§3-5 RADICAL EQUATIONS

Procedure

Solving Radical Equations

1. Isolate the radical (or one of the radicals).
2. Exponentiate to eliminate the isolated radical.
3. Repeat steps 1 and 2 if there are still radicals.
4. Solve the resulting equation.
5. Check your answers using the original equation.

Example 1 Solve $\sqrt{3x+1}-3=7$ for x.

Solution

$$\begin{aligned}
\sqrt{3x+1}-3 &= 7 \\
\sqrt{3x+1} &= 10 && \text{Isolate} \\
\left(\sqrt{3x+1}\right)^2 &= (10)^2 && \text{Exponentiate} \\
3x+1 &= 100 && \text{Solve} \\
3x &= 99 \\
x &= 33
\end{aligned}$$

$$\begin{aligned}
\sqrt{3x+1}-3 &\stackrel{?}{=} 7 && \text{Check} \\
\sqrt{3\cdot 33+1}-3 &\stackrel{?}{=} 7 \\
\sqrt{100}-3 &\stackrel{?}{=} 7 \\
10-3 &\stackrel{?}{=} 7 \\
7 &= 7\surd
\end{aligned}$$

$x = 33$ is the solution of $\sqrt{3x+1}-3=7$.

Example 2 Solve $\sqrt{19-3x}-1=3x$ for x.

Solution

$$\begin{aligned}
\sqrt{19-3x}-1 &= 3x \\
\sqrt{19-3x} &= 3x+1 && \text{Isolate} \\
\left(\sqrt{19-3x}\right)^2 &= (3x+1)^2 && \text{Exponentiate} \\
19-3x &= 9x^2+6x+1 && \text{Solve} \\
0 &= 9x^2+9x-18 \\
0 &= 9(x^2+x-2) \\
0 &= 9(x+2)(x-1)
\end{aligned}$$

$x - 1 = 0$ or $x + 2 = 0$

$x = 1$ $\quad x = -2$

$$\begin{aligned}
\sqrt{19-3x}-1 &\stackrel{?}{=} 3x \\
\sqrt{19-3\cdot 1}-1 &\stackrel{?}{=} 3\cdot 1 \\
\sqrt{16}-1 &\stackrel{?}{=} 3 \\
4-1 &\stackrel{?}{=} 3 \\
3 &= 3\surd
\end{aligned}$$

$$\begin{aligned}
\sqrt{19-3x}-1 &\stackrel{?}{=} 3x && \text{Check} \\
\sqrt{19-3\cdot(-2)}-1 &\stackrel{?}{=} 3\cdot(-2) \\
\sqrt{25}-1 &\stackrel{?}{=} -6 \\
5-1 &\stackrel{?}{=} -6 \\
4 &\neq -6
\end{aligned}$$

Therefore $x = 1$ is the only solution of $\sqrt{19-3x}-1=3x$.

Example 3 Solve $\sqrt{2x-15} = \sqrt{x+5}$ for x.

Solution

$$\begin{aligned} \sqrt{2x-15} &= \sqrt{x+5} && \text{Isolate} \\ \left(\sqrt{2x-15}\right)^2 &= \left(\sqrt{x+5}\right)^2 && \text{Exponentiate} \\ 2x - 15 &= x + 5 \\ 2x - x &= 5 + 15 \\ x &= 20 && \text{Solve} \end{aligned}$$

Check

$$\begin{aligned} &\sqrt{2x-15}\ \sqrt{x+5} \\ &\sqrt{2\cdot 20-15}\ \sqrt{20+5} \\ &\sqrt{40-15}\ \sqrt{25} \\ &\sqrt{25}\ \ 5 \\ &5\ 5\surd \end{aligned}$$

$x = 20$ is the solution to $\sqrt{2x-15} = \sqrt{x+5}$.

Example 4 Solve $\sqrt{x+2} + \sqrt{x-1} = 3$ for x.

Solution

$$\begin{aligned} \sqrt{x+2} + \sqrt{x-1} &= 3 \\ \sqrt{x+2} &= 3 - \sqrt{x-1} && \text{Isolate} \\ \left(\sqrt{x+2}\right)^2 &= \left(3 - \sqrt{x-1}\right)^2 && \text{Exponentiate} \\ x + 2 &= 3^2 - 2\cdot 3\cdot\sqrt{x-1} + \left(\sqrt{x-1}\right)^2 \\ x + 2 &= 9 - 6\sqrt{x-1} + (x-1) \\ x + 2 &= x + 8 - 6\sqrt{x-1} \\ 6 &= -6\sqrt{x-1} \\ \\ 1 &= \sqrt{x-1} && \text{Isolate} \\ (1)^2 &= \left(\sqrt{x-1}\right)^2 && \text{Exponentiate} \\ 1 &= \left(\sqrt{x-1}\right)^2 \\ 1 &= x - 1 && \text{Solve} \\ 2 &= x \end{aligned}$$

Check

$$\begin{aligned} \sqrt{x+2} + \sqrt{x-1} &\ ?\ 3 \\ \sqrt{2+2} + \sqrt{2-1} &\ ?\ 3 \\ \sqrt{4} + \sqrt{1} &\ ?\ 3 \\ 2 + 1 &\ ?\ 3 \\ 3 &= 3\surd \end{aligned}$$

$x = 2$ is the solution to $\sqrt{x+2} + \sqrt{x-1} = 3$.

§3-5 PROBLEM SET

Solve each radical equation.

1. $\sqrt{2x+1}=3$

2. $\sqrt{2-y}+1=5$

3. $5-\sqrt{2k}=3$

4. $9-\sqrt{t+2}=5$

5. $3-\sqrt{x+1}=0$

6. $\sqrt[3]{r}=2$

7. $\sqrt{x}-3=5$

8. $3-\sqrt{y+3}=0$

9. $4-\sqrt{x+1}=5$

10. $5-\sqrt{x+3}=3$

11. $t=\sqrt{6t-9}$

12. $t=2\sqrt{t-1}$

13. $x+2\sqrt{x+1}=7$

14. $x=\sqrt{6x+18}-3$

15. $x+2=\sqrt{2x+3}$

16. $3\sqrt{x-2}+2=x$

17. $x+3\sqrt{x-2}=12$

18. $a-4=2\sqrt{a-5}$

19. $\sqrt{x^2+3x-2}-x=1$

20. $x-1+\sqrt{x^2+3}=0$

21. $\sqrt{x^2-3x-1}=3$

22 $\sqrt{x}+\sqrt{x-7}=7$

23. $2=\sqrt{x-5}-\sqrt{x+16}$

24. $\sqrt{x}+\sqrt{x+11}=11$

25. $\sqrt{x+3}+\sqrt{x}=5$

26. $\sqrt{x+1}=2-\sqrt{x}$

27. $3\sqrt{c}-1=\sqrt{c}+1$

28. $\sqrt{m+10}-\sqrt{m-6}=2$

29. $\sqrt{2x+4}=3-\sqrt{2x}$

30. $2\sqrt{3w-5}-3\sqrt{w+1}=0$

31. $\sqrt{4s+3}=2\sqrt{s-1}+1$

32. $\sqrt{x}-\sqrt{x+8}=8$

33. $\sqrt{3+x}+\sqrt{x}=\dfrac{6}{\sqrt{3+x}}$

34. $\dfrac{5}{\sqrt{x-1}}+\dfrac{\sqrt{x+4}}{2}=2\sqrt{x-1}$

35. $\sqrt{x+7}=2-\sqrt{x-5}$

36. $2\sqrt{x+1}-\sqrt{2x}=\sqrt{x-4}$

37. $2\sqrt{x}-\sqrt{4x-22}=\sqrt{2}$

38. $\sqrt{x+9}-\sqrt{x+2}=\sqrt{4x-27}$

§3-5 PROBLEM SOLUTIONS

1.	4	**2**.	-14	**3**.	2	**4**.	14
5.	8	**6**.	8	**7**.	64	**8**.	6
9.	no solution	**10**.	1	**11**.	3	**12**.	2
13.	3	**14**.	± 3	**15**.	-1	**16**.	{ 2, 11 }
17.	6	**18**.	6	**19**.	3	**20**.	-1
21.	{ -2, 5 }	**22**	16	**23**.	no solution	**24**.	25
25.	$\frac{121}{25}$	**26**.	$\frac{9}{16}$	**27**.	1	**28**.	15
29.	no solution	**30**.	$\frac{29}{3}$	**31**.	$\frac{13}{4}$	**32**.	no solution
33.	1	**34**.	5	**35**.	no solution	**36**.	$\frac{21 \pm 2\sqrt{154}}{7}$
37.	18	**38**.	no real solutions				