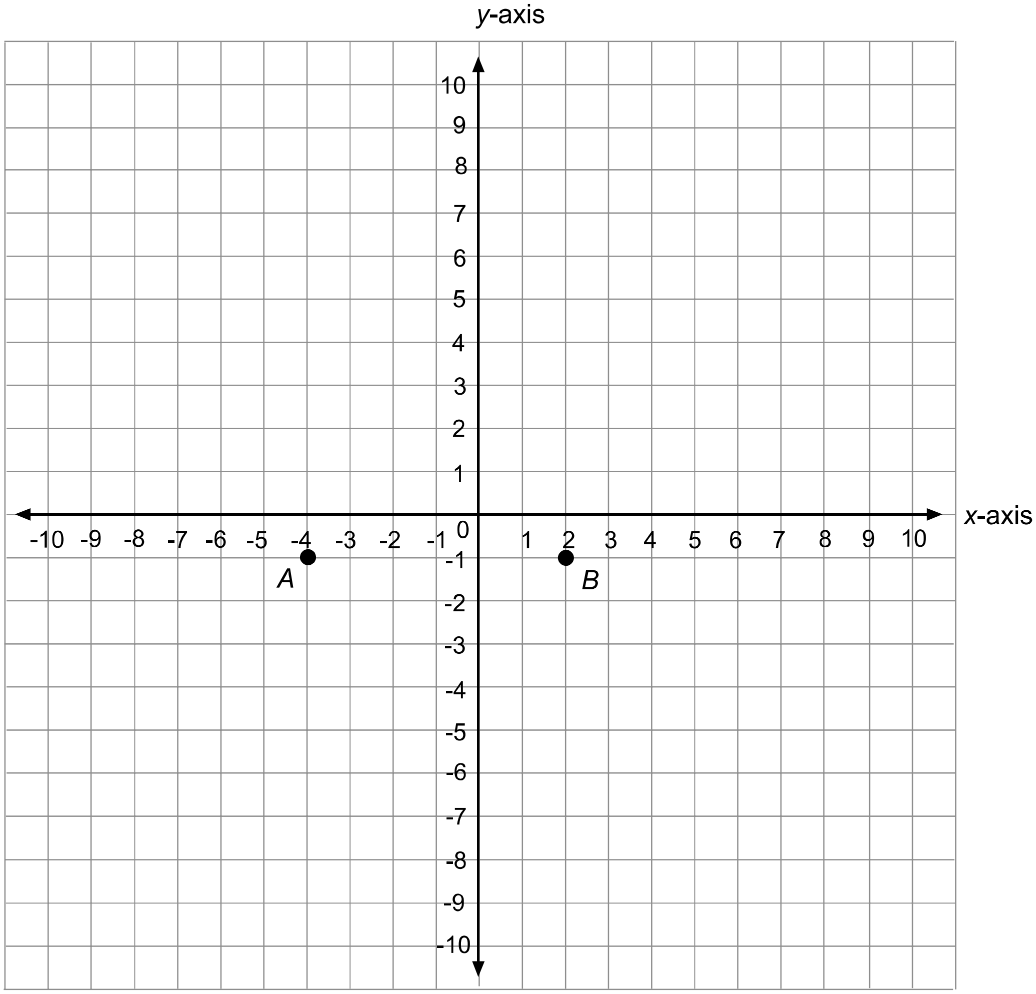
Distance

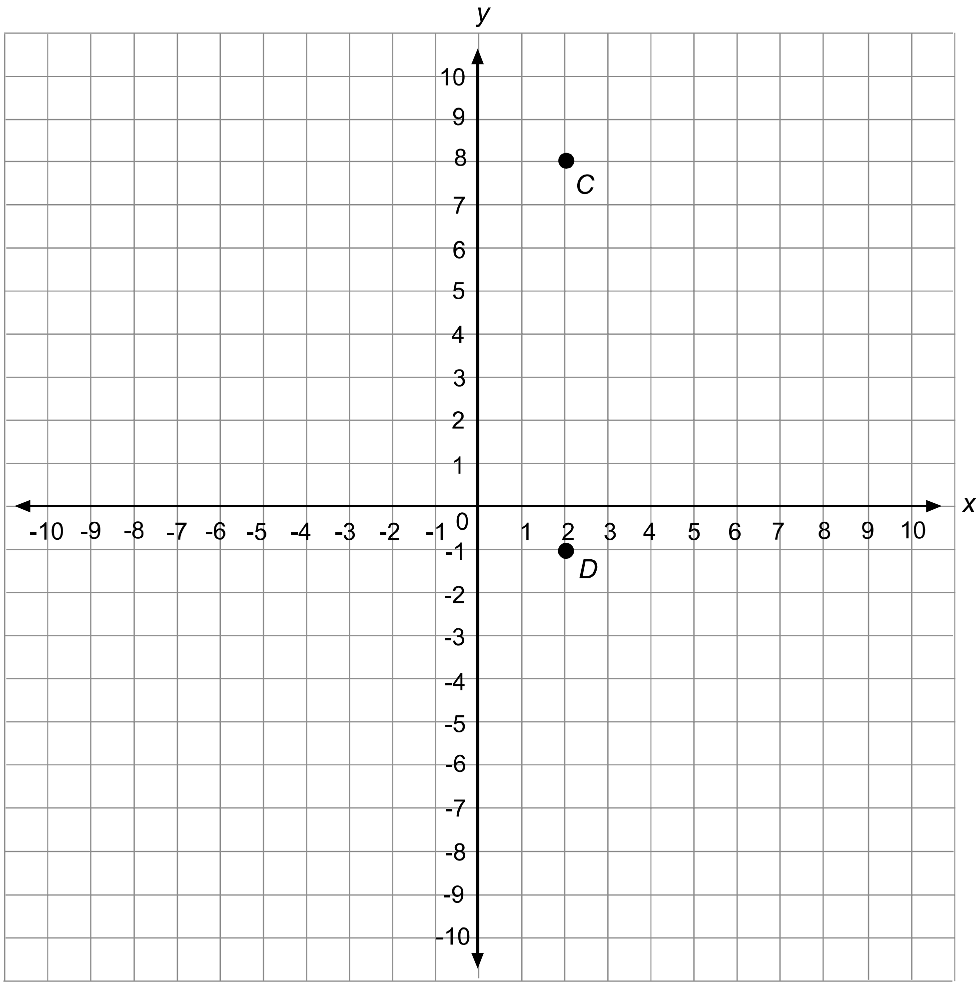
Look at the following **coordinate grids** or **planes**. The horizontal number line on a *rectangular coordinate system* is the ***x*-axis**. The vertical line on a coordinate system is the ***y*-axis**. We can easily find the **distance** between the given **graphs of the points** below. The *graph of a point* is the **point** assigned to an **ordered pair** on a *coordinate plane*.

Graph of Points *A* and *B*



Because the *points* on the **graph** above are on the same **horizontal (⟷) line**, we can count the spaces from one *point* to the other. So, the *distance* from *A* to *B* is 6.

Graph of Points *C* and *D*



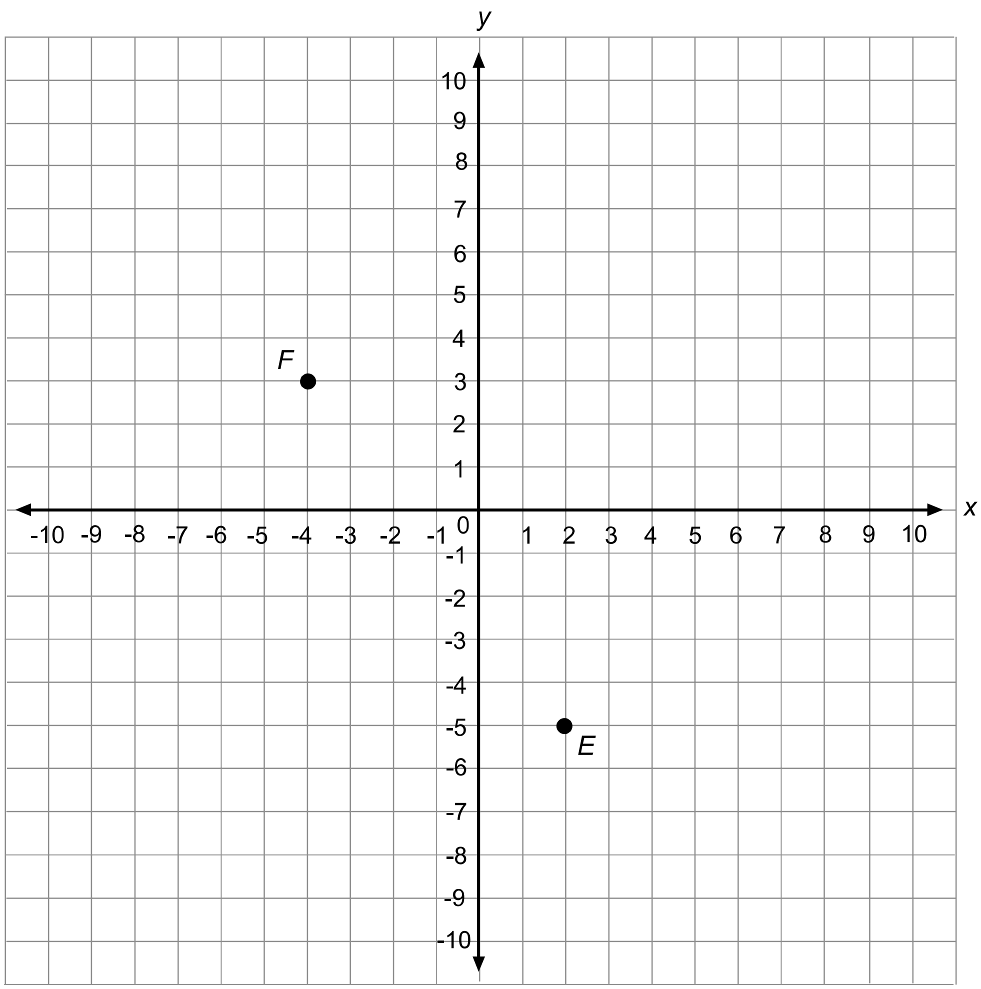
Because the points on the graph above are on the same **vertical (↕)** *line*, we can count the spaces from one point to the other. So, the distance from *C* to *D* is 9.

Remember: Distance is always a **positive number**. Even when you back your car down the driveway, you have covered a *positive* distance. If you get a **negative number**, simply take the **absolute value** of the number.

In many instances, the points we need to identify to find the distance between are not on the same *horizontal* or *vertical* line. Because we would have to count points on a *diagonal*, we would not get an accurate measure of the distance between those points. We will examine two methods to determine the distance between any two points.

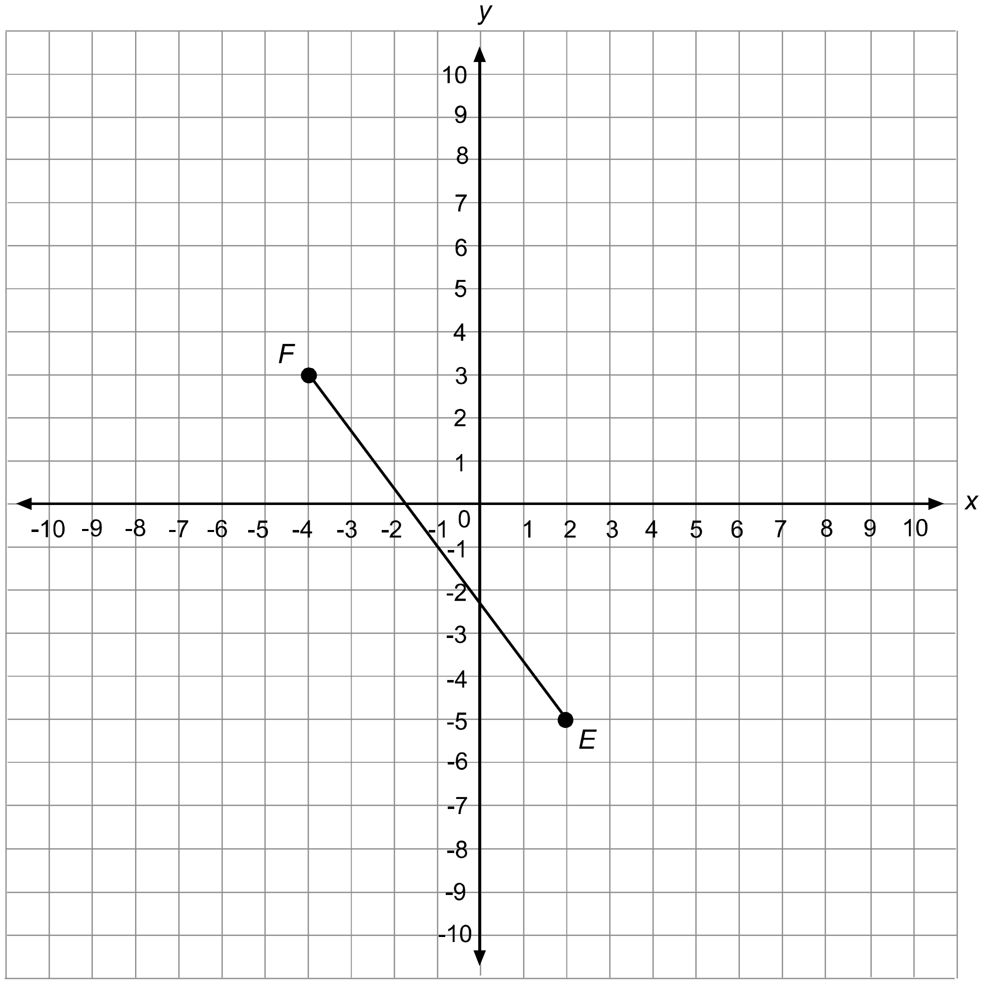
Look at the graph below. We want to find the distance between point *E* (2, -5) and *F* (-4, 3).

Graph of Points *E* and *F*



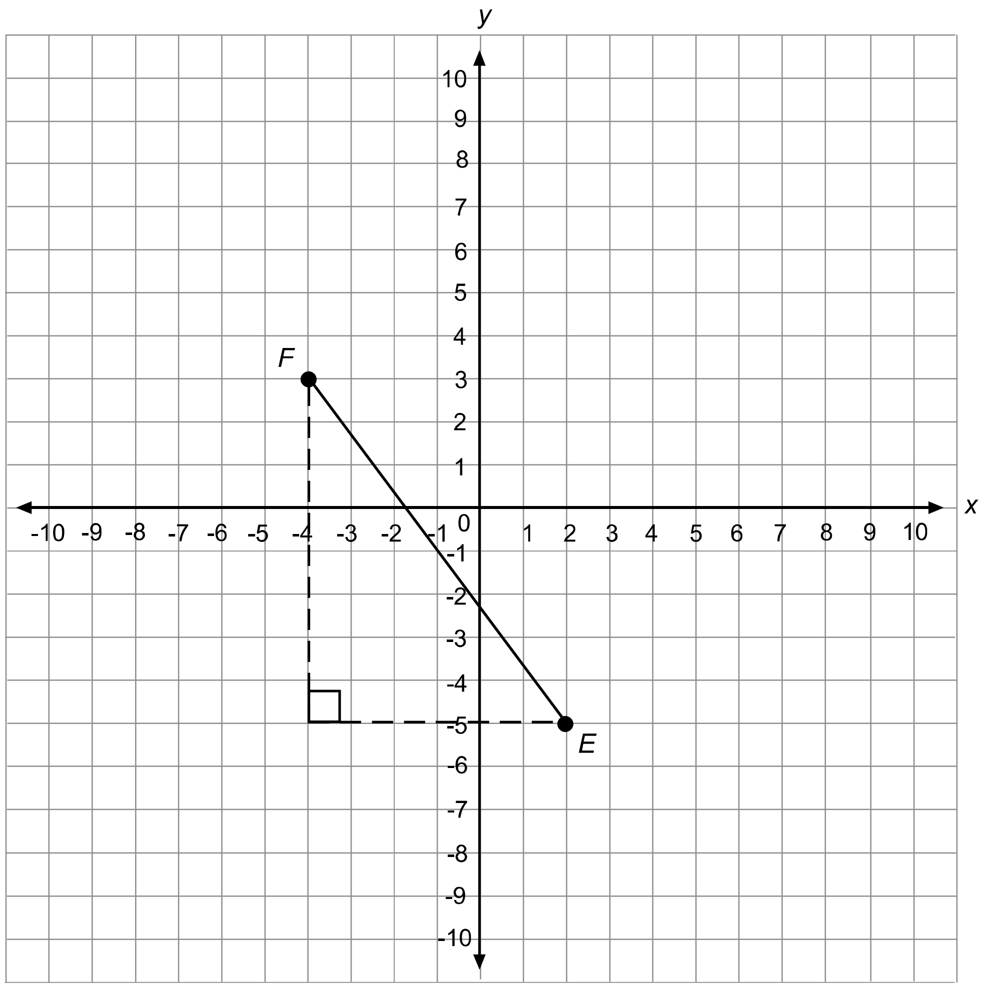
Notice that the distance between *E* and *F* looks like the **hypotenuse** of a **right triangle**.

Graph of Points *E* and *F*



Let's sketch the rest of the **triangle** and see what happens.

Graph of Points *E* and *F*



By completing the sketch of the *triangle*, we see that the result is a *right triangle* with one horizontal **side** and one vertical *side*. We can count to find the **lengths (*l*)** of these two sides, and then use the **Pythagorean theorem** to find the distance from *E* to *F*.

Remember: The *Pythagorean theorem* is the **square** of the *hypotenuse* (c) of a right triangle and is equal to the **sum** of the *squares* of the **legs** (a and b), as shown in the **equation** $a^2 + b^2 = c^2$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ \sqrt{100} &= c \\ 10 &= c \end{aligned}$$

Remember:

- The opposite of *squaring* a number is called *finding the* **square root**. For example, the *square root* of 100, or $\sqrt{100}$, is 10.
- The square root of a number is shown by the symbol $\sqrt{\ }$, which is called a **radical sign** or *square root sign*.
- The number underneath is called a **radicand**.

radical sign → $\sqrt{100}$ ← radicand

radical

- The **radical** is an **expression** that has a **root**. A *root* is an equal **factor** of a number.
- $\sqrt{100}$ is a **radical expression**. It is a numerical *expression* containing a *radical sign*.

$\sqrt{100} = 10$ because $10^2 = 100$

$\sqrt{9} = 3$ because $3^2 = 9$

$\sqrt{121} = 11$ because $11^2 = 121$

Using the Distance Formula

Sometimes, it is inconvenient to *graph* when finding the distance. So, another method we often use to find the distance between two points is the distance **formula**.

The distance *formula* is as follows.

distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The little 1s and 2s that are *subscripts* to the *x*'s and *y*'s signify that they come from different *ordered pairs*.

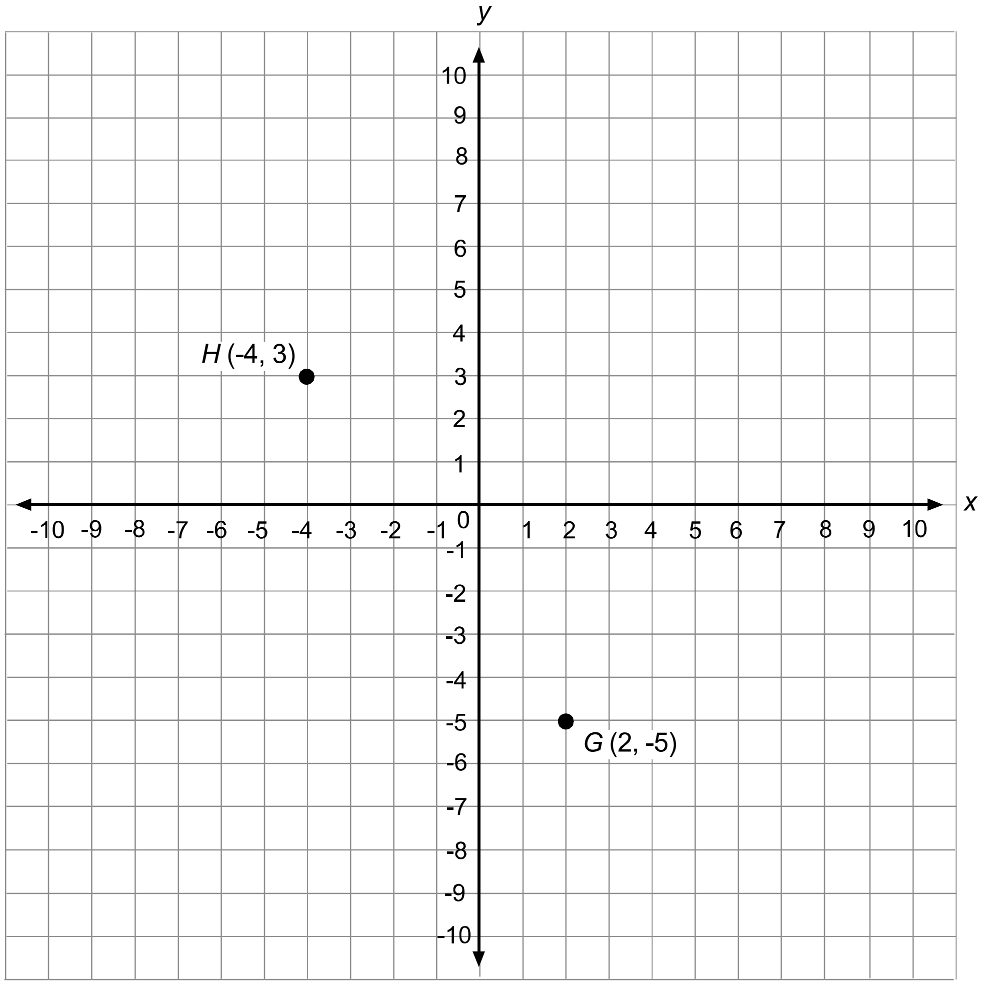
Example

(x_1, y_1) is one *ordered pair* and (x_2, y_2) is another ordered pair.

Note: Be consistent when putting the **values** into the formulas.

Let's look at the same example of *G* (2, -5) and *H* (-4, 3), and use the distance formula. See the graph on the following page.

Graph of Points *G* and *H*



$x_1 = 2$
$y_1 = \text{-}5$
$x_2 = \text{-}4$
$y_2 = 3$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(\text{-}4 - 2)^2 + (3 - \text{-}5)^2} =$$

$$\sqrt{(\text{-}6)^2 + (8)^2} =$$

$$\sqrt{36 + 64} =$$

$$\sqrt{100} =$$

$$10$$

Compare the numbers in the distance formula to the numbers used in the *Pythagorean theorem*.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
6^2 + 8^2 &= c^2 \\
36 + 64 &= c^2 \\
100 &= c^2 \\
\sqrt{100} &= c \\
10 &= c
\end{aligned}
$$

You should always get the same answer using either method.