

SURFACES AREAS OF CONES

A **cone** is a three-dimensional shape that has a circular base and a single vertex (the tip of the cone). It can be thought of as a pyramid with a circular base instead of a polygonal one. The surface area of a cone is the total area of all the surfaces that make up the cone, which includes two parts:

1. **The area of the circular base.**
2. **The area of the curved surface** (called the lateral surface area).



Surface Area Formula for a Cone

The **surface area** of a cone is the sum of the area of the circular base and the area of the curved surface. The formula for the surface area A of a cone is:

$$A = \pi r^2 + \pi r l$$

Where:

- r is the radius of the circular base of the cone,
- l is the slant height of the cone, which is the distance from the edge of the base to the tip of the cone along the curved surface,
- π is a constant approximately equal to 3.1416.

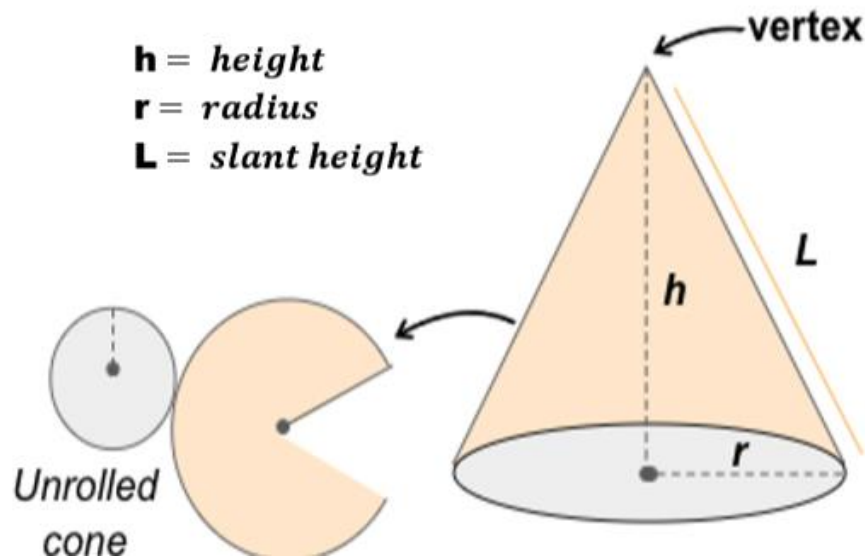
Breaking Down the Formula

1. Area (πr^2)

The first part of the surface area formula, πr^2 , is the area of the circular base of the cone. It is the area of a circle with radius r . The formula for the area of a circle is $A = \pi r^2$

2. Lateral Surface Area ($\pi r l$):

The second part, $\pi r l$, is the lateral (curved) surface area of the cone. To understand this, imagine wrapping a flat piece of paper around the cone. This piece of paper would form a sector of a circle. The curved surface area depends on both the radius of the base and the slant height of the cone.



How to Calculate the Surface Area

Step 1: Identify the Values

- **Radius (r):** This is the distance from the center of the base to the edge. It is typically given in the problem.
- **Slant Height (l):** This is the length from the edge of the base to the tip (vertex) of the cone, measured along the side. Sometimes this is not given directly, and you may need to find it using the Pythagorean theorem if you know the height h of the cone and the radius r .

Step 2: Calculate the Base Area

Use the formula πr^2 to find the area of the base.

Step 3: Calculate the Lateral Surface Area

Use the formula $\pi r l$ to find the lateral surface area.

Step 4: Add the Two Areas Together

Once you have both areas, add them together to get the total surface area.

Example Problem

Problem:

A cone has a radius of 5 cm and a slant height of 12 cm. Find its surface area.

Solution:

1. **Base Area:**

$$A_{\text{base}} = \pi r^2 = \pi(5)^2 = 25\pi \approx 25(3.1416) \approx 78.54 \text{ cm}^2.$$

2. **Lateral Surface Area:**

$$A_{\text{lateral}} = \pi r l = \pi(5)(12) = 60\pi \approx 60(3.1416) \approx 188.4 \text{ cm}^2.$$

3. **Total Surface Area:**

Now, add the base area and the lateral surface area:

$$A = 78.54 \text{ cm}^2 + 188.4 \text{ cm}^2 = 266.94 \text{ cm}^2$$

Thus, the surface area of the cone is approximately **266.94 cm²**.

Finding the Slant Height

If the slant height l is not given and only the vertical height h and the radius r are provided, you can use the **Pythagorean theorem** to find the slant height. The relationship is:

$$l = \sqrt{r^2 + h^2}$$

Where:

- l is the slant height,
- r is the radius,
- h is the vertical height (the perpendicular distance from the base to the tip).

Example Using Pythagorean Theorem

Problem:

A cone has a radius of 6 cm and a vertical height of 8 cm. Find the slant height and the surface area.

Solution:

1. Find the slant height, use the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$$

2. **Base Area:**

$$A_{\text{base}} = \pi r^2 = \pi(6)^2 = 36\pi \approx 36(3.1416) \approx 113.10 \text{ cm}^2.$$

3. **Lateral Surface Area:**

$$A_{\text{lateral}} = \pi r l = \pi(6)(10) = 60\pi \approx 60(3.1416) \approx 188.4 \text{ cm}^2.$$

4. **Total Surface Area:**

$$A = 113.10 \text{ cm}^2 + 188.4 \text{ cm}^2 = 301.5 \text{ cm}^2$$

Thus, the surface area of the cone is approximately 301.5 cm^2 .

- The surface area of a cone includes the area of the base and the lateral surface area.
- The formula for surface area is:

$$A = \pi r^2 + \pi r l$$

- If the slant height is not given, you can calculate it using the Pythagorean theorem if you know the radius and vertical height.