Problem Solving with the Pythagorean Theorem

Earlier in this unit, we used similar triangles to indirectly measure the height of a flagpole. The Pythagorean theorem is frequently used for measuring objects indirectly.

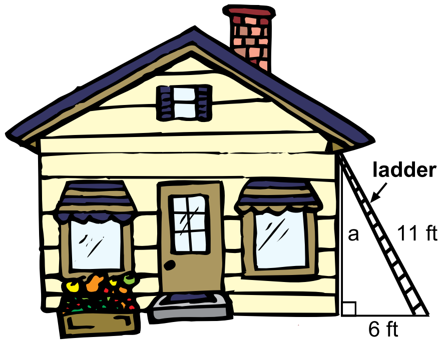
Example:

An 11-foot ladder is placed against a wall so that it reaches the top of the wall. The bottom of the ladder is 6 feet from the wall. How high is the wall? Round to the nearest whole number.

Write the Pythagorean theorem.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + 6^2 &= 11^2 \\
a^2 + 36 &= 121 \\
a^2 + 36 - 36 &= 121 - 36 \\
a^2 &= 85 \\
a &= \sqrt{85} \\
a &= 9.219544457 \\
a &\approx 9
\end{aligned}
$$

Use a drawing.



Rounding to the nearest whole number, we find that the wall is approximately equal to (≈) 9 feet high.

Practice

Use the **Pythagorean theorem** ($a^2 + b^2 = c^2$) *to solve each problem.* **Illustrate** *the problem. Use a* **calculator** *or the* **square root table** *in* **Appendix A**. **Round answers to the nearest whole number.** *Show all your work.*

1. A boat travels 8 miles west and then 15 miles south. How far is it from its starting point?

   Answer: ____________

2. The bases on a softball diamond are 60 feet apart. How long is it from home plate to second base?

   Answer: ____________

3. A square, empty lot is 40 yards on a side. If Jason walks diagonally across the lot, how far does he walk?

   Answer: ____________

4. A cable is fastened to the top of an 80-foot television tower and to a stake that is 30 feet from the base of the tower. How long is the cable?

   Answer: ____________

5. A 13-foot ladder is placed against a wall. The bottom of the ladder is 5 feet from the base of the wall. How high up the wall does the ladder reach?

   Answer: ____________

6. A 12-foot rope is fastened to the top of a flagpole. The rope is anchored at a point on the ground 6 feet from the base of the flagpole. What is the height of the flagpole?

   Answer: ____________

7. An airplane is 4 miles directly above the airport runway. A person on the ground is 10 miles from the runway. How far is the person from the airplane?

   Answer: ____________

Practice

Use the list below to write the correct term for each definition on the line provided.

formula	**Pythagorean theorem**
hypotenuse	**square units**
legs	**square (of a number)**

________________________ 1. the square of the hypotenuse (*c*) of a right triangle is equal to the sum of the squares of the legs

________________________ 2. the longest side of a right triangle; the side opposite the right angle in a right triangle

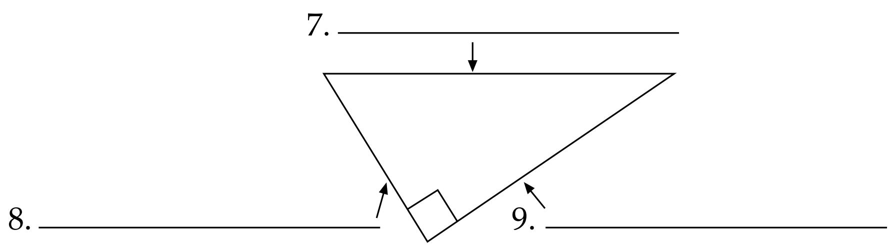
________________________ 3. in a right triangle, one of the two sides that form the right angle

________________________ 4. a way of expressing a relationship using variables or symbols that represent numbers

________________________ 5. the result when a number is multiplied by itself or used as a factor twice

________________________ 6. units for measuring area; the measure of the amount of an area that covers a surface

Label *the right triangle's* **legs** *and* **hypotenuse**.



Lesson Three Purpose

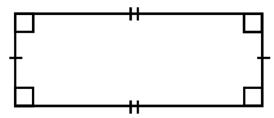
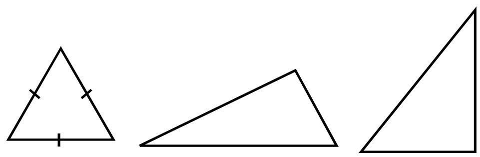
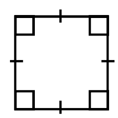
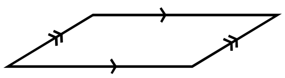
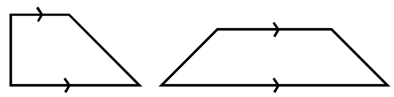
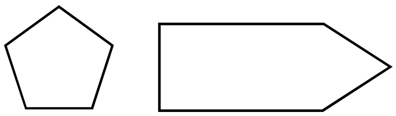
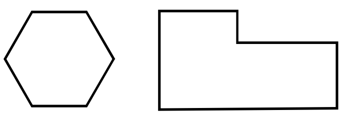
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, and absolute value. (MA.A.1.4.4)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding perimeter, area, and circumference of two-dimensional shapes. (MA.B.1.4.1)
- Solve real-world and mathematical problems, involving estimates of measurements, including length, perimeter, and area and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems including ratio and proportion. (MA.C.3.4.1)

Polygons

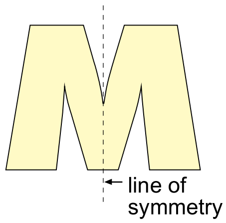
We have worked with rectangles, triangles, and squares in this unit. These shapes are considered *polygons* (closed shapes with straight sides). There are many other shapes which are included in the broad category of polygons.

This chart identifies many geometric shapes that are polygons.

Polygons

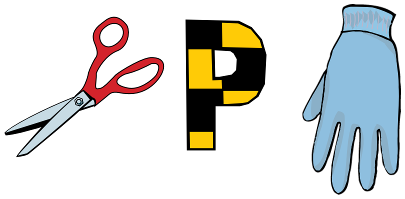
Name of Polygon	Description	Examples
rectangle	4 sides 4 right angles	
triangle	3 sides	
square	4 sides the same length 4 right angles (A square is also a rectangle.)	
parallelogram	4 sides 2 pairs of parallel sides (A rectangle is also a parallelogram.)	
trapezoid	4 sides 1 pair of parallel sides	
pentagon	5 sides	
hexagon	6 sides	

Symmetry

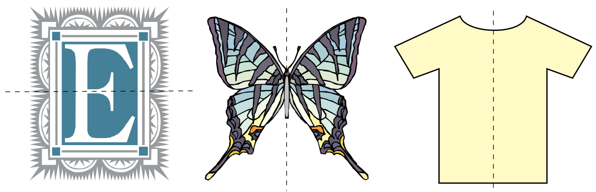


If a figure can be folded along a line so that it has two parts that are *congruent* and match exactly, that figure has *line symmetry*. Line symmetry is often just called *symmetry*. The *fold line* is called the **line of symmetry**. Sometimes more than one line of symmetry can be drawn.

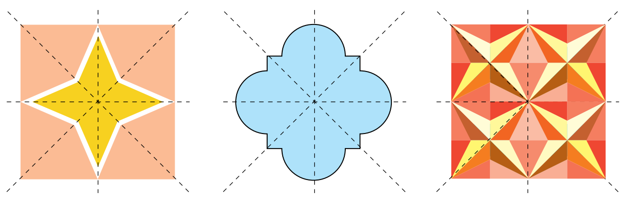
A figure can have no lines of symmetry,



one line of symmetry,

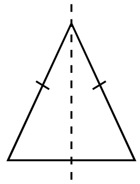


or more than one line of symmetry.

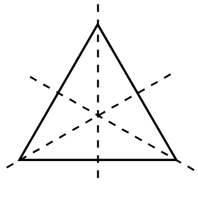


Consider the following polygons and their line(s) of symmetry.

isosceles triangle



equilateral triangle



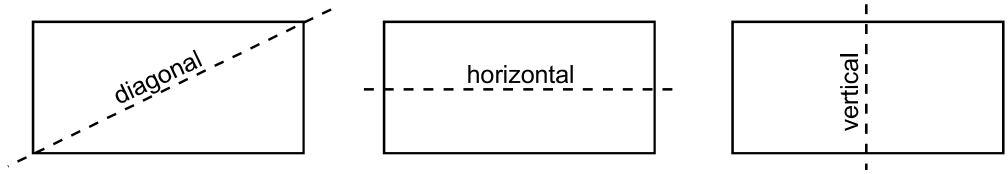
Notice three lines of symmetry.

Try this.

Which of these three dashed lines on the rectangles below create a line of symmetry?

- Get a sheet of paper and cut out three rectangles that are not squares.
- Make one fold on each rectangle as indicated by the dashed lines on the figures below.

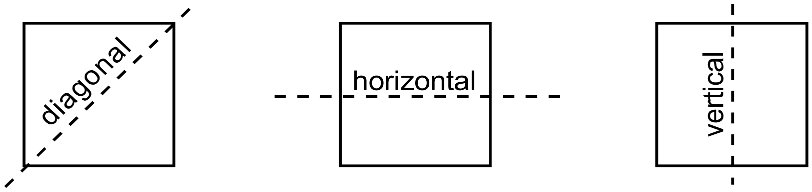
rectangle



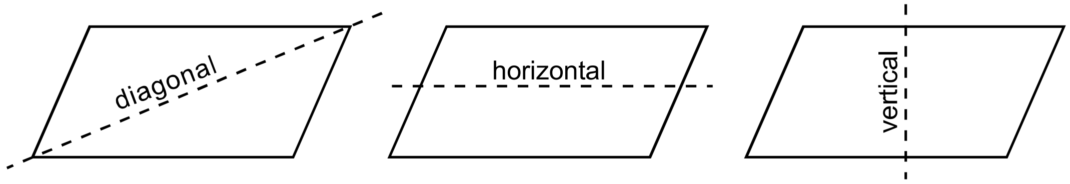
In each case, does one half of the folded paper fit exactly over the other half?

- Is the diagonal a line of symmetry? No.
- Do the horizontal and vertical lines create lines of symmetry? Yes.

If you had used a square, would your results be different? Yes, all three would result in lines of symmetry.



What if you used a parallelogram? Could you fold the parallelogram so that one half fits exactly over the other? No.

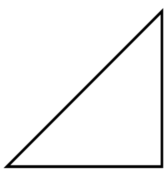


Practice

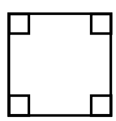
Use the chart on **polygons** *on page 462 and the list below to* **name** *each* **shape**. **One or more terms will be used more than once.**

hexagon	**rectangle**	**trapezoid**
parallelogram	**square**	**triangle**
pentagon		

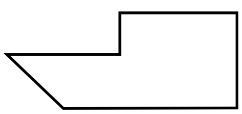
1. ______________________



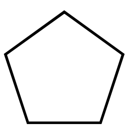
2. ______________________



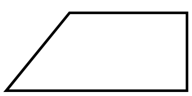
3. ______________________



4. ______________________



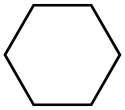
5. ______________________



6. ______________________

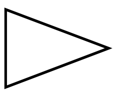
7. ______________________

8. ______________________



9. ______________________

10. ______________________



Answer the following.

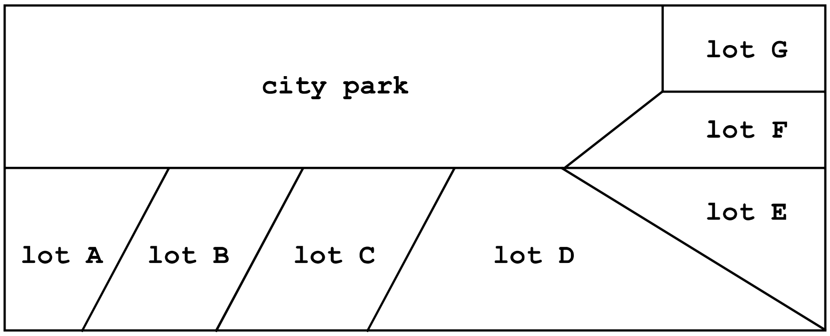
11. Which of the 10 shapes on the previous page can you draw one or more lines of symmetry? ________________________________

____

12. Select four polygons and draw their lines of symmetry.

Use the **map of a city block** *below to answer the following.*

Map of City Block



13. Which lot is a triangle? ________________________________

14. Which lot is a rectangle? ________________________________

15. Which lots are parallelograms? ________________________________

____

16. Which lots are squares? ________________________________

17. Which lots are trapezoids? ________________________________

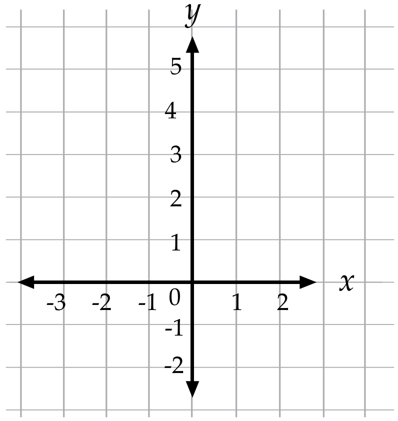
____

18. Which part of the city block is a pentagon? ____________________

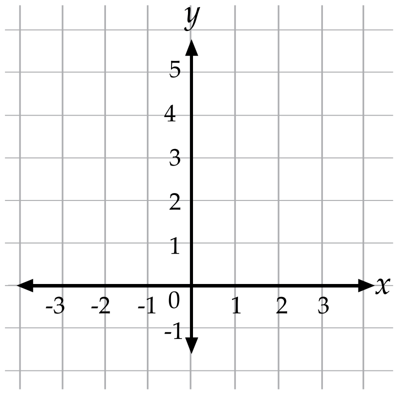
19. What is the shape of the city block? __________________________

Use the **coordinate grid** *below each problem to* **plot the ordered pairs** *listed.* **Connect the points** *as you plot them. When the shape is complete,* **draw** *any appropriate* **lines of symmetry**.

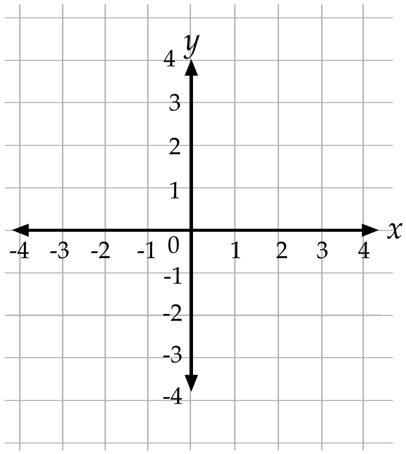
20. (-2, 1), (1, 1), (1, 4), (-2, 4), (-2, 1)



21. (3, 0), (0, 5), (-3, 0), (3, 0)



22. (-3, -1), (-3, 1), (-1, 1), (-1, 3), (1, 3), (1, 1), (3, 1), (3, -1), (1, -1), (1, -3), (-1, -3), (-1, -1), (-3, -1)



Use the **alphabet** *printed below to draw* **lines of symmetry** *through as many letters as possible.*

23. Which letters do *not* have symmetry?

____

A B C D E F G

H I J K L M N

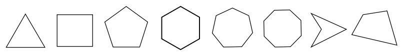
O P Q R S T U

V W X Y Z

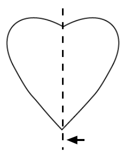
Practice

Match each definition with the correct term. Write the letter on the line provided.

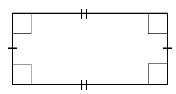
______ 1. a closed plane figure whose sides are straight and do not cross



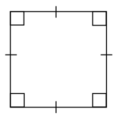
______ 2. a line that divides a figure into two congruent halves that are mirror images of each other



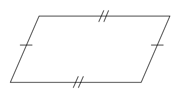
______ 3. a parallelogram with four right angles



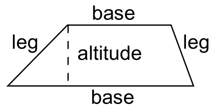
______ 4. a rectangle with four sides the same length



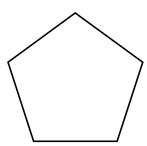
______ 5. a quadrilateral with two pairs of parallel sides



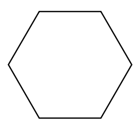
______ 6. a quadrilateral with just one pair of opposite sides parallel



______ 7. a polygon with five sides



______ 8. a polygon with six sides



A. hexagon

B. line of symmetry

C. parallelogram

D. pentagon

E. polygon

F. rectangle

G. square

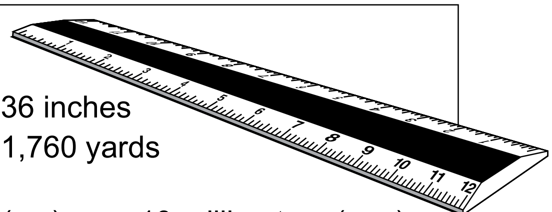
H. trapezoid

Perimeter of Polygons

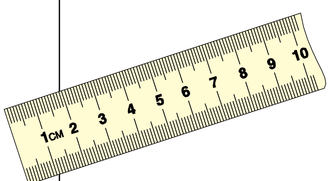
To compute the **perimeter** (*P*) of a polygon we determine the distance around the polygon. Lengths of **sides** and various formulas are used. *Sides* are the edges of **two-dimensional** geometric figures. *Two-dimensional* figures have two dimensions: length (*l*) and width (*w*).

A brief review of conversions between metric units of length and U.S. customary measures of length may be helpful.

Common Conversion Factors



1 foot (ft)	=	12 inches (in.)		
1 yard (yd)	=	3 feet	=	36 inches
1 mile (mi)	=	5,280 feet	=	1,760 yards



1 centimeter (cm)	=	10 millimeters (mm)
1 meter (m)	=	100 centimeters
1 meter	=	1000 millimeters
1 kilometer (km)	=	1000 meters

Note: Appendix C has a list of conversions.

When we change from a *large* unit to a *smaller* unit, we *multiply* by the conversion factor.

	larger to smaller	conversion factor	multiply
a.	2 yd = ? ft	1 yd = 3 ft	2 x 3
b.	5 ft = ? in.	1 ft = 12 in.	5 x 12
c.	2 m = ? cm	1 m = 100 cm	2 x 100
d.	3 km = ? m	1 km = 1000 m	3 x 1000

Answers: (a) 6 ft (b) 60 in. (c) 200 cm (d) 3000 m

When we change from a *small* to a *larger* unit, we *divide* by the conversion factor.

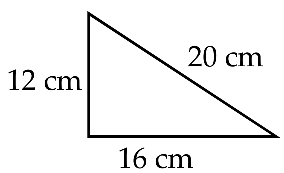
	smaller to larger	conversion factor	divide
e.	300 cm = ? m	100 cm = 1 m	300 by 100
f.	45 mm = ? cm	10 mm = 1 cm	45 by 10
g.	25000 m = ? km	1000 m = 1 km	25000 by 1000

Answers: (e) 3 m (f) 4.5 cm (g) 25 m

Formulas are used to find perimeter of triangles, squares, and rectangles. For the polygons in this unit, such as trapezoids and pentagons, we simply add the lengths of all the sides (s) to get perimeter (P).

Examples **Formulas**

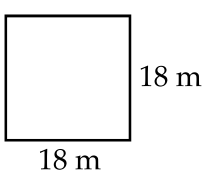
triangle



Perimeter = the sum of the lengths of the three sides—a, b, and c.

$P = a + b + c$
$P = 12 \text{ cm} + 16 \text{ cm} + 20 \text{ cm}$
$P = 48 \text{ cm}$
The perimeter is 48 centimeters.

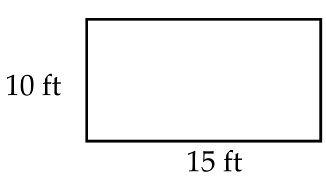
square



Perimeter = 4 times the length (l) of one side (s = side length).

$P = 4s$
$P = 4(18 \text{ m})$
$P = 72 \text{ m}$
The perimeter is 72 meters.

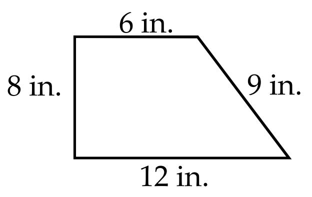
rectangle



Perimeter = 2 times the length (l) plus 2 times the width (w).

$P = 2l + 2w$
$P = 2(15 \text{ ft}) + 2(10 \text{ ft})$
$P = 30 \text{ ft} + 20 \text{ ft}$
$P = 50 \text{ ft}$
The perimeter is 50 feet.

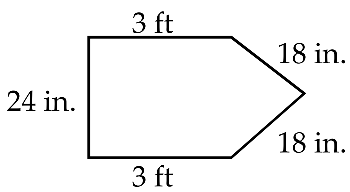
trapeziod



Perimeter = the sum of the lengths of all the sides.

$P = s_1 + s_2 + s_3 + s_4$
$P = 8 \text{ in.} + 6 \text{ in.} + 9 \text{ in.} + 12 \text{ in.}$
$P = 35 \text{ in.}$
The perimeter is 35 inches.

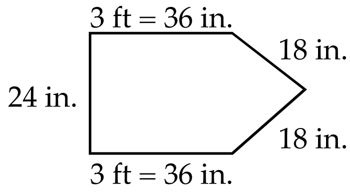
pentagon



Perimeter = the sum of the lengths of all the sides.
$P = s_1 + s_2 + s_3 + s_4 + s_5$

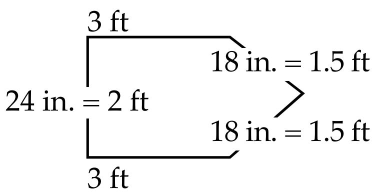
Alert: The units are *not* the same. Some are expressed in feet and others are expressed in inches. We *must* change feet to inches *or* inches to feet.

Example: Change *feet to inches*.



$P = 36 \text{ in.} + 18 \text{ in.} + 18 \text{ in.} + 36 \text{ in.} + 24 \text{ in.}$
$P = 132 \text{ in.}$
The perimeter is 132 inches.

Example: Change *inches to feet*.

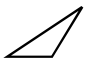
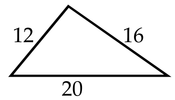
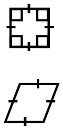
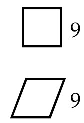
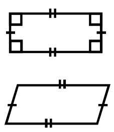
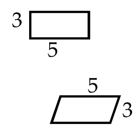
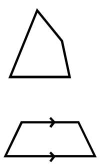
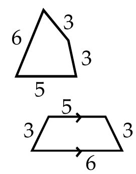
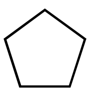
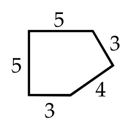
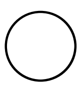
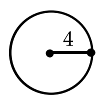


$P = 3 \text{ ft} + 1.5 \text{ ft} + 1.5 \text{ ft} + 3 \text{ ft} + 2 \text{ ft}$
$P = 11 \text{ ft}$
The perimeter is 11 feet.

So the perimeter of the pentagon above can be expressed as 132 inches *or* 11 feet. Either answer is correct.

Let's Summarize:

Perimeter (P) is the distance around a figure.

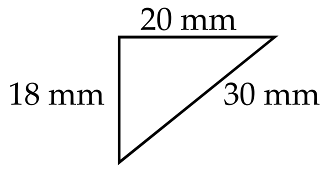
Mathematical Formulas for Perimeter (P)		
figure	**formula**	**example**
 **triangle**	$P = a + b + c$	 $P = 12 + 16 + 20$ $P = 48$ units
 **square** and **rhombus**	$P = 4s$	 $P = 4(9)$ $P = 36$ units
 **rectangle** and **parallelogram**	$P = 2l + 2w$	 $P = 2(5) + 2(3)$ $P = 16$ units
 **quadrilateral** and **trapezoid**	$P = s_1 + s_2 + s_3 + s_4$	 $P = 5 + 3 + 3 + 6$ $P = 17$ units
 **pentagon**	$P = s_1 + s_2 + s_3 + s_4 + s_5$	 $P = 5 + 5 + 3 + 4 + 3$ $P = 20$ units
 **circle**	$C = \pi d$ or $C = 2\pi r$	 $C \approx 2(3.14)(4)$ $C \approx 25.12$ units

Key				
C = circumference	d = diameter	l = length	P = perimeter	r = radius
π = pi Use 3.14 or $\frac{22}{7}$ for π.		s = side	w = width	

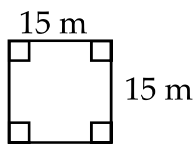
Practice

Find the **perimeter** *of each* **polygon**. *Refer to the conversion chart on page 470 if units are* not *the same.*

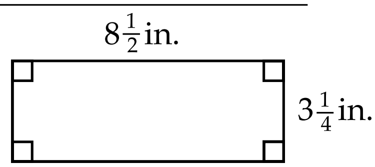
1. ______________________



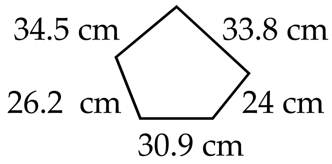
2. ______________________



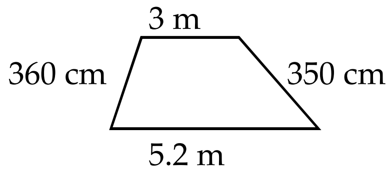
3. ______________________



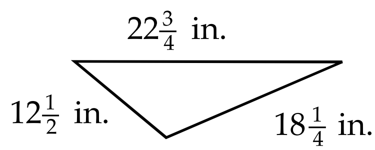
4. ______________________



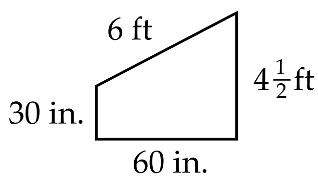
5. ______________________



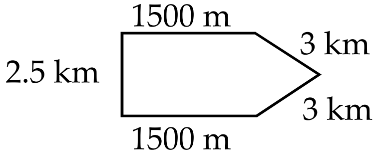
6. ______________________



7. ______________________



8. ______________________



Use the **appropriate formula** *to find the* **missing measure**. *Refer to* **formulas** *in unit as needed.*

9. square
   $s = 8$ m

   $P =$ ____________

11. rectangle
    $l = 3.5$ m
    $w = 1.8$ m

    $P =$ ____________

10. rectangle
    $l = 12$ cm
    $w = 6$ cm

    $P =$ ____________

12. triangle
    $a = 13$ m
    $b = 22$ m
    $c = 12$ m

    $P =$ ____________

---

13. rectangle
    $l = 9$ km

    $w =$ ____________
    $P = 26$ km

    Hint: Use $P = 2(\text{length}) + 2(\text{width})$ and solve for w.
    $26 = 2(9) + 2w$

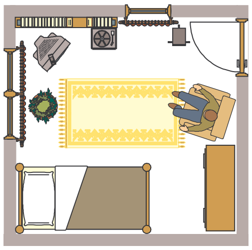
14. triangle
    $a = 250$ in.
    $b = 168$ in.

    $c =$ ____________
    $P = 562$ in.

    Hint: Use $P = a + b + c$ and solve for c.

15. The perimeter of a square room is 50 meters.

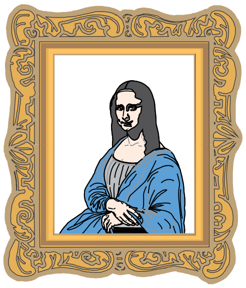
What is the length of each side of the room? __________________



$P = 50$ m

16. How many centimeters of framing are needed to frame a painting

that is 70 centimeters long and 55 centimeters wide? ___________



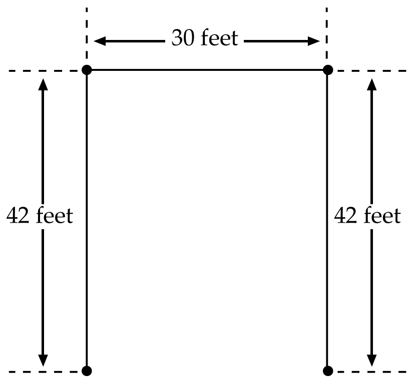
17. The perimeter of a triangular yard is 240 meters. The longest side is 110 meters and the shortest side is 50 meters.

How long is the third side? _________________________________

Answer the following.

18. Green Thumb Yard Design will plant viburnum shrubs in rows across the back and down two sides of the yard sketched below. At each of the 4 corners, a shrub will be planted. All shrubs will be planted 3 feet apart.

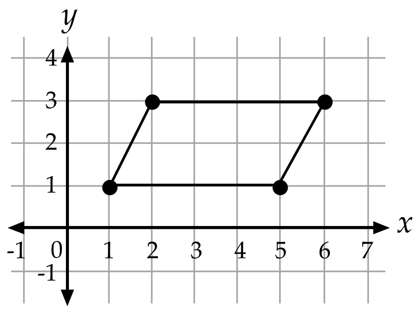
    How many shrubs need to be planted? ______________________



Finding Areas of Special Shapes

Finding the Area of Parallelograms

Remember, a parallelogram is a 4-sided figure with opposite sides parallel. Let's investigate how to find its area (A). *Area* is the inside region of a two-dimensional figure and is measured in *square units*. Let's take a piece of graph paper and plot these points (1, 1), (5, 1), (6, 3), and (2, 3). We will connect them with line segments so that a parallelogram is formed.

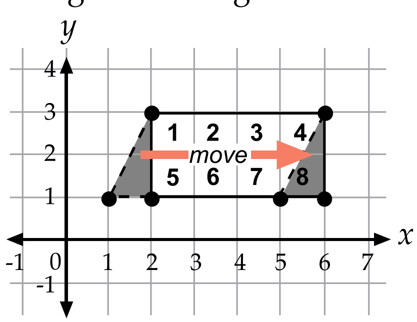


What do you think the area is of the parallelogram above?

Remember: Area is measured by the number of squares it takes to cover the region.

Let's look below and find the area.

In this case, there are 6 full squares, and then bits and pieces of other squares. What if we cut off the triangle formed by (1, 1), (2, 1), and (2, 3) and move it and place it on top of the triangle formed by (5, 1), (6, 1), and (6, 3). Notice the two triangles are congruent.



We now have a rectangle which has the same area as the parallelogram, and there are clearly 8 squares. The area of the parallelogram is 8 square units.

Remember: To find the area of a rectangle we must use the following formula and multiply the length times the width

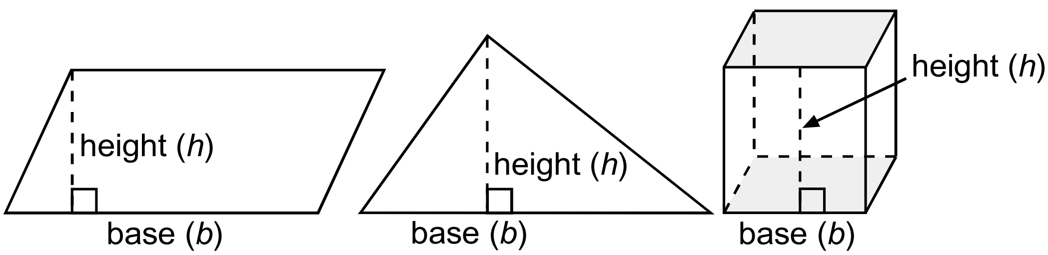
$A = lw$.

Any parallelogram can be *rearranged* to form a rectangle. Therefore, the formula for area of a parallelogram is closely related to the formula for area of a rectangle.

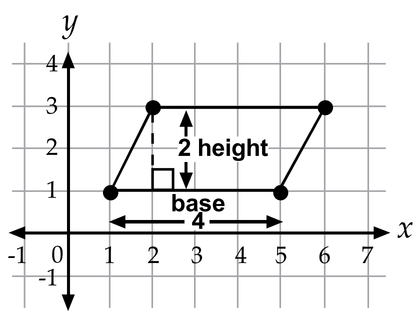
To find the area of a parallelogram we must multiply the **base (*b*)** times the **height (*h*)**.

$$A = bh$$

Let's make sure that we understand that the *base* (b) can be any side. The base is only the line or plane upon which a figure is thought of as resting. The *height (h)* is a little trickier. The height is the length of an **altitude**, a line segment perpendicular to the base (forming a right angle), drawn from the *opposite* side.

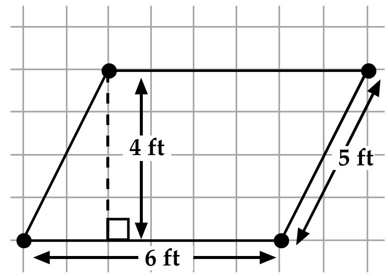


Look at the original parallelogram below. Can you see that the base is 4 units and the height is 2 units? Therefore the area is 8 square units.



Example:

Find the area of the parallelogram below.



Solution:

$A = bh$
$A = (6)(4)$
$A = 24$ square feet

Note: The 5 was *not* used to find the area of the parallelogram because only the base and height are needed.

What is the perimeter of this parallelogram?

Remember: The perimeter of a polygon is the sum of all the sides.

Solution:

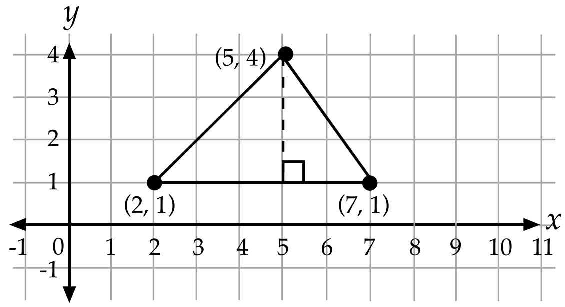
$P = s_1 + s_2 + s_3 + s_4$
$P =$ 6 ft + 5 ft + 6 ft + 5 ft
$P =$ 22 ft
$P =$ 22

Note: The 4 is *not* used to find the perimeter of the parallelogram because only the sides are needed.

Finding the Area of Triangles

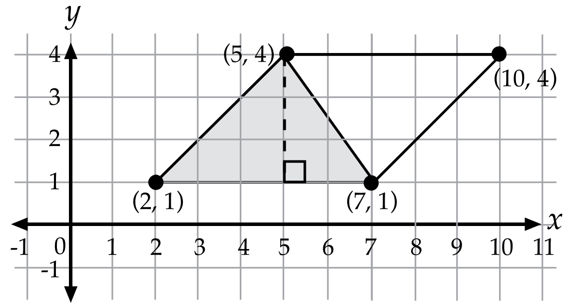
Challenge:

Plot these 3 points: (2, 1), (7, 1), and (5, 4) Connect them to make a triangle. Again it is difficult to be sure about the area.



Can you plot a 4^{th} point to make it a parallelogram instead of a triangle?

Yes. The point is (10, 4).



We know that to find the area of the parallelogram we will use the following formula.

$A = bh$
$A = (5)(3)$
$A = 15$ square units

What part of the parallelogram is the triangle? If you guessed $\frac{1}{2}$, you are correct.

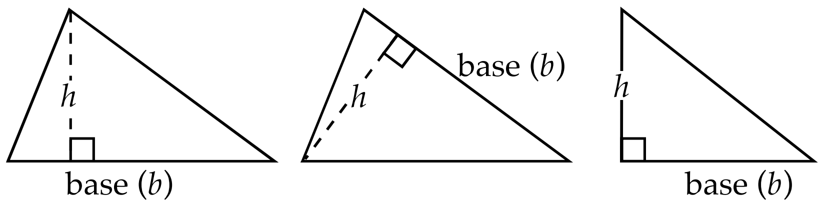
The area of our triangle = $\frac{1}{2}$(area of our parallelogram) = $\frac{1}{2}(15) = 7\frac{1}{2}$ square units.

To find the area of a triangle, we use the following formula:

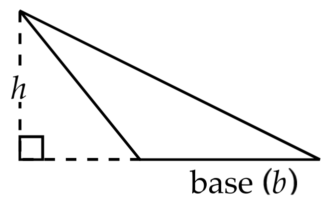
$$A = \frac{1}{2}bh$$

Remember: Any side of the triangle can be used as a base. The height is the length of the line segment draw from the *opposite* vertex perpendicular to the base (forming a right angle).

Study the following pictures.

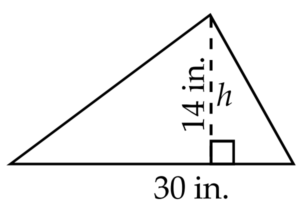


Note: Sometimes the height falls outside the triangle.



Example:

Find the area of the triangle below.



There are a few ways to do this problem.

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(30)(14)$
$A = (15)(14)$
$A = 210$ square units

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(30)(14)$
$A = \frac{1}{2}(420)$
$A = 210$ square units

$A = \frac{1}{2}bh$
$A = (0.5)(30)(14)$
$A = 210$ square units

Hint:
$\frac{1}{2} = 0.5$

Remember: The **associative property** lets us group *without* changing the order. The **commutative property** lets us change the order.

Commutative Property and Associative Properties Revisited

You can add or multiply two numbers in any *order* without affecting the result. (See the **Rules for Order of Operations** chart in Unit 1.)

Addition	**Multiplication**
$a + b = b + a$	$a \bullet b = b \bullet a$
$3 + 7 = 7 + 3$	$3 \bullet 7 = 7 \bullet 3$

Think of the *commutative property* of addition and multiplication as a "sequence" or *order* making *no* difference in the outcome. For example, when getting dressed, you could put your shoes on *before* your belt.

However, the commutative property does *not* apply to subtraction and division.

$6 - 3 \neq 3 - 6$ $\qquad$ $6 \div 3 \neq 3 \div 6$

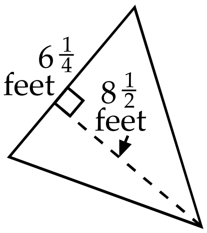
In other words, in this case, order *does* matter. You would *not* put your socks on *after* your shoes were already on your feet.

The *associative property* lets us add or multiply three or more numbers by grouping.

addition	multiplication
$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$

Example:

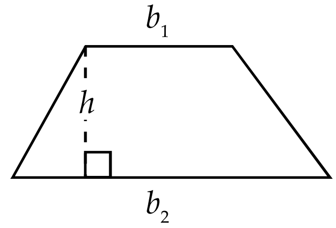
Find the area of the triangle below.



$$
\begin{aligned}
A &= \tfrac{1}{2}bh \\
&= \tfrac{1}{2}(6\tfrac{1}{4})(8\tfrac{1}{2}) \\
&= \tfrac{425}{16} \\
&= 26.6 \text{ square feet}
\end{aligned}
$$

(rounded to nearest tenth)

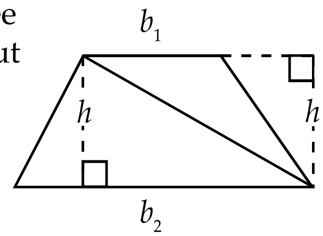
How to Find the Area of a Trapezoid



Remember, a trapezoid is a four-sided figure with exactly two parallel sides, called *bases*. Since the bases aren't the same length, we denote them by using subscripts: b_1 (b sub one) and b_2 (b sub 2). **Note: These are not exponents!**

The height of a trapezoid is the length of a line segment drawn from one base perpendicular to the other base.

If we draw a diagonal from opposite corners, we see that we have two triangles with the same height, but different bases. The area of the trapezoid would be the sum of the areas of the two triangles.



$$\text{area of a trapezoid} = \tfrac{1}{2}b_1h + \tfrac{1}{2}b_2h$$

The **distributive property** would allow us to write the formula this way:

$$\text{area of a trapezoid} = \tfrac{1}{2} \bullet h(b_1 + b_2)$$

Distributive Property Revisited

You can *distribute* the numbers to write an *equivalent* or equal expression.

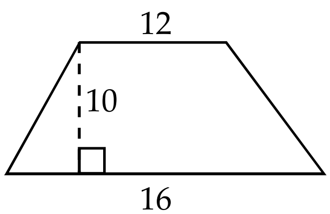
For all numbers a, b, and c:

$$a(b + c) = ab + bc$$
$$3(7 + 4) = 3 \bullet 7 + 3 \bullet 4$$

Think of the distributive property of multiplication as "spreading" things out or *distributing* things out to make the problem easier to work with—yet making *no* difference in the outcome.

Example:

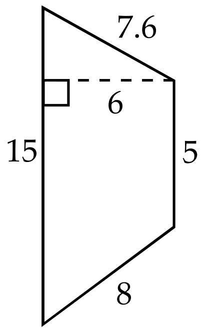
Find the area of the trapezoid below.



$$A = \tfrac{1}{2} \bullet h(b_1 + b_2)$$
$$A = \tfrac{1}{2}(10)(12 + 16)$$
$$A = \tfrac{1}{2}(10)(28)$$
$$A = 140 \text{ square units}$$

Example:

Find the area of the trapezoid below.

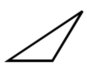
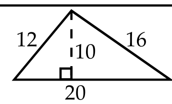
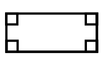
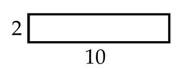
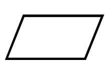
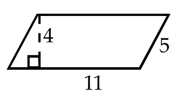
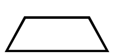
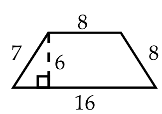


$$
\begin{aligned}
A &= \tfrac{1}{2} \bullet h(b_1 + b_2)\\
A &= \tfrac{1}{2}(6)(5 + 15)\\
A &= \tfrac{1}{2}(6)(20)\\
A &= \tfrac{1}{2}(120)\\
A &= 60 \text{ square units}
\end{aligned}
$$

Note: The measures of the other two sides do *not* matter!

Let's Summarize:

Area (A) is the number of square units a two-dimensional figure contains. Area is measured in square units.

Mathematical Formulas for Area (A)			
figure	**formula**	**example**	
 **triangle**	$A = \frac{1}{2}bh$		$A = \frac{1}{2}(20)(10)$ A = 100 square units
 **rectangle**	$A = lw$		A = (2)(10) A = 20 square units
 **parallelogram**	$A = bh$		A = 11(4) A = 44 square units
 **trapezoid**	$A = \frac{1}{2}h(b_1 + b_2)$		$A = \frac{1}{2}(6)(8 + 16)$ A = 72 square units

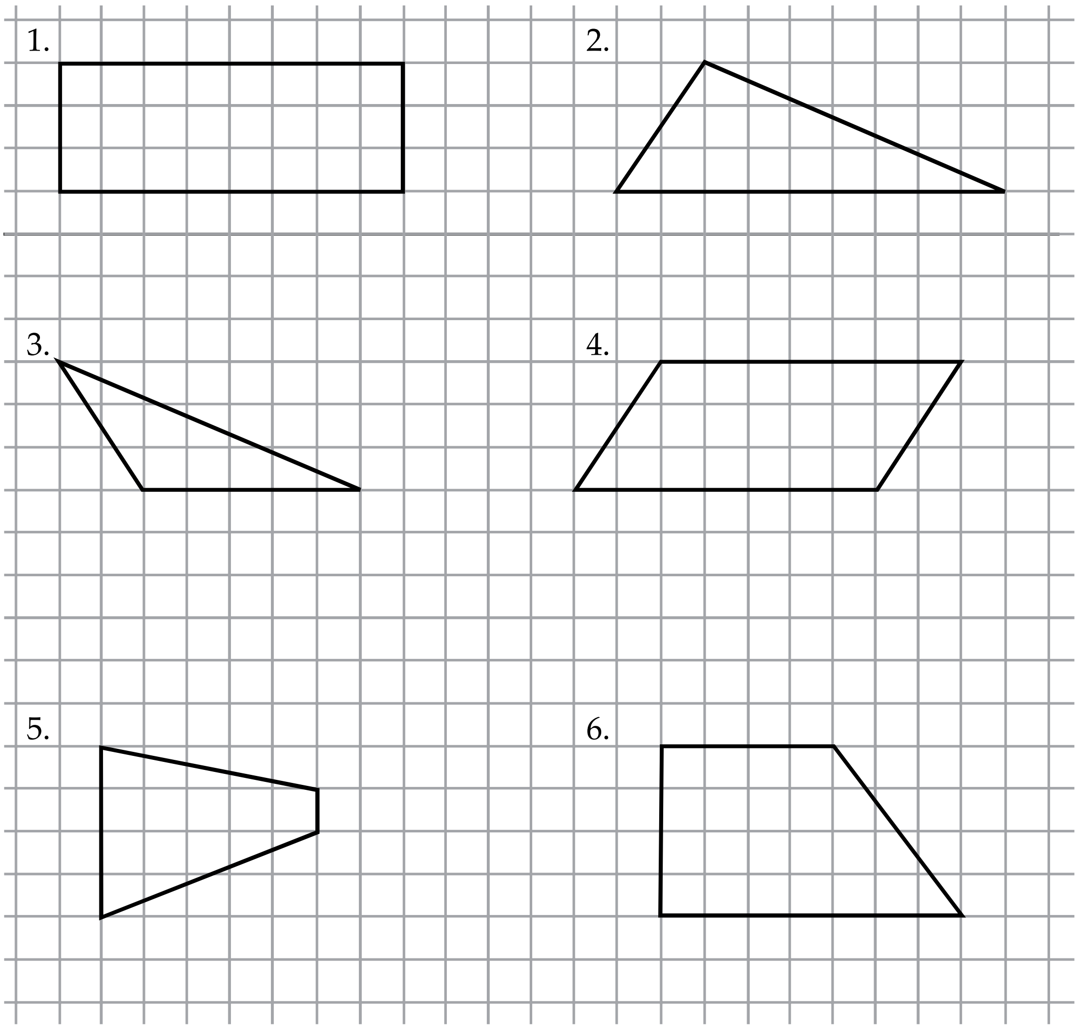
Key				
A = area	b = base	h = height	l = length	w = width

Note: Appendix C contains a list of formulas.

Practice

Find the **area** *of the following figures using the grid below.*

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________



Find the **area** *of the following. Refer to the* **formulas** *in this unit or in* **Appendix C** *as needed.*

7. A 4-inch square. ____________________

8. A rectangle with a length of $6\frac{1}{2}$ inches and a width of $3\frac{1}{4}$ inches.

   ____

9. A parallelogram with a base of $3\frac{1}{4}$ feet and a height of 4 feet.

   ____

10. A triangle with a base of 10 yards and a height of 7 yards.

    ____

11. A trapezoid with bases of $10\frac{1}{2}$ inches and $12\frac{1}{2}$ inches and a height of 4 inches. ____________________

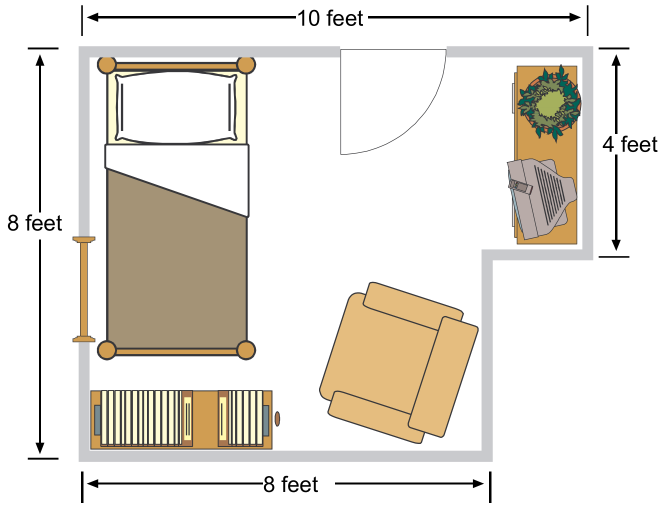
12. A trapezoid with bases of 0.3 kilometers and 0.5 kilometers and a height of 0.2 kilometers. ____________________

Practice

Answer the following. Refer to **formulas** *in unit or in* **Appendix C** *as needed.*

1. Here is a blueprint of Jamie's bedroom. She would like to carpet this room.
   (*Hint*: Can you divide this space into two rectangles?)

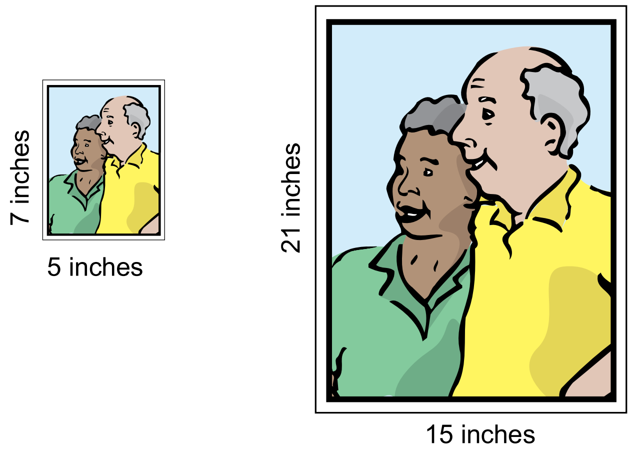
   Find the area. ____



She would also like to put a wallpaper border around the top of the room.

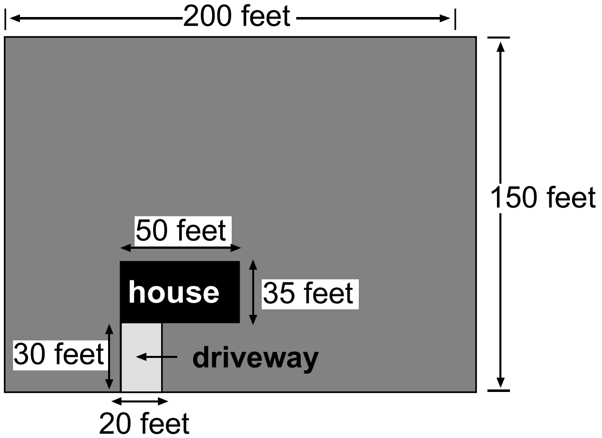
Find the perimeter. ___________________________________

2. Latasha is having some pictures enlarged at a local copy store. She wants to enlarge a photo that is 5 inches by 7 inches so that the dimensions will be 15 inches by 21 inches.



How many times larger that the original photo will the area of the new photo be? ____

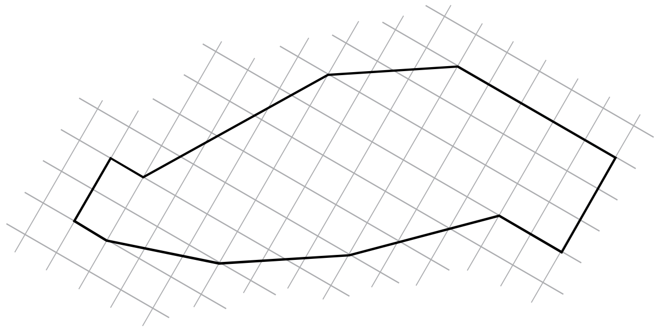
3. Tom wants to reseed his yard. A diagram of his lot is shown below. (**Hint:** Tom will not seed the house nor the driveway.)



How many square feet would he need to reseed his yard?

____

4. Below is an irregular shape. Find the area by dividing the shape into 5 smaller regions. ____________________

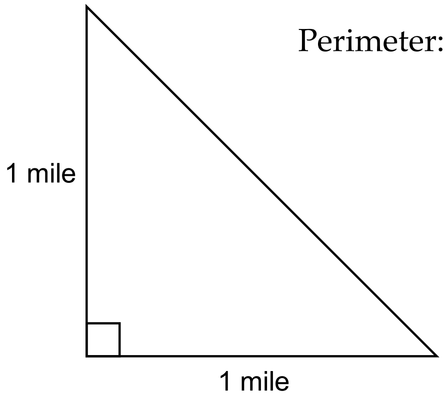


5. Find the area and perimeter of this right triangle. Use the Pythagorean theorem ($a^2 + b^2 = c^2$) and the square root table in Appendix A.

Round each answer to the nearest tenth.

Area: ____________________

Perimeter: ____________________



Practice

Use the list below to complete the following statements.

altitude	**commutative**	**perimeter (*P*)**
area (*A*)	**property**	**sides (*s*)**
associative property	**distributive property**	**two-dimensional**
base (*b*)	**height (*h*)**	**width (*w*)**

1. The ________________________ is the sum of the sides.

2. ________________________ are the edges of two-dimensional figures.

3. Figures that have length and width are ________________________ figures.

4. ________________________ is measured by the number of squares it takes to cover the region.

5. To find the area of a rectangle we must multiply the length times the ________________________ .

6. The ________________________ of a shape can be any side—it is the line or plane upon which a figure is thought to be resting.

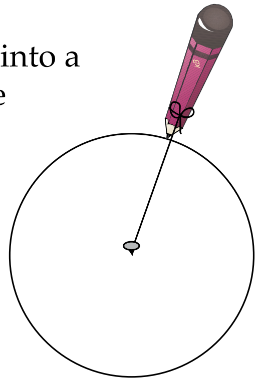
7. The ________________________ is the length of a(n) ________________________ , a line segment perpendicular to the base, drawn from the *opposite* side.

8. The ____________________ lets us change the order without affecting the result.

9. Think of the ____________________ of multiplication as "spreading" things out to make it easier to work with—yet making *no* difference in the outcome.

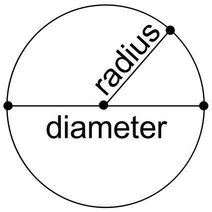
10. The ____________________ lets us group without changing the order and without affecting the result.

Circles



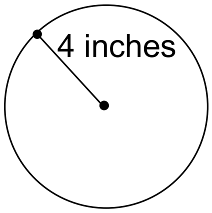
Suppose that you tie a string to a thumbtack and push it into a piece of cardboard. Next you tie a marker or pencil to the other end of the string and move the string around the thumbtack. The curved line drawn by the marker or pencil is a **circle**. The thumbtack is the **center of the circle**. The *center of the circle* is the point from which all the points on the circle are the same distance.

A *circle* is the set of all points that are the same distance from the center. The distance of any line segment drawn from the center to any point on the circle is called a **radius (*r*)**. A **diameter (*d*)** of a circle is the distance of a line segment from one point of the circle to another, through the center.



The *diameter* of a circle is *twice* its radius. $d = 2r$

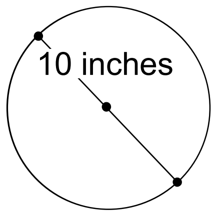
Example:



If the radius of this circle measures 4 inches, what is the diameter?

Solution: $d = 2r$
$d = 2(4)$
$d = 8$ inches

Example:

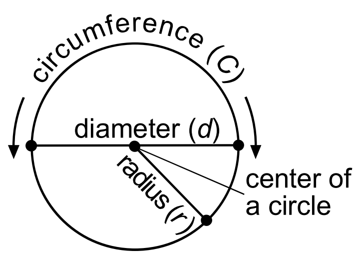


If the diameter of this circle measures 10 inches, what is the radius?

$$\begin{aligned} \textit{Solution}\text{:}\quad d &= 2r \\ 10 &= 2r \\ 5 &= \mathrm{r} \end{aligned}$$

Circles have their own name for perimeter. The perimeter of a circle is called the **circumference (*C*)**. The *circumference* of a circle is the distance around the circle. The diameter is twice, or two times, the radius.

Circles



circumference (*C*) - the distance around a circle or the perimeter (*P*) of a circle

diameter (*d*) - a line segment that passes through the center of a circle to another point on a circle

center of a circle - the point from which all points on a circle are the same distance

radius (*r*) - any line segment from the center of a circle to a point on a circle

Practice

Complete the chart below.

A class measured the **diameters** and **circumferences** of several round objects. Here are their results.

- Find four more circular objects and the **circumference** and **diameter** of each.
- **Divide the circumference by the diameter**. Use a calculator and round each answer to the nearest hundredth.

Measurements of Round Objects

Item	Diameter	Circumference	Circumference Divided by Diameter
cookie tin	$5\frac{3}{4}$ inches	18 inches	3.13
auto tire	29 inches	91 inches	____
plate	9 inches	$28\frac{1}{4}$ inches	____
pail	10 inches	31.5 inches	____
____	____	____	____
____	____	____	____
____	____	____	____
____	____	____	____

Circumference

Centuries ago mathematicians found that the answer is always the same when a circumference is divided by its diameter. Your answers on the last practice were probably very close to 3.14.

$$\frac{C}{d} \approx 3.14$$

If we multiply both sides of this equation by d, we get the formula for finding the circumference of a circle.

$$C \approx 3.14d$$

The number 3.14 is represented by the Greek letter π, which is pronounced **pi**.

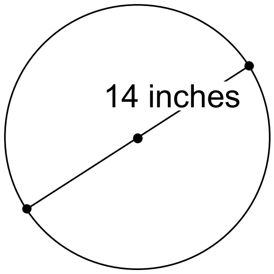
To find the circumference of a circle use the following formula:	
$C = \pi d$	Since $d = 2r$, we can also use $C = 2\pi r$.
$C = 2\pi r$	The commutative property allows us to rearrange the letters.

If you look on a scientific calculator, you will find a key for π. If you press it you will probably get 3.141592654.

π is neither 3.14 nor the long number which the calculator gave us. One of the great challenges faced by mathematicians through the ages was trying to find an *exact value* for π. However, there isn't one! The value of π is an *unending* decimal. In this section we will use 3.14 for π. Our answers will be *approximate* (≈)! Be aware that the fraction $\frac{22}{7}$ is also used for π.

Example:

Find the circumference of a circle whose *diameter* is 14 inches.



Solution: Since we know the diameter we will use this formula.

$C = \pi d$	
$C = \pi(14)$ or	
$C = 14\pi$	some people stop here
$C \approx 14(3.14)$	
$C \approx 43.96$ inches	The circumference is about 43.96 inches.

Example:

Find the circumference of a circle whose *radius* is 14 inches.

Solution: Since we know the radius we will use this formula.

$C = 2\pi r$	
$C \approx 2(3.14)(14)$	multiply in any order
$C \approx 87.92$ inches	The circumference is about 87.92 inches.

Example:

Jim bought a new bicycle. The wheel is 26 inches in diameter. How many inches will Jim go in just 1 revolution or turn of the bicycle tire?

Solution:

$C = \pi d$	26 is the diameter of the bicycle wheel.
$C \approx (3.14)(26)$	
$C \approx 81.64$ inches	

To find the area of circle, use the following formula: $A = \pi r^2$

Example:

Find the area of a circle whose *radius* is 10 inches.

Solution:

$A = \pi r^2$

$A \approx (3.14)(10)^2$

$A \approx (3.14)(100)$

$A \approx 314$ square inches

Remember: Order of operations—square first

Example:

Find the area of a circle whose *diameter* is 10 inches.

Solution: The formula for area of a circle uses the radius, so we must first find the radius using what we know about the diameter.

$d = 2r$

$10 = 2r$

$5 = r$

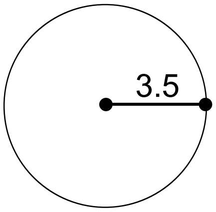
$A = \pi r^2$

$A \approx (3.14)(5)^2$ remember to square first

$A \approx (3.14)(25)$

$A \approx 78.5$ square inches

Let's summarize how to do various problems using this circle.



Mathematical Formulas Diameter (*d*), Circumference (*C*), and Area (*A*)			
measurement	**formula**	**explanation**	**example**
diameter	$d = 2r$	*diameter* is twice the radius	$d = 2(3.5)$ $d = 7$
circumference	$C = \pi d$ *or* $C = 2\pi r$	*circumference* is distance around	$C \approx 2(\frac{22}{7} \cdot \frac{7}{2})$ *or* $2(3.14 \cdot 3.5)$ $C \approx 22$ square units
area	$A = \pi r^2$	*area* is squares on the inside	$A \approx (\frac{22}{7})(\frac{7}{2})^2$ *or* $(3.14)(3.5)^2$ $A \approx 38\frac{1}{2}$ square units

Key			
A = area	C = circumference	d = diameter	r = radius
π = pi Use 3.14 or $\frac{22}{7}$ for π.		$\approx$ = approximately equal to (e.g., 3.14 is an approximation for pi.)	

Note: Appendix B has a list of mathematical symbols and their meanings and Appendix C has a list of formulas.

Practice

Fill in the chart below. **Round answers to the nearest hundredth.** *Refer to* **formulas** *in unit or in* **Appendix C** *as needed.*

Measurements

	Diameter	Circumference
1.	6 meters	________
2.	15 centimeters	________
3.	6.8 inches	________
	Radius	**Circumference**
4.	21 millimeters	________
5.	6 inches	________
6.	48 centimeters	________
	Radius	**Area**
7.	24 inches	________
8.	$3\frac{1}{2}$ yards	________
9.	$5\frac{1}{4}$ feet	________
	Diameter	**Area**
10.	28 inches	________
11.	72 yards	________
12.	126 inches	________

✓ **Check yourself:** Use the list of **scrambled answers** below and check your answers to problems 1, 4, 7, and 10 above

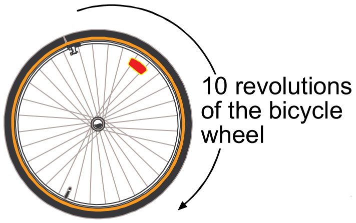
1,808.64 **131.88** **615.44** **18.84**

Practice

Answer the following. *Refer to the* **formulas** *in the unit or in* **Appendix C** *as needed.*

1. If the diameter of a circular flower bed is 28 feet, what is the circumference of the flower bed?

2. Each spoke in a wheel is 63 centimeters from the center to the outside of the rim. How long is each full rotation of the wheel? How far will this wheel move in 10 revolutions?



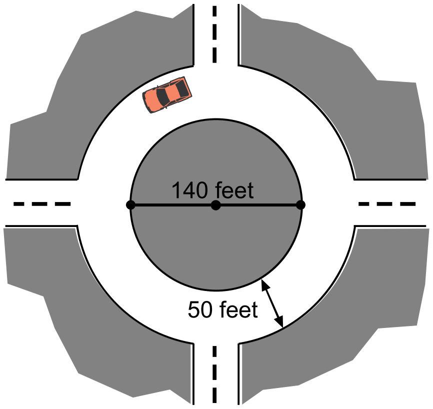
3. The diameter of a circular island on a roundabout is 140 feet, as shown below.

- How far is it around the outside edge of this island?

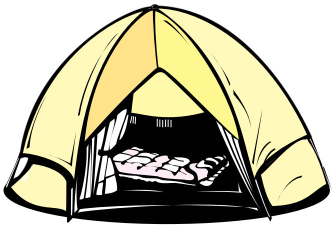
Suppose that the street around the island is 50 feet wide.

- What is the *outside* diameter measuring from one outer side of the roundabout to the other outer side?

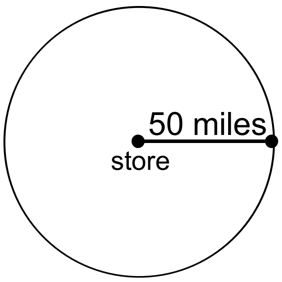
- What is the circumference (disregarding the entrances) of the outside curb of the street?



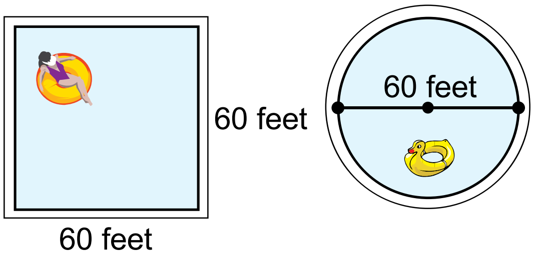
4. A circular tent has a diameter of 14 feet. How many square feet of ground space does it cover?



5. Mr. Brown, a merchant, says that some people drive 50 miles to shop at his store. How many square miles of territory lie inside the circle with center at his store and a radius of 50 miles?



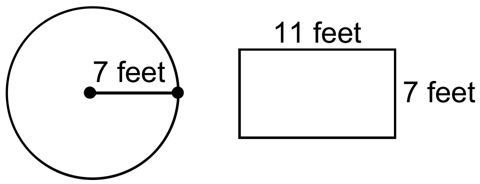
6. You have a choice of two different pools for your new pool. One is a square pool measuring 60 feet to a side. The other is a circular pool 60 feet in diameter. Find the area of each, and determine which pool is larger.



Circle the letter of the correct answer.

7. Compare the circle to the rectangle below. What is the ratio of the area of the circle to the area of the rectangle?

   a. 2:1
   b. 4:1
   c. 22:7
   d. 11:22



8. If the radius of a circle is 10 meters (m), then how long is the circumference?

   a. 10π m
   b. 100π m
   c. 20π m
   d. 250π m

Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

__________ 1. A *circle* is the set of all points that are the same distance from the center.

__________ 2. The *center of the circle* is the point from which all points on the circle are the same distance.

__________ 3. The distance of any line segment drawn from the center to any point on the circle is called a *radius (r)*.

__________ 4. A *diameter (d)* of a circle is the distance of a line segment from one point of the circle to another, through the center.

__________ 5. The diameter of a circle is four times its radius, $d = 4r$.

__________ 6. The *circumference (C)* of a circle is the distance around the circle.

__________ 7. The *perimeter* (P) of a circle is called the diameter.

__________ 8. When the circumference of a circle is divided by its diameter the answer is close to 3.14 or π.

__________ 9. The formula for finding the circumference of a circle is $C = \pi d$ or $C = 2\pi r$.

__________ 10. There is an *exact* value for pi and it is 3.141.