Unit 5: How Radical Are You?

Introduction

We will see that radical expressions can be rewritten to conform to the mathematical definitions of simplest terms. We will then be able to perform the operations of addition, subtraction, multiplication and division on these reformatted expressions. We will also explore the effects of multiplying a radical expression by its conjugate.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1
  Simplify radical expressions.

Simplifying Radical Expressions

A **radical expression** is any mathematical **expression** that contains a **square root** symbol. Look at the following examples:

$$\sqrt{5} \qquad \frac{\sqrt{6}}{3} \qquad \frac{3}{\sqrt{6}} \qquad \frac{7}{5+\sqrt{2}} \qquad \sqrt{36}$$

Certain numbers can be reformatted to make them easier to work with. To do so, mathematicians have rules that make working with numbers uniform. If we all play by the same rules, we should all have the same outcome.

With this in mind, here are the two basic rules for working with *square roots*.

1. Never leave a **perfect square factor** under a **radical sign (**$\sqrt{\ }$**)**.
   Why? Because if you do, the radical expression is *not* simplified.

2. Never leave a *radical sign* in a **denominator**.
   Why? Because if you do, the radical expression is *not* simplified.

Important! Do **not** use your calculator with the *square roots*. It will change the numbers to **decimal** approximations. We are looking for exact answers.

Let's explore each of the rules…one at a time.

Rule One

First, let's review the idea of *perfect squares*. Perfect squares happen whenever you multiply a number times itself. In the following examples,

$3 \times 3 = 9 \qquad 7 \times 7 = 49 \qquad 9 \times 9 = 81$

9, 49, and 81 are all perfect squares.

It will be helpful to learn the chart below. You will be asked to use these numbers many times in this unit and in real-world applications. The chart shows the perfect squares underneath the radical sign, then gives the square root of each perfect square.

Perfect Squares:
Square Root = Whole Number

$\sqrt{1}$	= 1
$\sqrt{4}$	= 2
$\sqrt{9}$	= 3
$\sqrt{16}$	= 4
$\sqrt{25}$	= 5
$\sqrt{36}$	= 6
$\sqrt{49}$	= 7
$\sqrt{64}$	= 8
$\sqrt{81}$	= 9
$\sqrt{100}$	= 10
$\sqrt{121}$	= 11
$\sqrt{144}$	= 12
$\sqrt{169}$	= 13
$\sqrt{196}$	= 14
$\sqrt{225}$	= 15
$\sqrt{256}$	= 16
$\sqrt{289}$	= 17
$\sqrt{324}$	= 18
$\sqrt{361}$	= 19
$\sqrt{400}$	= 20

Any time you see a perfect square under a square root symbol, simplify it by writing it as the square root.

Sometimes, perfect squares are hidden in an *expression* and we have to search for them. At first glance, $\sqrt{45}$ looks as if it is in **simplest radical form**. However, when we realize that 45 has a *factor* that is a perfect square, we can rewrite it as

$$\sqrt{45} = \sqrt{9} \bullet \sqrt{5}.$$

From the information in the chart, we know that 9 is a perfect square and that

$\sqrt{9} = 3$. Therefore

$\sqrt{45} = 3 \bullet \sqrt{5}$ or $3\sqrt{5}$.

Let's look at some examples.

$$\begin{aligned}\sqrt{18} &= \sqrt{9} \bullet \sqrt{2}\\ &= 3 \bullet \sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \bullet \sqrt{5}\\ &= 2 \bullet \sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

Now you try some in the following practices. Study the chart of perfect squares on page 367 before you start the practices.

Rule Two

Now it's time to work on that second rule: never leave a square root in the *denominator*. Because if a square root is left in the denominator of a radical expression, the radical expression is *not* simplified.

If a **fraction** has a denominator that is a perfect square root, just rewrite the *fraction* using that square root. Let's look at examples.

$$\frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3} \qquad\qquad \frac{4}{\sqrt{81}} = \frac{4}{9}$$

Many times, however, that denominator will *not* be a perfect square root. In those cases, we have to reformat the denominator so that it is a perfect square root. This is called **rationalizing the denominator** or the bottom number of the fraction. To do this, we make it into a **rational number** by using a method to eliminate **radicals** from the denominator of a fraction. Remember, we aren't concerned about what may happen to the format of the **numerator**, just the denominator.

To reformat an **irrational** denominator (one with a square root in it), we find a number to multiply it by that will produce a perfect square root.

Follow the explanation of this example carefully.

$\frac{2}{\sqrt{7}}$ ⟵ Yikes! This denominator is irrational!
I need to *rationalize* it.

$\frac{2}{\sqrt{7}} \bullet \frac{\sqrt{7}}{\sqrt{7}}$ ⟵ Look what happens if I multiply the denominator by itself. (Since, $\frac{\sqrt{7}}{\sqrt{7}}$ = 1, I have *not* changed the value of the original fraction.)

$\frac{2}{\sqrt{7}} \bullet \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}}$ ⟵ Because I remember the perfect square roots from the chart on page 240, I see that $\sqrt{49}$ is a perfect square root…and therefore rational!

$\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$ ⟵ This may *not* look like a simpler expression than I started with, but *it does conform to the second rule*.

Follow along with this example.

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = \frac{2\sqrt{3}}{1} = 2\sqrt{3}$$

In the above example, notice that we reduced the "real 6" and the "real 3," but not with the square root of 3. Do *not* mix a rational number with an irrational number, sometimes referred to as a *non-rational* number when you are reducing…they are *not* **like terms**!

It's time for you to practice.