The Polygon Angle-Sum Theorems

The Polygon Angle-Sum Theorems help us calculate the sum of the interior and exterior angles of polygons. Here's an in-depth explanation:

1. Interior Angle-Sum Theorem

The **Interior Angle-Sum Theorem** states:

The sum of the measures of the interior angles of a polygon with n sides is given by the formula:

$$\text{sum} = (n - 2) * 180^\circ$$

Why does this work?

1. A polygon with n sides can be divided into $n-2$ triangles by drawing diagonals from one vertex.

    a. For example:

- A **triangle** ($n=3$) has 1 triangle. Its angle sum is $1 \cdot 180^\circ = 180^\circ$
- A **quadrilateral** ($n=4$) can be divided into 2 triangles. Its angle sum is
- $2 \cdot 180^\circ = 360^\circ$
- A **pentagon** ($n=5$) can be divided into 3 triangles. Its angle sum is $3 \cdot 180^\circ = 540^\circ$

2. Since each triangle has a sum of 180°, the total sum for the polygon is the number of triangles ($n-2$) * 180°.

Using the Formula

If you know the number of sides (n) of a polygon:

1. Subtract 2 from n.
2. Multiply the result by 180°

Example 1: Find the sum of interior angles of a hexagon (*n=6*).

$$\textbf{Sum} = (6-2)\cdot 180^\circ = 4\cdot 180^\circ = 720^\circ$$

Example 2: Find the measure of each interior angle of a regular octagon (*n=8*).

First, find the total sum of interior angles:

$$Sum = (8-2)\cdot 180^\circ = 6\cdot 180^\circ = 1080^\circ$$

1. Divide the total sum by *n* to get the measure of one angle in a regular polygon:

$$\text{Each angle} = 1080^\circ / 8 = 135^\circ$$

2. Exterior Angle-Sum Theorem

The **Exterior Angle-Sum Theorem** states:

The sum of the measures of the exterior angles of a polygon, one at each vertex, is always 360°, regardless of the number of sides.

Why does this work?

1. The exterior angles of a polygon are the angles formed by extending one side of the polygon and measuring the angle between this extension and the adjacent side.
2. As you "walk around" the polygon, the exterior angles represent one complete rotation (a full 360°).

- This is true for any polygon, whether it has 3 sides or 100 sides.

Using the Theorem

You can find the measure of each exterior angle in a regular polygon by dividing 360° by the number of sides (*n*).

Example 1: Find the measure of each exterior angle of a regular hexagon (n=6).

$$\textbf{Each exterior angle} = 360^\circ/6 = 60^\circ$$

Example 2: If each exterior angle of a regular polygon measures 30°, how many sides does the polygon have?

$n = 360^\circ / 30^\circ = 12$

So the polygon is a dodecagon (12 sides).