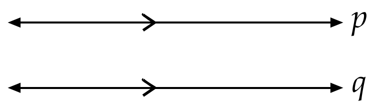
Exploring Parallel Lines

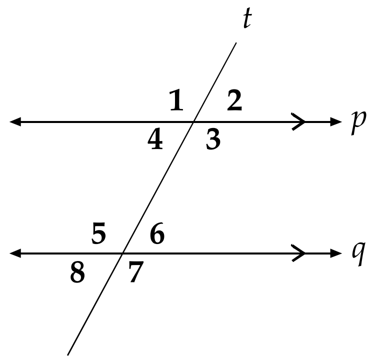
Two (or more) lines in a plane that do *not* intersect are said to be **parallel (||)**. **Parallel lines** remain the same distance apart. They will *never* intersect, even if extended.



Lines can be labeled using a lower case letter.
The bold arrows (>) on the lines indicate that the lines are parallel.

Line *p* is *parallel* to line *q*. The symbol (||) means *is parallel to*. Therefore, we can use the following notation: *p* || *q*

Suppose we draw a third line *t* which intersects the two *parallel lines* (||) *p* and *q*. The third line *t* is called a **transversal**. A *transversal* is a line that intersects two or more (usually parallel) lines.



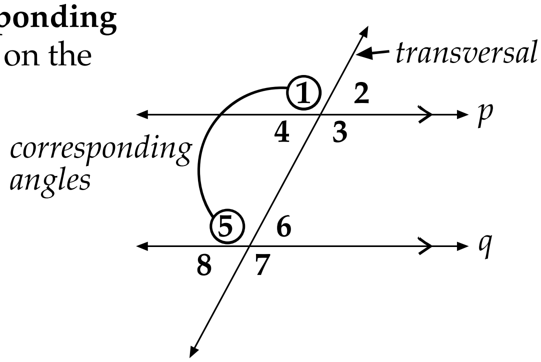
Notice that 8 angles are formed. Look at the 4 angles at the top.

$\angle 1$ is in the upper left
$\angle 2$ is in the upper right
$\angle 3$ is in the lower right
$\angle 4$ is in the lower left

Now look at the bottom 4.

$\angle 5$ is in the upper left, just like $\angle 1$
$\angle 6$ is in the upper right, just like $\angle 2$
$\angle 7$ is in the lower right, just like $\angle 3$
$\angle 8$ is in the lower left, just like $\angle 4$

When parallel lines are cut by a transversal, angles in the same relative or matching position are called **corresponding angles**. *Corresponding angles* also lie on the same side of a transversal.



Therefore

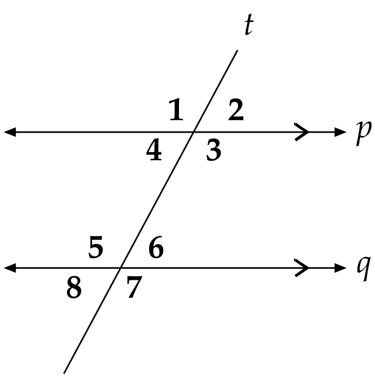
∠1 corresponds to ∠5;
∠6 corresponds to ∠2;
∠7 corresponds to ∠3;
and ∠8 corresponds to ∠4.

Measure each pair of angles with a protractor. Label each angle with its measure.

Did you find the following?

If you have parallel lines cut by a transversal, then the measures of the corresponding angles are equal.

Look at the angles again.



Since ∠1 and ∠5 are corresponding angles, we know that

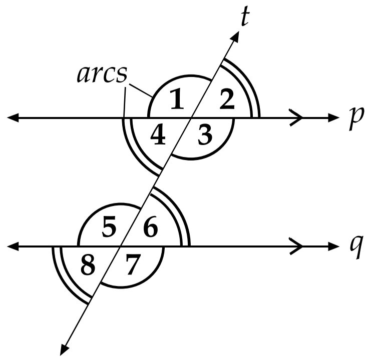
$m\angle 1 = m\angle 5$

We also know that ∠1 and ∠3 are vertical angles, and ∠5 and ∠7 are vertical angles. Remember that the measures of vertical angles are equal. Therefore,

$m\angle 1 = m\angle 3$
$m\angle 5 = m\angle 7$

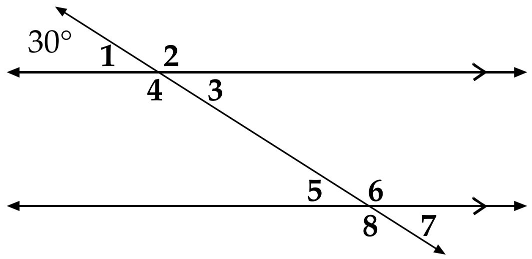
Logically, we can say that $m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7$.
Similar logic would allow us to say that $m\angle 2 = m\angle 4 = m\angle 6 = m\angle 8$.

To summarize what has been said we will mark all angles having the same measure with *arcs*. Use the arcs marked on the angles as your guide to which angles have the same measure.



What is the relationship between ∠1 and ∠2? If you said that the angles are supplementary, then you are correct! The two angles together make a straight angle. A straight angle is an angle that measures 180 degrees.

Now see if you can unravel the following mystery:



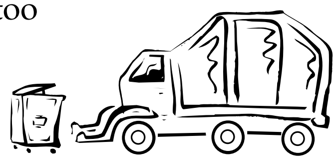
Find the measures of all angles if you know that m∠1 = 30°.

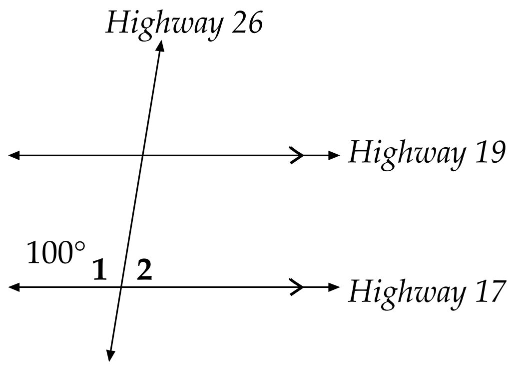
Solution: If m∠1 = 30°, then m∠2 = 150°.
∠1 and ∠2 are supplementary, and their measures add to 180°.

m∠1 = m∠3 = m∠5 = m∠7 = 30°
m∠2 = m∠4 = m∠6 = m∠8 = 150°

Here is another problem to consider:

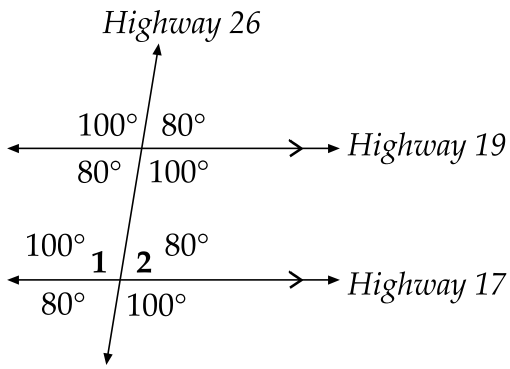
The city of Big Hill, Florida is considering expanding garbage service to a nearby community. There is one problem. The city has to be sure that the angles at the intersections will not be too sharp for the garbage trucks to make the turns. A garbage truck cannot turn at an angle that is less than 70°. Can the garbage service be instituted if we know that one of the corners measures 100°?





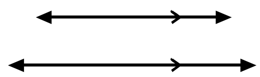
Solution: Label all angles whose measures equal 100 degrees. (corresponding and vertical). Remember, $\angle 1$ and $\angle 2$ are supplementary, so $m\angle 2$ must be 80°.

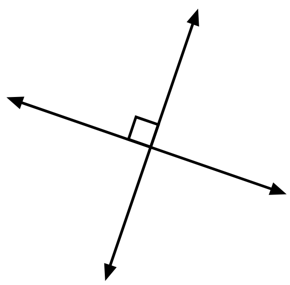
Now label the corresponding vertical and corresponding angles. You see that there are no corners where an angle measures less than 70°, so the garbage service can be extended.

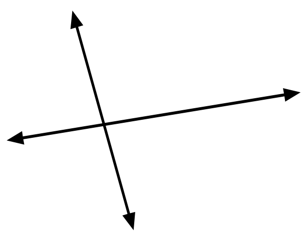


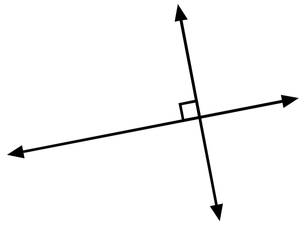
Practice

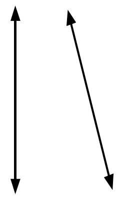
Write || *if the lines are* **parallel**. *Write* ⊥ *if the lines are* **perpendicular**. *If the lines are **neither** parallel **nor** perpendicular, write* **neither**.

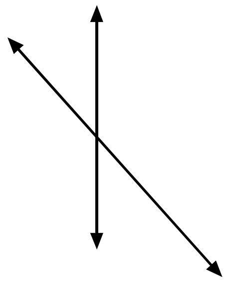
1. ____________ 

2. ____________ 

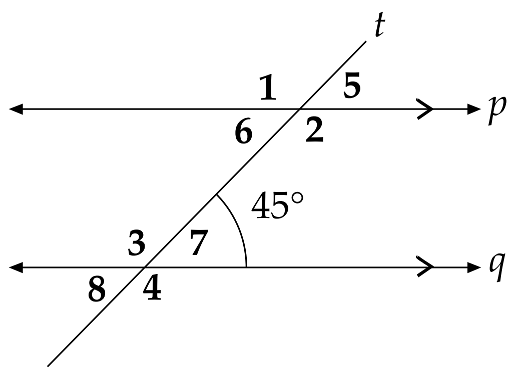
3. ____________ 

4. ____________ 

5. ____________ 

6. ____________ 

Use the figure below to answer the following.



7. Which two lines are parallel? ______________________________

8. Name four pairs of corresponding angles. ____________________

____

9. Name four pairs of vertical angles. __________________________

____

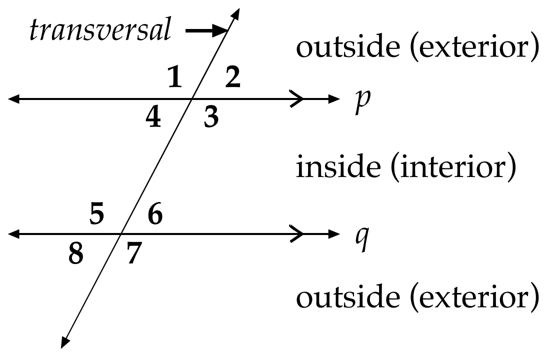
10. Find the m∠1. ____

11. Find the m∠2. ____

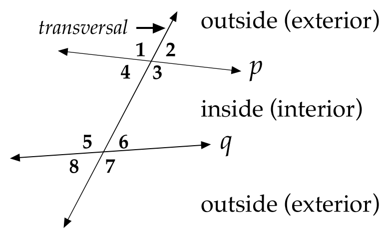
12. Find the m∠6. ____

Angles Formed by a Transversal

When a transversal intersects two lines, it forms eight angles. These angles are **alternative angles**. *Alternative angles* lie on opposite sides and at opposite ends of a transversal. The four angles lying between or *inside* the two lines are *alternate interior angles*. The four angles lying *outside* the two lines are *alternate exterior angles*.



Note: Even when lines cut by a transversal are *not* parallel, we still use the same vocabulary.



However, there are special properties when the lines intersected by a transversal are parallel.

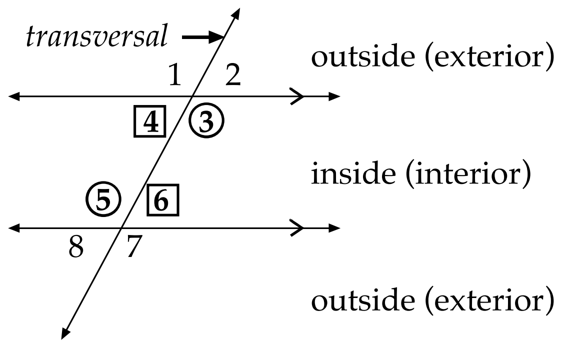
Remember that we have shown that when a transversal intersects two parallel lines:

$m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7$ *and*
$m\angle 2 = m\angle 4 = m\angle 6 = m\angle 8$

Alternate Interior and Alternate Exterior Angles

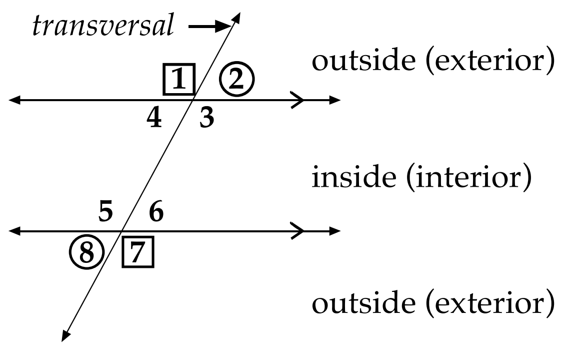
Angles 3, 4, 5, and 6 are called *alternate interior angles* because they are on the *inside* of the parallel lines. Angles 1, 2, 7, and 8 are called *alternate exterior angles* because they are on the *outside* of the parallel lines.

∠3 and ∠5
and
∠4 and ∠6 are alternate interior angles (opposite inside).



Alternate exterior angles are on *opposite sides* of the transversal and on the *outside* of the lines it intersects. There are 2 pairs of alternate exterior angles.

∠1 and ∠7
and
∠2 and ∠8 are alternate exterior angles (opposite outside).

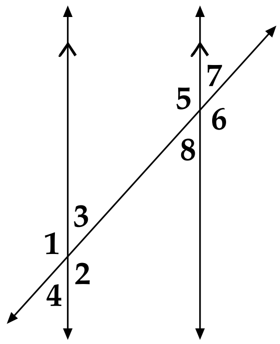


If we have parallel lines cut by a transversal, then the following is true:

- the measures of alternate interior angles are equal
- the measures of alternate exterior angles are equal.

Practice

Use the figure below to answer the following.



1. Name two pairs of alternate interior angles. ____________________

   ____

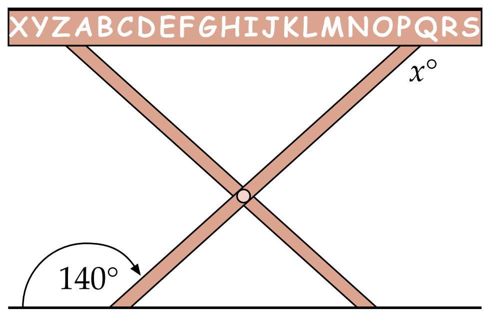
2. Name two pairs of alternate exterior angles. ___________________

   ____

3. If angle 4 measures 30°, what is the measure of angles 6 and 7? __

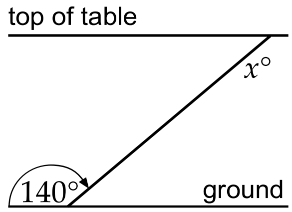
   ____

4. Max is building a child's play table. Here is an end view of the table.



The top of the table will be parallel to the ground. What is the measure of angle x?

Hint: Redraw the picture.

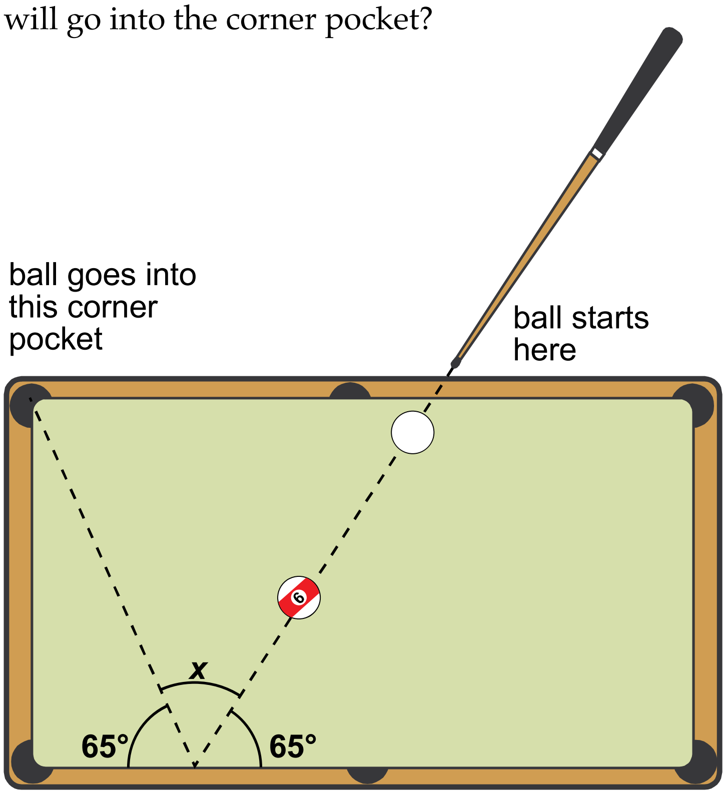


a. 140°
b. 40°
c. 180°
d. 90°

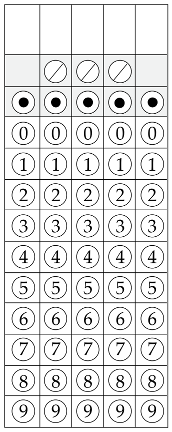
Number 5 is a **gridded-response item**.

Write answer along the top of the grid and correctly mark it below.

5. Chip is playing pool. He wants to hit the ball into the corner pocket as shown below. What angle (x) must the path of the ball take so it will go into the corner pocket?



Mark your answer on the grid to the right.



Practice

Use the list below to complete the following statements.

alternate angles	**parallel**
corresponding angles	**parallel lines**
alternate exterior angles	**transversal**
alternate interior angles	

1. ____________________ are always the same distance apart and will never intersect, even if extended.

2. A ____________________ is a line that intersects two or more (usually parallel) lines.

3. When parallel lines are cut by a *transversal*, angles in the same relative or matching position that lie on the same side of a transversal are called ____________________ .

4. When a transversal intersects two lines, it forms eight angles. The four angles lying between or *inside* the two lines are ____________________ .

5. Two (or more) lines in a plane that do *not* intersect are said to be ____________________ .

6. When a transversal intersects two lines, it forms eight angles. The four angles lying *outside* the two lines are ____________________ .

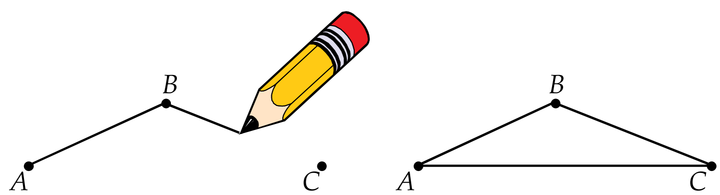
7. A pair of angles that lie on the *opposite side* and at opposite ends of a transversal are called ____________________ .

Lesson Two Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Relate the concepts of measurement to similarity and proportionality in real-world situations. (MA.B.1.4.3)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems including ratio and proportion. (MA.C.3.4.1)
- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two-dimensional figures. (MA.C.3.4.2)

Triangles

Draw three points on a piece of paper. Make sure they do not lie on the same line. Label one point *A*, one point *B*, and one point *C*. Connect the points. The figure formed is, of course, a **triangle**. In a *plane* (a flat surface), a closed figure formed by three line segments is a *triangle*.

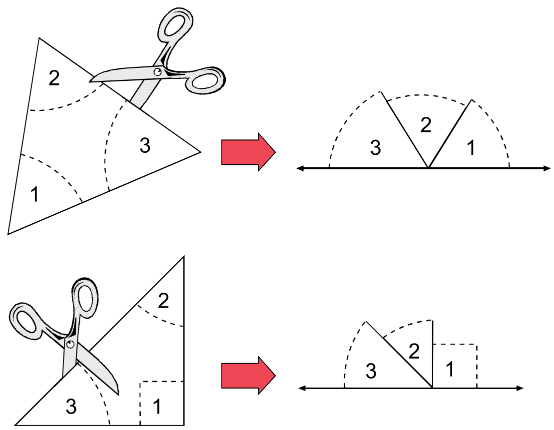


The points, A, B, and C are called the *vertices of the triangle*. If we want to talk about just one point, we would use the word *vertex*, the singular for *vertices*. The triangle we drew is triangle *ABC*. Vertices are named in a clockwise or counterclockwise manner.

Note: $\triangle ABC$ means triangle *ABC*.

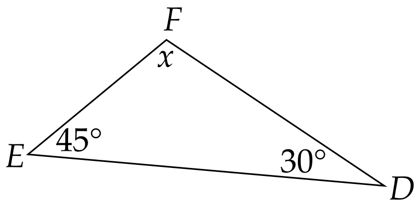
Angles of a Triangle

Let's investigate the three angles of a triangle. Take a piece of paper and draw a large triangle. Put numbers in each angle. Cut the triangle out. Now cut the angles off as shown below. Fit the angle pieces with the points together as shown below. What do the three angles make? You should have found that the angles make a straight angle.



This demonstrates that the sum of the measures of the three angles in *any* triangle is 180 degrees.

Example:



What is the $m\angle F$?

Solution:

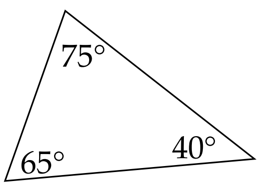
$$\begin{aligned} m\angle D + m\angle E + m\angle F &= 180^\circ \\ 30^\circ + 45^\circ + x^\circ &= 180^\circ \\ 75^\circ + x^\circ &= 180^\circ \\ 75^\circ - 75^\circ + x^\circ &= 180^\circ - 75^\circ \quad \text{subtract 75 from both sides} \\ x^\circ &= 105^\circ \end{aligned}$$

Classifying Triangles

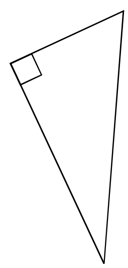
Triangles are **polygons** with three sides. *Polygons* are closed plane figures whose sides are straight and do not cross. Triangles are classified in two different ways. Triangles are classified either by the measure of their angles or the measure of their sides. However, no matter how a triangle is classified, the sum of the measures of the angles in a triangle is 180 degrees.

Triangles Classified by Their Angles—Acute, Right, Obtuse, and Equiangular

An **acute triangle** contains *all* acute angles with measures less than 90°.

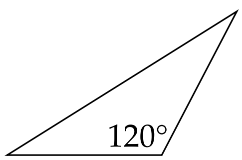


A **right triangle** contains *one* right angle with a measure of 90°.

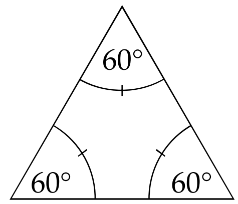


Note: right triangles are marked ⊾

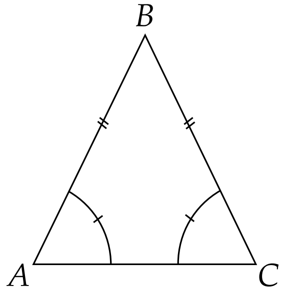
An **obtuse triangle** contains *one* obtuse angle with a measure of more than 90° but less than 180°.



An **equiangular triangle** contains *all* equal angles, each with a measure of 60°.



Note: *Tick marks* are used to denote *angles* or *sides* with the same measure. *Arcs* are also used to show *angles* with the same measure.

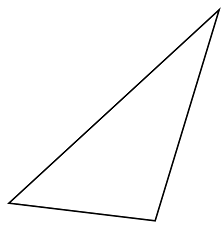


$m\angle A = m\angle C$

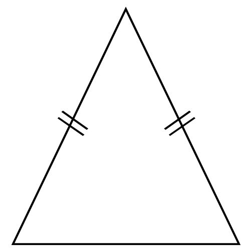
$AB = BC$

Triangles Classified by the Lengths of Their Sides—Scalene, Isosceles, and Equilateral

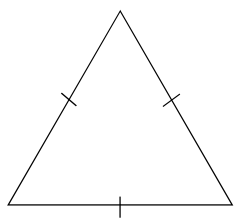
A **scalene triangle** has *no* congruent sides—no sides are the same length.



An **isosceles triangle** has at least *two* congruent sides—two or more sides are the same length.



An **equilateral triangle** has *three* congruent sides—*all* sides are the same length.



So, all triangles may be classified by their angles (acute, right, obtuse, or equiangular), by their sides (equilateral, isosceles, scalene), or both. See the chart below.

Triangles

Classification	Acute < 90°	Right = 90°	Obtuse > 90° and <180°	Equiangular all = 60°
Equilateral	✓			✓
Isosceles	✓	✓	✓	✓
Scalene	✓	✓	✓	

< means less than

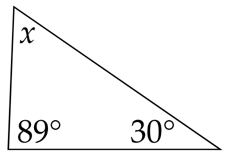
> means greater than

As you see in the above chart, a right triangle *may be* either isosceles or scalene, but is *never* acute, equilateral, or obtuse.

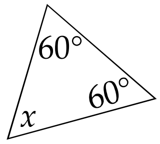
Practice

Find the **value of *x***. *Then classify each triangle as* **acute**, **right**, **obtuse**, *or* **equiangular**.

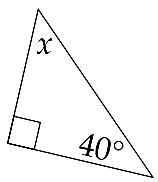
value of *x* **classification of triangle**

1. $x =$ ________ 

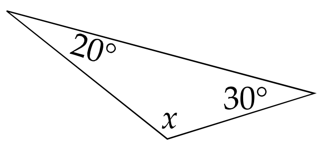
____

2. $x =$ ________ 

____

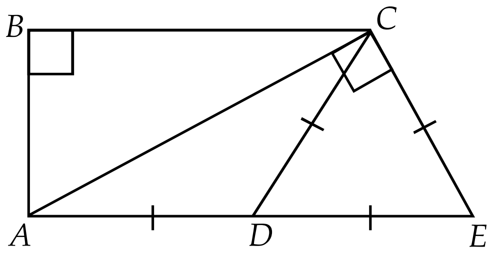
3. $x =$ ________ 

____

4. $x =$ ________ 

____

Use the figure below to answer the following.



5. Name all the right triangle(s). ________________________________

6. Name the isosceles triangle(s). ________________________________

7. Name the obtuse triangle(s). ______________________________

8. Name the equilateral triangle(s). ___________________________

Draw the following.

9. Draw an isosceles right triangle.

10. Draw an obtuse scalene triangle.

11. Draw a right scalene triangle.

Answer the following.

12. Can a triangle have two right angles? ______________________

    Why or why not? ____

    ____

    ____

13. Can a triangle have two obtuse angles? ____________________

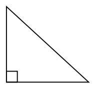
    Why or why not? ____

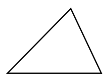
    ____

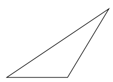
    ____

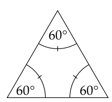
Practice

Match each **figure** *with the correct term.*

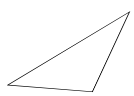
______ 1.  has one 90° angle — A. acute triangle

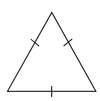
______ 2.  all angles are less than 90° — B. equiangular triangle

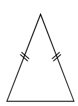
______ 3.  has one angle more than 90° but less than 180° — C. obtuse triangle

______ 4. 

all angles are equal — D. right triangle

______ 5.  has no congruent sides — A. equilateral triangle

______ 6.  has 3 congruent sides — B. isosceles triangle

______ 7.  has 2 congruent sides — C. scalene triangle

Practice

Use the list below to write the correct term for each definition on the line provided.

acute triangle	**isosceles triangle**	**right triangle**
equiangular triangle	**obtuse triangle**	**scalene triangle**
equilateral triangle	**polygon**	**triangle**

______________________ 1. a polygon with three sides; the sum of the measures of the angles is 180°

______________________ 2. a triangle with one right angle

______________________ 3. a triangle with one obtuse angle

______________________ 4. a triangle with three equal angles

______________________ 5. a triangle with three acute angles

______________________ 6. a triangle with three congruent sides

______________________ 7. a triangle with at least two congruent sides and two congruent angles

______________________ 8. a triangle with no congruent sides

______________________ 9. a closed plane figure whose sides are straight and do not cross

Similar Figures

Two figures that have the same shape are called **similar figures**. We will consider similar triangles as well as other similar shapes.

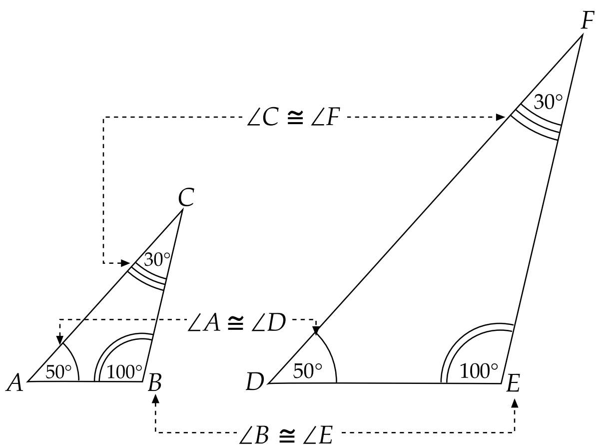
Similar figures are alike in a very specific way. All of the following must be true for two figures to be called similar figures.

- They have the same shape.
- The **corresponding angles** are congruent (have the same measure).
- The **ratios** (comparison of two quantities) of the lengths of the **corresponding sides** are equal—the sides are proportional in length.
- They may or may not have the same size or be in the same position.

We will use this knowledge and our skills in setting up and solving **proportions**—mathematical sentences stating two ratios are equal.

Look at these two triangles.

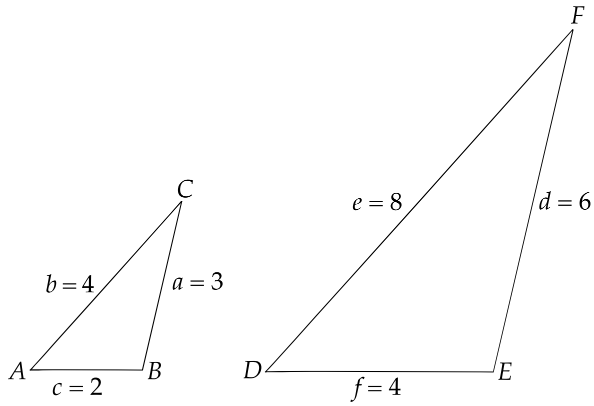
Remember: The symbol $\cong$ means is *congruent to*.



All of the following corresponding angles are equal.

$\angle A$ corresponds to $\angle D$
$\angle B$ corresponds to $\angle E$
$\angle C$ corresponds to $\angle F$

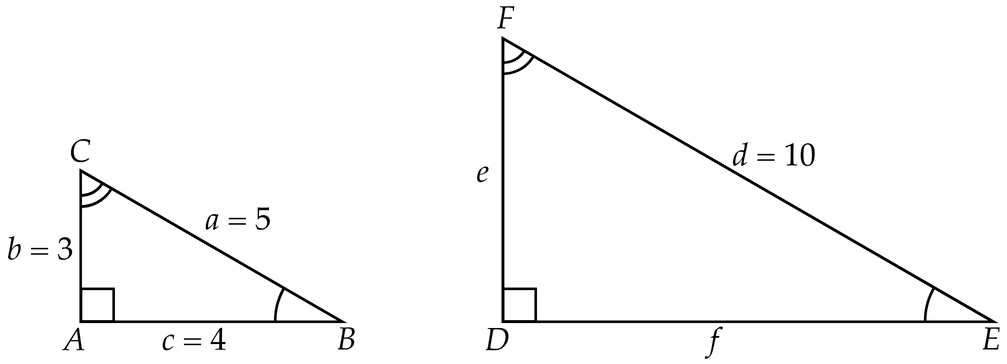
and



All of the following pairs of corresponding sides are *proportional* and have the same ratio.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$
$$\frac{3}{6} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

These triangles are similar.



The symbol ~ means *is similar to*.

Therefore $\triangle ABC \sim \triangle DEF$.

The single and double arcs indicate corresponding pairs of angles. ∠A corresponds to ∠*D*, ∠*B* corresponds to ∠*E*, and ∠*C* corresponds to ∠F.

$$\angle A \cong \angle D; \angle B \cong \angle E; \angle C \cong \angle F$$

We can write: $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ and replace known values to find length of the sides.

$$\frac{5}{10} = \frac{3}{e} = \frac{4}{f}$$

The $\frac{5}{10}$ is our **scale factor** and is used to set up proportions for finding the values of *e* and *f*. The *scale factor* is the ratio between the lengths of corresponding sides of two similar figures. And you know that pairs of corresponding sides in similar figures have equal ratios.

You can solve a proportion by using **cross products**. A *cross product* is the product of one *numerator* and the opposite *denominator* in a pair of fractions. The cross products of equivalent fractions will be equal.

$\frac{5}{10} = \frac{3}{e}$	and	$\frac{5}{10} = \frac{4}{f}$
$5e = 30$	divide both sides by 5	$5f = 40$
$\frac{5e}{5} = \frac{30}{5}$	← →	$\frac{5f}{5} = \frac{40}{5}$
$e = 6$		$f = 8$

When we substitute our values for *e* and *f* into the ratios above, we get

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

$$\frac{5}{10} = \frac{3}{6} = \frac{4}{8}$$

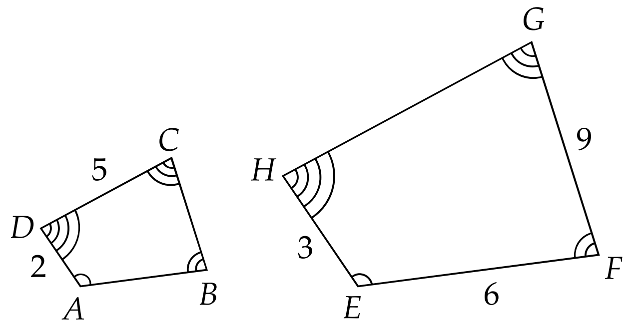
Notice that each ratio reduces to $\frac{1}{2}$.

$$\frac{5}{10} = \frac{1}{2}$$
$$\frac{3}{6} = \frac{1}{2}$$
$$\frac{4}{8} = \frac{1}{2}$$

Another way to describe our result is to say:

5 is to 10 as 3 is to 6 as 4 is to 8.

The properties of similar triangles also apply to similar **quadrilaterals** (polygons with four sides).



$m\angle A = m\angle E$
$m\angle B = m\angle F$
$m\angle C = m\angle G$
$m\angle D = m\angle H$

Also: $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Filling in the given side lengths gives us:

$$\frac{AB}{6} = \frac{BC}{9} = \frac{5}{GH} = \frac{2}{3}$$

From these ratios we see a scale factor of $\frac{2}{3}$, leading us to set up the following proportions.

$\frac{AB}{6} = \frac{2}{3}$	and	$\frac{BC}{9} = \frac{2}{3}$	and	$\frac{5}{HG} = \frac{2}{3}$
$3(AB) = 2 \bullet 6$		$3(BC) = 2 \bullet 9$		$5 \bullet 3 = 2(HG)$
$3(AB) = 12$		$3(BC) = 18$		$15 = 2(HG)$
$\frac{3(AB)}{3} = \frac{12}{3}$		$\frac{3(BC)}{3} = \frac{18}{3}$		$\frac{15}{2} = \frac{2(HG)}{2}$
$AB = 4$		$BC = 6$		$7\frac{1}{2} = HG$

Now substitute the values for *AB*, *BC*, and *HG* into the ratios above.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$
$$\frac{4}{6} = \frac{6}{9} = \frac{5}{7\frac{1}{2}} = \frac{2}{3}$$

Notice that each ratio reduces to $\frac{2}{3}$.

$$\frac{4}{6} = \frac{2}{3}$$
$$\frac{6}{9} = \frac{2}{3}$$
$$\frac{5}{7\frac{1}{2}} = \frac{2}{3}$$

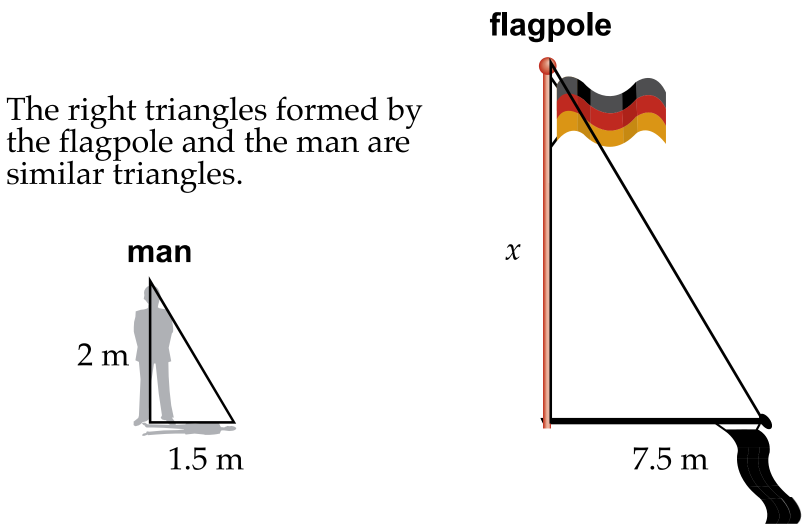
Another way to describe the results is to say: 4 is to 6 as 6 is to 9 as 5 is to $7\frac{1}{2}$.

Indirect Measurement

If you cannot measure a length directly, you can sometimes use similar triangles to make an indirect measurement.

Example:

Consider shadows made by a flagpole and a man. The man is 2 meters tall and casts a shadow 1.5 meters. The flagpole casts a shadow 7.5 meters in length. Let's find the height of the flagpole.



1. Identify two corresponding ratios.
   ratio of shadows: 1.5 to 7.5
   ratio of heights: 2 to x

2. Set up a proportion.

shadow of man → $\frac{1.5}{7.5} = \frac{2}{h}$ ← height of man
shadow of flagpole → ← height of flagpole

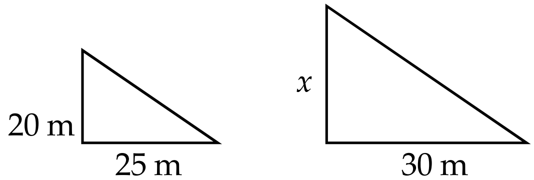
3. Solve the proportions.

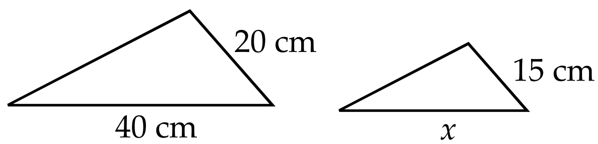
$\frac{1.5}{7.5}$	=	$\frac{2}{h}$	
1.5 x h	=	7.5 x 2	find cross products
1.5 x h	=	15	solve for h
$\frac{1.5h}{1.5}$	=	$\frac{15}{1.5}$	divide each side by 1.5
h	=	10	answer: height of the flagpole is 10 meters

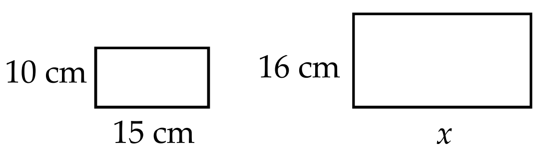
We can see that the *scale factor* is 5 because the shadow of the flagpole is 5 times as long as the shadow of the man. The flagpole will be 5 times as tall as the man or 5 times 2 or 10 meters.

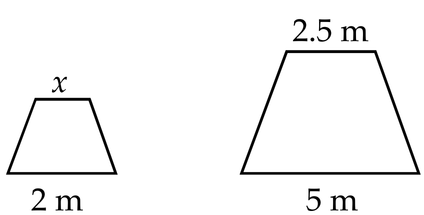
Practice

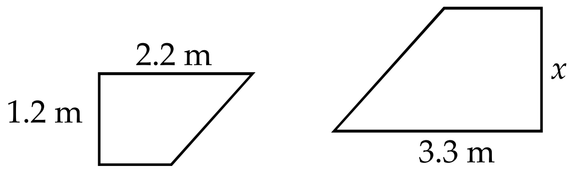
For each problem below, the two figures are **similar**. *Set up and solve a* **proportion** *to find the* **length** ***x***. *Show all your work.*

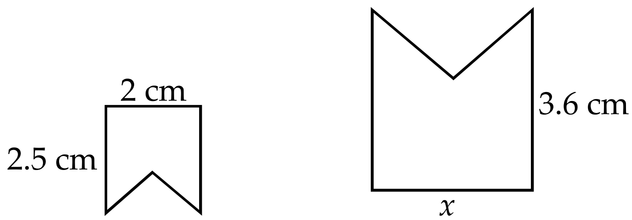
1. 

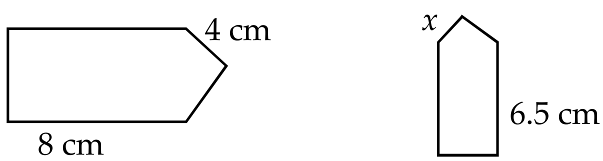
2. 

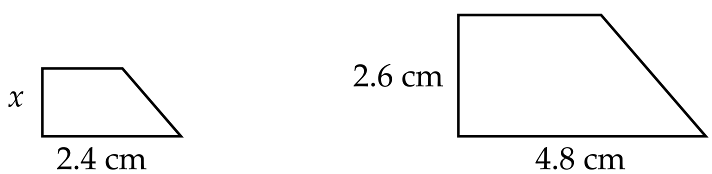
3. 

4. 

5. 

6. 

7. 

8. 

Draw a **diagram**, *set up a* **proportion**, *and* **solve** *the following.*

9. When a high-rise office building casts a 70-foot shadow, a man 6 feet tall casts a 3-foot shadow. How tall is the building?

10. A TV tower is 35 meters high and casts a 10-meter shadow. How tall is a nearby tree that casts a 6-meter shadow at the same time?

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the quotient of two numbers used to compare two quantities	A. corresponding angles and sides
______ 2. the product of one numerator and the opposite denominator in a pair of fractions	B. cross product
______ 3. polygon with four sides	C. proportion
______ 4. the matching angles and sides in similar figures	D. quadrilateral
______ 5. figures that have the same shape but not necessarily the same size	E. ratio
______ 6. a mathematical sentence stating that two ratios are equal	F. scale factor
______ 7. the ratio between the lengths of corresponding sides of two similar figures	G. similar figures

The Coordinate System

Before we can continue our study of geometry, we need to know how to **graph points** on a plane. You probably have studied graphing points before. Below is a model of a **coordinate grid or system**. A *coordinate grid or system* is a two-dimensional system used to locate points in a plane. Study the picture and see how much you remember!

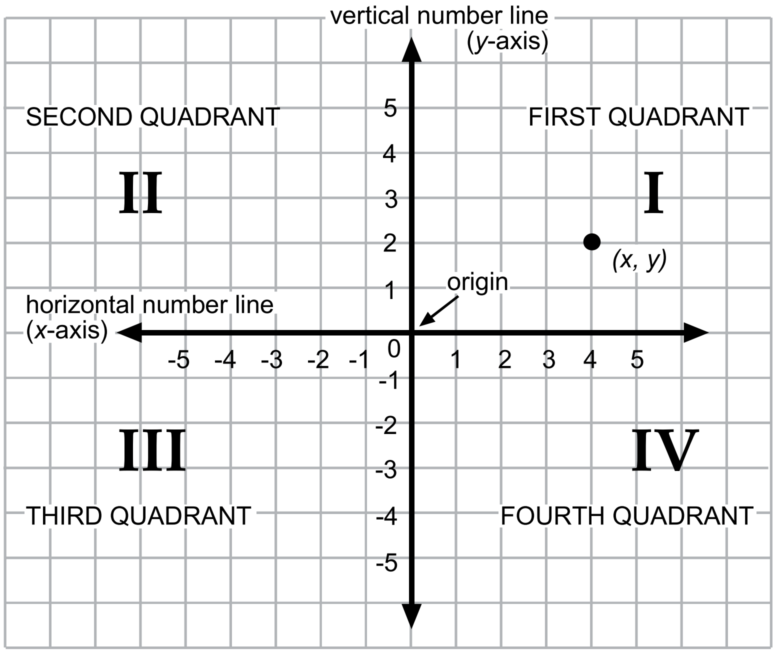
1. The coordinate system has a horizontal (↔) number line called the ***x*-axis** and a vertical (↕) number line called the ***y*-axis**.

2. The two number lines or **axes of a graph** are perpendicular and *intersect* (meet) at a point called the **origin**. The **coordinates** at the **intersection** of the *origin* are (0, 0).

3. The *axes* divide the plane into four parts called **quadrants**. We start in the upper right and label that part Quadrant I. The upper left part is Quadrant II, the lower left Quadrant III, and the lower right Quadrant IV. The axes and the origin are not in any quadrant.

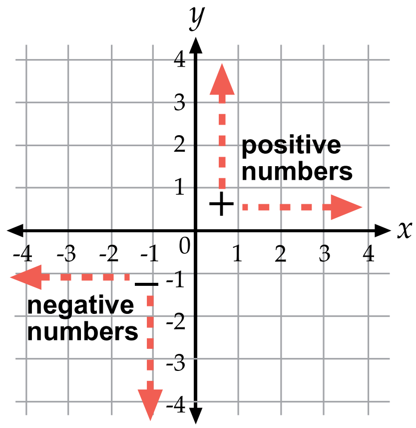
4. Each point in a coordinate plane can be represented by an **ordered pair** which describes the position of the point in relation to the *x*-axis and *y*-axis.

Coordinate Grid (or System)



Locating Points

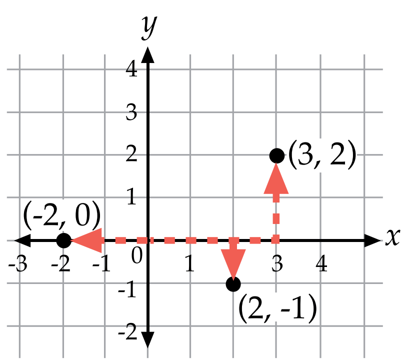
Let's look at the ordered pair (3, 2).



- Start at the *origin* (0, 0).
- The first number is the ***x*-coordinate**. The first number tells us whether to move left or right from the origin. If the number is *positive* we move *right*. If the number is *negative* we move *left*.
- The second number is the ***y*-coordinate**. The second number tells us whether to move up or down. If the number is *positive*, we move *up*. If the number is *negative*, we move *down*.

Study the following:

Graph (3, 2). Then graph (2, -1) and (-2, 0).



Start at the origin.

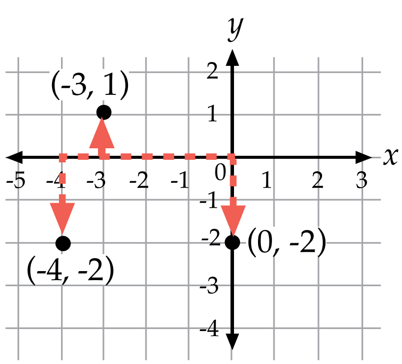
x-coordinate | *y*-coordinate

(3, 2): right 3, up 2

(2, -1): right 2, down 1

(-2, 0): left 2, do *not* move up or down

Graph (-3, 1). Then graph (-4, -2) and (0, -2).



Start at the origin.

x-coordinate | *y*-coordinate

(-3, 1): left 3, up 1

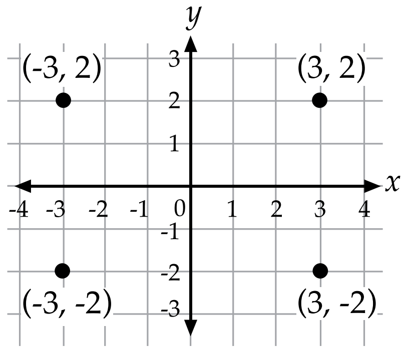
(-4, -2): left 4, down 2

(0, -2): do *not* move left or right, down 2

Let's Summarize:

When you graph a point, the signs of the coordinates tell which directions to move from the origin.

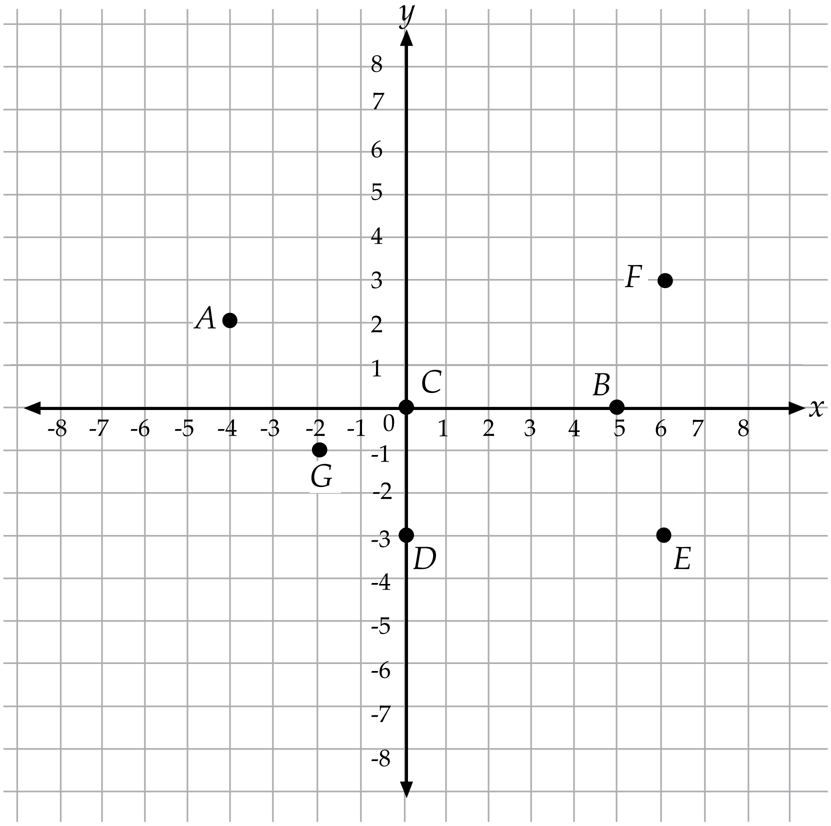
(3, 2)	(-3, 2)	(-3, -2)	(3, -2)
(+, +)	(- , +)	(- , -)	(+, -)
(right, up)	(left, up)	(left, down)	(right, down)



Note that the point (0, 0) is the origin.

Practice

Seven **points** *have been labeled on the* **coordinate grid** *below. Match each* **ordered pair** *with the* **correct point** *in the* **coordinate plane**. *Write the letter of the point on the line provided.*



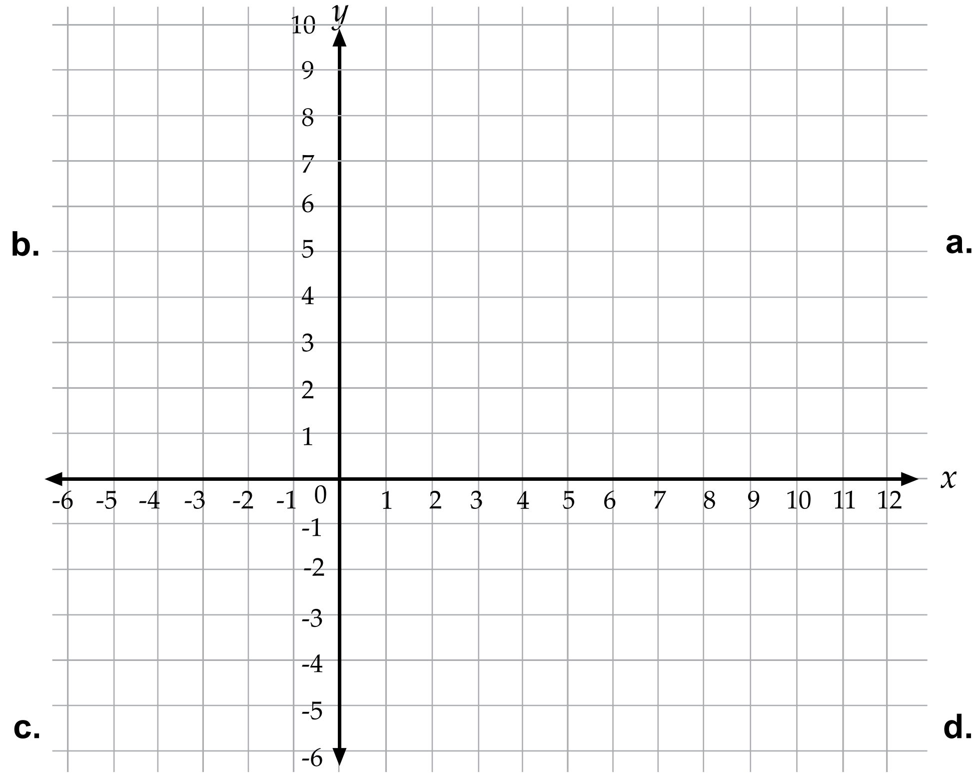
1. (5, 0) ____________
2. (6, 3) ____________
3. (0, 0) ____________
4. (-4, 2) ____________
5. (6, -3) ____________
6. (0, -3) ____________
7. (-2, -1) ____________

Answer the following.

8. What point(s) is (are) located in quadrant II? ____________
9. What point(s) is (are) located on the *x*-axis? ____________
10. What point(s) is (are) located on the *y*-axis? ____________

Complete the following.

11. On the coordinate system below, graph the three points and connect them with line segments. Classify the resulting triangle first by its sides (scalene, isosceles, equilateral), and then by its angles (obtuse, acute, right, equiangular).



a. (2, 1), (2, 5), (5, 1) ____________________________________

b. (-2, 1), (-4, 1), (-5, 4) ____________________________________

c. (-3, -1), (0, -3), (-6, -3) ____________________________________

d. (11, 0), (10, -5), (12, -5) ____________________________________

12. Using the same coordinate system that you used in number 11, plot these four points: (2, 7), (2, 9), (7, 9) and (7, 7). Connect the points so that a **rectangle** is formed. Label the rectangle number 12.

a. Find the width of the *rectangle*. ___________________________

b. Find the length of the rectangle. __________________________

c. Find the **area** of the rectangle. ____________________________

Remember: The *area* of the rectangle can be found by multiplying the length (l) times the width (w).
$A = lw$

d. How many little **squares** are inside the rectangle? _________

Did you get the same answer that you did for number 12c

above? ____

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the horizontal (↔) axis on a coordinate plane

______ 2. the vertical (↕) axis on a coordinate plane

______ 3. the graph of the intersection of the x-axis and y-axis in a coordinate plane, described by the ordered pair (0, 0)

______ 4. any of four regions formed by the axes in a rectangular coordinate system

______ 5. the point assigned to an ordered pair on a coordinate plane

______ 6. network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps

A. coordinate grid or system

B. graph of a point

C. origin

D. quadrant

E. x-axis

F. y-axis

Practice

Use the list below to write the correct term for each definition on the line provided.

area (A)	**ordered pair**
axes (of a graph)	**x-coordinate**
coordinates	**y-coordinate**
intersection	

________________________ 1. the point at which two lines meet

________________________ 2. the first number of an ordered pair

________________________ 3. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

________________________ 4. the second number of an ordered pair

________________________ 5. numbers that correspond to points on a graph in the form (x, y)

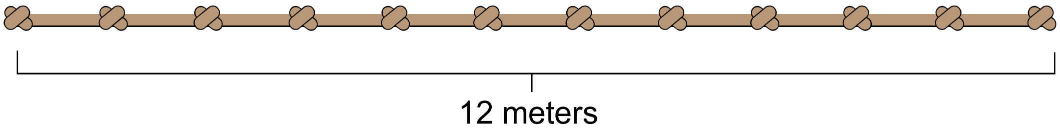
________________________ 6. the location of a single point on a rectangular coordinate system where the digits represent the position relative to the x-axis and y-axis

________________________ 7. the inside region of a two-dimensional figure measured in square units

Pythagorean Theorem

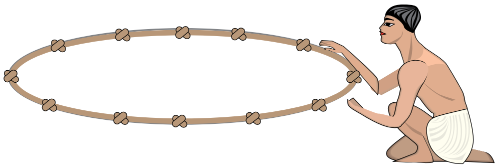
For thousands of years, concepts behind the **Pythagorean theorem** have been used. However, a Greek mathematician named Pythagoras is credited as the first person to write a proof of the theorem. Pythagoras lived during the sixth century B.C.

In about 2000 B.C., ancient Egyptian farmers reportedly used a 12-meter rope in surveying land. See the drawing below with knots equally spaced.

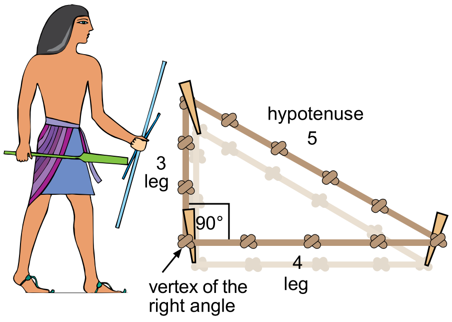


The farmers wanted to make square (90°) corners for their fields. They discovered a "magic 3–4–5" triangle that would help them do this.

- Workers took a rope, knotted it into 12 equal spaces, and formed it into a loop.



- Next they took three stakes and stretched the rope around them to make a triangle that had sides of 3, 4, and 5 units of equal space.

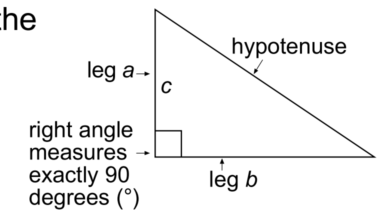


The stake supporting rope lengths of 3 and 4 turned out to be the *vertex* of a right angle, and a right triangle was formed. This method, in modified form, is still used by builders today. The sides forming the right angle are called **legs**, and the side opposite the right angle is the **hypotenuse**. The *hypotenuse* is always the longest side of a right triangle.

The ancient Greeks learned this trick from the Egyptians. Between 500 and 350 B.C., a group of Greek philosophers called the *Pythagoreans* studied the 3–4–5 triangle. They learned to think of the triangle's sides as the three sides of three squares. They generalized this to apply to *any* right triangle. This general statement became the *Pythagorean theorem*.

Pythagorean theorem: In a right triangle, the sum of the **squares** of the lengths of the legs, *a* and *b*, equals the square of the length of the hypotenuse *c*.

Algebraically: $a^2 + b^2 = c^2$

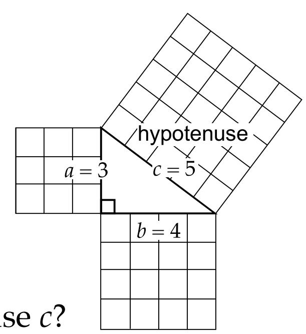


Remember: To *square* a number, multiply it by itself.
Example: The square of 6 or $6^2 = 6 \times 6 = 36$.

The Pythagorean theorem tells us that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.

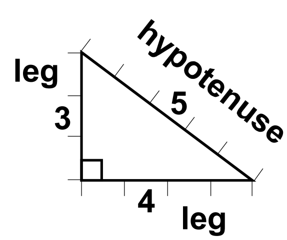
If the lengths of the legs are represented by *a* and *b* and the length of the hypotenuse is represented by *c*, then $a^2 + b^2 = c^2$.

Look at the figure at the right.



- What is the area (inside region) of the square on leg *a*?
- What is the area of the square on leg *b*?
- What is the area of the square on the hypotenuse *c*?

Is the sum of the areas on the legs equal to the area of the square on the hypotenuse?



We see that:

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

Remember: *Area* (*A*) is the inside region of a two-dimensional figure and is measured in **square units**.

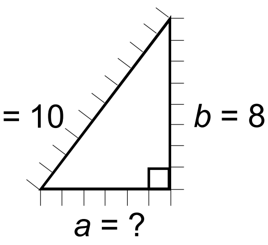
The Pythagorean theorem applies to all right triangles, not just the 3–4–5 right triangle. If you know the lengths of two sides of a right triangle, you can find the length of the hypotenuse or third side.

Example 1:

A right triangle has leg *b* with a unit measure of 8 and the hypotenuse with a unit measure of 10. What is the unit measure of leg *a*?

We can use the **formula** to determine the length of leg *a*.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 8^2 &= 10^2 \\ a^2 + 64 &= 100 \\ a^2 + 64 - 64 &= 100 - 64 \\ a^2 &= 36 \\ a &= \sqrt{36} \\ a &= 6 \end{aligned}$$



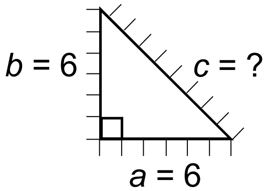
We see that *a* has a length of 6 units.

Example 2:

A right triangle has legs with a unit measure of 6. What is the measure of the hypotenuse?

We will use the same *formula* to determine the length of the hypotenuse.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 6^2 &= c^2 \\ 36 + 36 &= c^2 \\ 72 &= c^2 \\ \sqrt{72} &= c \\ 8.5 &\approx c \end{aligned}$$



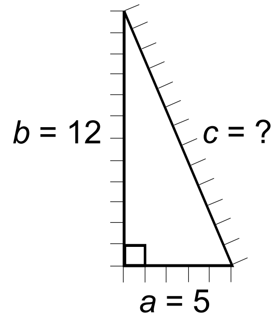
We see *c* has a length of about 8.5 units.

Note: Using our calculator, $\sqrt{72} = 8.485281374$ and **rounding** to the nearest tenth, is approximately equal to (≈) 8.5.

Example 3:

A right triangle has legs with unit measures of 12 and 5. What is the measure of the hypotenuse?

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
5^2 + 12^2 &= c^2 \\
25 + 144 &= c^2 \\
169 &= c^2 \\
\sqrt{169} &= c \\
13 &= \mathrm{c}
\end{aligned}
$$

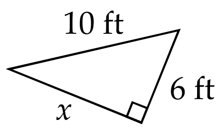


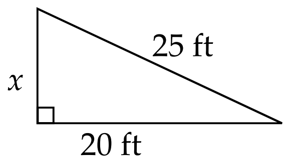
We see c has a length of 13 units.

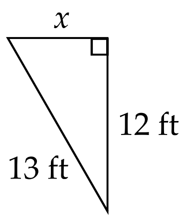
Practice

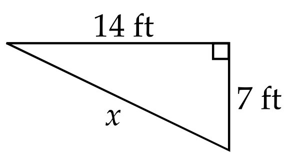
Use the **Pythagorean theorem** ($a^2 + b^2 = c^2$) *to solve each problem. Use a* **calculator** *or the* **square root table** *in* **Appendix A**. **Round answers to the nearest tenth.** *Show all your work.*

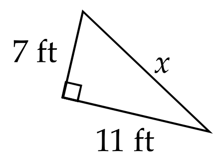
Find ***x*** *for the following.*

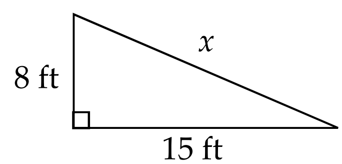
1. 

2. 

3. 

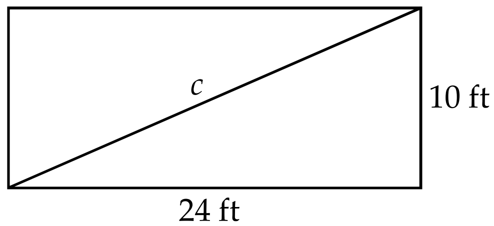
4. 

5. 

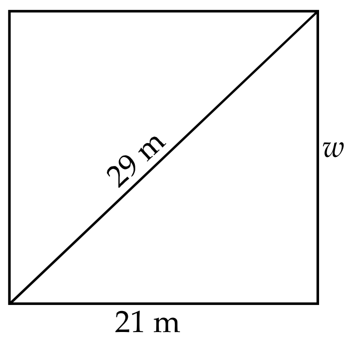
6. 

Solve the following. Show all your work.

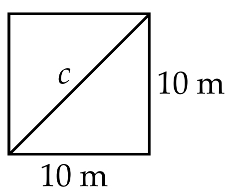
7. Find the length of the diagonal of the rectangle. ____________



8. Find the width of the rectangle. ____________



9. Find the diagonal of the square. ____________



10. Find the length of the rectangle. ____________