- MA.912.A.3.10
  Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

Geometry Body of Knowledge

Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Point-Slope Form

We know that if we have two points we can draw a line that connects them. But did you know we can also produce the equation of that line using those points?

To do this, we will use yet another format for the equation of a line. It is called the point-slope form. Notice that it looks a bit like the slope-intercept format, but it has a little extra.

$(y - y_1) = m(x - x_1)$ point-slope form

(x_1, y_1) is one of the coordinates given

m = slope

Let's see how this works.

Example 1

Find the equation of the line which passes through points (3, 5) and (-2, 1).

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope using the two points.

$$m = \frac{1-5}{-2-3} = \frac{-4}{-5} = \frac{4}{5} \quad \longleftarrow \text{ the slope is } \frac{4}{5}$$

- Select one of the given points (3, 5).

- Replace x_1 and y_1 with the coordinates from the point you selected, and then replace m with the slope ($\frac{4}{5}$) that you found.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{4}{5}(x - 3)$$

- Simplify.

$$y - 5 = \frac{4}{5}(x - 3)$$
$$y - 5 = \frac{4}{5}x - \frac{12}{5} \quad \longleftarrow \text{ distribute } \frac{4}{5}$$
$$y = \frac{4}{5}x - \frac{12}{5} + 5 \quad \longleftarrow \text{ add 5 to both sides}$$
$$y = \frac{4}{5}x - \frac{12}{5} + \frac{25}{5} \quad \longleftarrow \text{ get a } \textbf{common denominator}$$
$$y = \frac{4}{5}x + \frac{13}{5} \quad \longleftarrow \text{ simplify}$$

This is the equation of the line in slope-intercept form: $y = mx + b$ with $m = \frac{4}{5}$, $b = \frac{13}{5}$.

We could also transform this equation to standard form of $ax + by = c$ using a bit of algebra.

$$y = \frac{4}{5}x + \frac{13}{5}$$
$$-\frac{4}{5}x + y = \frac{13}{5} \quad \longleftarrow \text{ subtract } \frac{4}{5}x \text{ from both sides}$$
$$4x - 5y = -13 \quad \longleftarrow \text{ multiply both sides by -5}$$

How about another example before you try this yourself?

Example 2

Find the equation of the line in both y-intercept and standard form that passes through the points (-4, 0) and (-2, 2).

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope.

$$m = \frac{2-0}{-2--4} = \frac{2}{2} = \frac{1}{1} = 1 \longleftarrow \text{the slope is 1}$$

- Select a point.

(-2, 2)

- Replace x_1, y_1, and m.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= 1(x - -2) \end{aligned}$$

- Simplify.

$$\begin{aligned} y - 2 &= 1(x - -2) \\ y - 2 &= 1(x + 2) \\ y - 2 &= x + 2 \\ y &= x + 4 \end{aligned} \quad \longleftarrow \text{equation in slope-intercept form}$$

- Transform $y = x + 4$ into standard form.

$$\begin{aligned} -x + y &= 4 && \longleftarrow \text{multiply both sides by -1} \\ x - y &= -4 && \longleftarrow \text{equation in standard form} \end{aligned}$$

Look at some other situations when using the point-slope format is helpful.

Example 3

Write an equation in point-slope form of the line that passes through (2, -3) and has a slope of $-\frac{3}{8}$.

$$y - y_1 = m(x - x_1)$$

- We can skip finding the slope—it is already done for us!

$$m = -\frac{3}{8}$$

- There is no need to select a point because we only have one to choose.

(2, -3)

- Replace x_1, y_1, and m, and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= -\frac{3}{8}(x - 2) \\ y + 3 &= -\frac{3}{8}(x - 2) \end{aligned}$$

Ta-da! We are finished! We have written the equation in point-slope form.

Example 4

Write an equation in point-slope form for a horizontal line passing through the point (-4, 2).

$$y - y_1 = m(x - x_1)$$

- Slope = 0 (horizontal lines have zero slope)

$$m = 0$$

- Use the point (-4, 2), replace x_1, y_1, and m, and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= 0(x - -4) \\ y - 2 &= 0(x + 4) \\ y - 2 &= 0x + 0 \\ y - 2 &= 0 \\ y &= 2 \end{aligned}$$

All *horizontal* lines have equations that look like y = "a number." That number will *always* be the y-coordinate from any point on the line.

Time for some practice…here we go!