqrt{3}) =$$
$$6\sqrt{5} + 6\sqrt{3}$$

Example 2

$$\sqrt{3}(2\sqrt{5} - 4\sqrt{3}) =$$
$$2\sqrt{15} - 4\sqrt{9} =$$
$$2\sqrt{15} - 4 \cdot 3 =$$
$$2\sqrt{15} - 12$$

Example 3

$$6\sqrt{3}(2\sqrt{2} + 5\sqrt{6}) =$$
$$12\sqrt{6} + 30\sqrt{18} =$$
$$12\sqrt{6} + 30\sqrt{9}\sqrt{2} =$$
$$12\sqrt{6} + 30 \cdot 3\sqrt{2} =$$
$$12\sqrt{6} + 90\sqrt{2}$$

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous pages as needed.*

1. $\sqrt{2}(\sqrt{6}+\sqrt{5})$

2. $\sqrt{2}(\sqrt{6}+\sqrt{5})$

3. $3\sqrt{2}(5\sqrt{3}-4\sqrt{2})$

4. $\sqrt{6}(3\sqrt{8}+5\sqrt{2})$

5. $\sqrt{6}(3\sqrt{8}-5\sqrt{2})$

6. $-2(\sqrt{5} + 7)$

7. $2\sqrt{3}(\sqrt{7} + \sqrt{10})$

8. $4(2\sqrt{3} - 5\sqrt{2})$

9. $4\sqrt{3}(2\sqrt{3} - 5\sqrt{2})$

10. $8\sqrt{6}(2\sqrt{6} + 5\sqrt{8})$

The FOIL Method

Another reliable method we can use when multiplying two radical expressions with multiple terms is the **FOIL method:** multiplying the **f**irst, **o**utside, **i**nside, and **l**ast terms. We use that same process in problems like these.

Example 1

$$(\sqrt{6}-5)(\sqrt{3}+4) =$$

$$\sqrt{6}\cdot\sqrt{3} + \sqrt{6}\cdot 4 - 5\cdot\sqrt{3} - 5\cdot 4 =$$ ← Multiply the **first** terms, the **outside** terms, the **inside** terms, and then the **last** terms.

$$\sqrt{18} + 4\sqrt{6} - 5\sqrt{3} - 20 =$$ ← Carefully write out the **products**.

$$3\sqrt{2} + 4\sqrt{6} - 5\sqrt{3} - 20$$ ← Simplify each term and combine like terms (if needed).

Example 2

$$(\sqrt{3}+\sqrt{2})(\sqrt{7}-\sqrt{11}) =$$

$$\sqrt{3}\sqrt{7} - \sqrt{3}\sqrt{11} + \sqrt{2}\sqrt{7} - \sqrt{2}\sqrt{11} =$$

$$\sqrt{21} - \sqrt{33} + \sqrt{14} - \sqrt{22}$$ ← Notice that no term has a perfect square as a factor. Therefore, there is no further simplifying to be done.

Time to try the following practice.

Practice

Simplify *each of the following. Refer to the* **examples** *on page 395 as needed.*

1. $(\sqrt{6} - 2)(\sqrt{5} + \sqrt{7})$
2. $(5 - \sqrt{3})(2 + \sqrt{7})$
3. $(4 + 5\sqrt{2})(2 - \sqrt{2})$
4. $(2\sqrt{5} - 3)(\sqrt{5} + 6)$

5. $(4 - 3\sqrt{10})(2 - \sqrt{10})$

6. $(2\sqrt{7} - 3)(5\sqrt{7} + 1)$

7. $(\sqrt{5} - 7)(3\sqrt{5} + 7)$

8. $(\sqrt{10} - \sqrt{6})(\sqrt{7} - \sqrt{13})$

9. $(3\sqrt{6} + 2\sqrt{3})(\sqrt{5} - 2)$

10. $(4\sqrt{3} - \sqrt{5})(3\sqrt{3} - \sqrt{5})$

11. $(3 + \sqrt{10})(3 - \sqrt{10})$

12. $(6\sqrt{5} + 4)(6\sqrt{5} - 4)$

Two-Term Radical Expressions

At the beginning of this unit, we learned that there are two rules we must remember when simplifying a radical expression. Rule one requires that we never leave a perfect square factor under a radical sign. Rule two insists that we never leave a radical in the denominator. With that in mind, let's see what to do with two-term radical expressions.

Looking at a problem like $\frac{2+\sqrt{7}}{5-\sqrt{6}}$, we see that we must rationalize the denominator (reformat it without using a square root). At first glance, it may seem to you that multiplying that denominator by itself makes the square roots disappear. But when we try that, we realize that new square roots appear as a result of the FOILing.

$$(5-\sqrt{6})(5-\sqrt{6}) =$$

$$25 - 5\sqrt{6} - 5\sqrt{6} + \sqrt{6}\sqrt{6} =$$

$$25 - 10\sqrt{6} + 6$$

So there must be a better way to rationalize this denominator. Try multiplying $(5-\sqrt{6})$ by its **conjugates** $(5+\sqrt{6})$. These numbers are *conjugates* because they match, except for the signs between the terms. Notice that one has a "+" and the other has a "–".

$$(5-\sqrt{6})(5+\sqrt{6}) =$$

$$25 + 5\sqrt{6} - 5\sqrt{6} - \sqrt{6}\sqrt{6} =$$

$$25 - \sqrt{36} =$$

$$25 - 6 =$$

$$19$$

Remember, we only need to rationalize the denominator. It is acceptable to leave simplified square roots in the numerator. Now, let's take a look at the entire problem.

$$\frac{2+\sqrt{7}}{5-\sqrt{6}} \cdot \frac{5+\sqrt{6}}{5+\sqrt{6}} =$$

← reformat the fraction by multiplying it by 1

$$\frac{5+\sqrt{6}}{5+\sqrt{6}} = 1$$

$$\frac{(2)(5)+2\sqrt{6}+5\sqrt{7}+\sqrt{7}\sqrt{6}}{(5)(5)+5\sqrt{6}-5\sqrt{6}-\sqrt{6}\sqrt{6}} =$$

← FOIL the numerator and denominator

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-\sqrt{36}} =$$

← simplify

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-6} =$$

← simplify again

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{19}$$

← and again, if necessary

Follow along with this one!

$$\frac{3+\sqrt{2}}{4+\sqrt{8}} = \frac{3+\sqrt{2}}{4+\sqrt{8}} \cdot \frac{4-\sqrt{8}}{4-\sqrt{8}}$$ ← reformat the fraction by multiplying it by 1

$$\frac{4-\sqrt{8}}{4-\sqrt{8}} = 1$$

$$\frac{(3)(4) - 3\sqrt{8} + 4\sqrt{2} - \sqrt{2}\sqrt{8}}{(4)(4) - 4\sqrt{8} + 4\sqrt{8} - \sqrt{8}\sqrt{8}}$$ ← FOIL the numerator and denominator

$$\frac{12 - 3\sqrt{4 \cdot 2} + 4\sqrt{2} - \sqrt{16}}{16 - \sqrt{64}}$$ ← simplify

$$\frac{12 - 3 \cdot 2\sqrt{2} + 4\sqrt{2} - 4}{16 - 8}$$ ← simplify again

$$\frac{12 - 6\sqrt{2} + 4\sqrt{2} - 4}{8}$$ ← and again

$$\frac{8 - 2\sqrt{2}}{8} = \frac{2(4 - \sqrt{2})}{8}$$ ← and again

$$\frac{4-\sqrt{2}}{4}$$ ← and again, if necessary

With more practice, you will be able to mentally combine some of those simplifying steps and finish sooner.

So let's practice on the following page.

Practice

Simplify *each of the following.*

1. $\frac{\sqrt{5}+2}{-\sqrt{1}}$ 3

2. $\frac{\sqrt{6}+5}{3\sqrt{6}-2}$

$\sqrt{\;}$
$\sqrt{\;}$

$\sqrt{\;}$ $\sqrt{\;}$
$\sqrt{\;}$ $\sqrt{\;}$

5. $\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}+3\sqrt{\ }}$

6. $\frac{\sqrt{2}+3\sqrt{2}}{2\sqrt{5}}$

7. $\frac{\sqrt{5}-3}{\sqrt{\ }+7}$

8. $\frac{\sqrt{6}+5\sqrt{6}}{\sqrt{3}\ 5\sqrt{\ }}$

Practice

Simplify *each of the following.*

1. $\dfrac{4-\sqrt{7}}{3+\sqrt{7}}$

2. $4\ 2-\sqrt{3}\quad 2+\sqrt{3}$
3

3. $\dfrac{6\sqrt{5}-2}{\sqrt{5}+\sqrt{2}}$

4. $\frac{6\sqrt{2}+5}{1+\sqrt{5}}$

5. $\frac{5+3\sqrt{2}}{1-\sqrt{2}}$

6. $\frac{\sqrt{6}+2}{2\sqrt{6}+1}$

7. $\frac{\sqrt{5}+2\sqrt{7}}{\sqrt{5}+\sqrt{7}}$

Practice

Match each **symbol or expression** *with the appropriate* **description**.

________	1. 7	A. coefficient in the expression 5 x $\sqrt{\ }$
________	2. $\sqrt{\ }$	B. conjugate of $x + 4$
________	3. $\sqrt{\ }2\ 2\ 7\ 7\ \frac{2\sqrt{7}}{\sqrt{49}}\ 7\ \ \frac{\sqrt{\ }}{}$	C. perfect square of 11
________	4. $x - 4$	D. radical expression
________	5. 5	E. radical sign
________	6. $3x\ \ 6\sqrt{\ }$	F. rationalizing the denominator
________	7. 121	G. square root of 49

Unit Review

Simplify *each of the following.*

1. $\sqrt{75}$

5. $\sqrt{\frac{5}{8}}$

2. $-\sqrt{40}$

6. $\frac{1}{\sqrt{7}}$

3. $5\sqrt{27}$

7. $\sqrt{\frac{3}{8}}$

4. $\frac{3}{\sqrt{36}}$

8. $\frac{5\sqrt{6}}{\sqrt{5}}$

9. $6\sqrt{3} - 8\sqrt{3}$

10. $4\sqrt{8} - 5\sqrt{2} + 3\sqrt{32}$

11. $\sqrt{75} + \sqrt{45} - \sqrt{80}$

12. $2\sqrt{50} + 3\sqrt{45} + \sqrt{32} + \sqrt{80}$

13. $\sqrt{5} + \sqrt{2} + \sqrt{8} + \sqrt{125}$

14. $\sqrt{2x}\sqrt{10}$

18. $\sqrt{\frac{6}{30}}$

15. $\frac{\sqrt{18}}{\sqrt{3}}$

19. $\frac{\sqrt{60}}{3\sqrt{5}}$

16. $\sqrt{6}\cdot\sqrt{2}$

20. $\sqrt{8}\ 3\cdot\sqrt{2}\ 8$

17. $\frac{2\sqrt{6}}{\sqrt{24}}$

21. $\frac{18-\sqrt{12}}{\sqrt{6}}$

22. $(3+5\sqrt{6})(3-5\sqrt{6})$

23. $\dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}}$

24. $\dfrac{5+2\sqrt{3}}{2+\sqrt{5}}$

25. $\dfrac{6\sqrt{6}-12}{\sqrt{6}+2}$