READING MATH

Proofs

When you solve an equation by factoring, you are using a deductive argument. Each step can be justified by an algebraic property.

Solve $4x^2 - 324 = 0$.

$4x^2 - 324 = 0$	Original equation
$(2x)^2 - 18^2 = 0$	$4x^2 = (2x)^2$ and $324 = 18^2$
$(2x + 18)(2x - 18) = 0$	Factor the difference of squares.
$2x + 18 = 0$ or $2x - 18 = 0$	Zero Product Property
$x = -9$ $\quad x = 9$	Solve each equation.

Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reasons for each statement. A *two-column proof* is a deductive argument that contains statements and reasons.

Two-Column Proof

Given: a, x, and y are real numbers such that $a \neq 0$, $x \neq 0$, and $y \neq 0$.
Prove: $ax^4 - ay^4 = a(x^2 + y^2)(x + y)(x - y)$

There is a reason for each statement.

The first statement contains the given information.

The last statement is what you want to prove.

Statements	Reasons
1. a, x, and y are real numbers such that $a \neq 0$, $x \neq 0$, and $y \neq 0$.	**1.** Given
2. $ax^4 - ay^4 = a(x^4 - y^4)$	**2.** The GCF of ax^4 and ay^4 is a.
3. $ax^4 - ay^4 = a[(x^2)^2 - (y^2)^2]$	**3.** $x^4 = (x^2)^2$ and $y^4 = (y^2)^2$
4. $ax^4 - ay^4 = a(x^2 + y^2)(x^2 - y^2)$	**4.** Factor the difference of squares.
5. $ax^4 - ay^4 = a(x^2 + y^2)(x + y)(x - y)$	**5.** Factor the difference of squares.

Reading to Learn

1. Solve $\frac{1}{16}t^2 - 100 = 0$ by using a two-column proof.
2. Write a two-column proof using the following information. (*Hint:* Group terms with common factors.)

   Given: c and d are real numbers such that $c \neq 0$ and $d \neq 0$.

   Prove: $c^3 - cd^2 - c^2d + d^3 = (c + d)(c - d)(c - d)$
3. Explain how the process used to write two-column proofs can be useful in solving Find the Error exercises, such as Exercise 37 on page 451.