1-1 Solving Linear Equations

I CAN... create and solve linear equations with one variable.

MA.912.AR.2.1–Given a real-world context, write and solve one-variable multi-step linear equations.

MA.K12.MTR.1.1, MTR.4.1, MTR.6.1

MODEL & DISCUSS

Joshua is going kayaking with a group during one of his vacation days. In his vacation planning, he budgeted $50 for a kayak rental.

A. How can Joshua determine the number of hours he can rent a kayak for himself? Describe two different options.

B. Joshua found out that there is a $25 nonrefundable equipment fee in addition to the hourly rates. How does this requirement change the mathematics of the situation?

C. **Choose Efficient Methods** How do the processes you used for parts A and B differ? How are they the same?

ESSENTIAL QUESTION

How do you create equations and use them to solve problems?

CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Solve Linear Equations

What is the value of x in the equation $\frac{2(x+4)}{3} - 8 = 32$?

Method 1

$\frac{2(x+4)}{3} - 8 = 32$

$2(x+4) - 24 = 96$ — Multiply each side by 3 first.

$2x + 8 - 24 = 96$

$2x - 16 = 96$

$2x = 112$

$x = 56$

OR

Method 2

$\frac{2(x+4)}{3} - 8 = 32$

$\frac{2(x+4)}{3} = 40$ — Add 8 to each side first.

$2(x+4) = 120$

$x + 4 = 60$

$x = 56$

VOCABULARY

Remember, a *variable* is an unknown quantity, or a quantity that can vary. An *equation* is a mathematical statement with two expressions set equal to each other. A *solution of an equation* is a value for the variable that makes the equation a true statement.

Each solving method yields the same solution. Is one method better than the other?

Look at how the expression on the left side of the original equation is built up from x.

$$x \rightarrow x+4 \rightarrow 2(x+4) \rightarrow \frac{2(x+4)}{3} \rightarrow \frac{2(x+4)}{3} - 8$$

Notice how Method 2 applies these steps in reverse to isolate x. This is often a good strategy and can lead to simpler solution methods.

Try It! **1.** Solve the equation $4 + \frac{3x-1}{2} = 9$. Explain the reasons why you chose your solution method.

EXAMPLE 2 Solve Consecutive Integer Problems

The sum of three consecutive integers is 132. What are the three integers?

Write an equation to model the problem. Then solve.

$$x + (x + 1) + (x + 2) = 132$$

The three integers are consecutive, so each is 1 greater than the previous.

$$3x + 3 = 132$$

Combine like terms.

$$3x + 3 - 3 = 132 - 3$$

$$\frac{3x}{3} = \frac{129}{3}$$

$$x = 43$$

REPRESENT AND CONNECT
How would the equation and solution be different if you let x be the middle number?

The first of the three consecutive numbers is 43.
The three consecutive numbers whose sum is 132 are 43, 44, 45.

Try It! **2.** The sum of three consecutive odd integers is 57. What are the three integers?

APPLICATION

EXAMPLE 3 Use Linear Equations to Solve Mixture Problems

A lab technician needs 25 liters of a solution that is 15% acid for a certain experiment, but she has only a solution that is 10% acid and a solution that is 30% acid. How many liters of the 10% and the 30% solutions should she mix to get what she needs?

Formulate Write an equation relating the number of liters of acid in each solution. Represent the total number of liters of one solution with a variable, like x. Then the total number of liters of the other solution must be $25 - x$.

25 L of 15% solution	=	x L of 10% solution	+	$(25 - x)$ L of 30% solution
$(0.15)(25)$	=	$0.10x$	+	$0.30(25 - x)$

Compute

$$3.75 = 0.1x + 7.5 - 0.3x$$

$$3.75 - 7.5 = 0.1x - 0.3x + 7.5 - 7.5$$

Subtract 7.5 from each side.

$$-3.75 = -0.2x$$

$$(-1)(-3.75) = (-1)(-0.2x)$$

Multiply each side by −1, then divide each side by 0.2.

$$3.75 = 0.2x$$

$$\frac{3.75}{0.2} = \frac{0.2x}{0.2}$$

$$18.75 = x$$

Interpret Since x represents the number of liters of the 10% acid solution, the lab technician should use 18.75 liters of the 10% solution. Since $25 - x$ represents the number of liters of the 30% acid solution, she should use $25 - 18.75$, or 6.25 liters of the 30% solution.

Try It! **3.** If the lab technician needs 30 liters of a 25% acid solution, how many liters of the 10% and the 30% acid solutions should she mix to get what she needs?

APPLICATION

EXAMPLE 4 Use Linear Equations to Solve Problems

Four friends use an online coupon to get discounts on concert tickets. They spent $108 for the four tickets. What was the price of one ticket without the discount?

Your online order is complete.

Your order details are shown below for your reference.

ORDER # 328
Sec B, Row 10, Seats 13-16

	Quantity	Price
Tickets	4	?
Discount	$15.00	4 x $15.00
Order Total		**$108**

Step 1 Write an equation to represent the problem situation.

Let p represent the original ticket price.

4 • original ticket price minus $15 = $108

$$4(p - 15) = 108$$

Step 2 Solve the equation.

$$4(p - 15) = 108$$

$$\frac{4(p - 15)}{4} = \frac{108}{4}$$

$$p - 15 = 27$$

$$p - 15 + 15 = 27 + 15$$

$$p = 42$$

The ticket price without the discount was $42.

COMMON ERROR
Subtract 15 from the price of each ticket, not from the total cost of four undiscounted tickets.

Try It! **4.** The same four friends buy tickets for two shows on consecutive nights. They use a coupon for $5 off each ticket. They pay a total of $416 for 8 tickets. Write and solve an equation to find the original price of the tickets.

APPLICATION

EXAMPLE 5 Solve Work and Time Problems

LaTanya will walk her bike from her house to the bike shop, which is 1.5 mi from her house, to get the bike fixed. After getting her bike repaired, she rides home. If the whole errand took one hour, how much time did LaTanya spend at the bike shop?

Step 1 Write an equation to represent the situation.

Time walking + Time at the shop + Time biking = Total time

$$\frac{1.5 \text{ miles}}{3 \text{ miles per hour}} + t + \frac{1.5 \text{ miles}}{10 \text{ miles per hour}} = 1 \text{ hour}$$

The equation $\frac{1.5}{3} + t + \frac{1.5}{10} = 1$ represents the situation.

ANALYZE AND PERSEVERE
Look for relationships between the distance traveled and the rate when you write the equation.

Step 2 Solve for t.

$$\frac{1.5}{3} + t + \frac{1.5}{10} = 1$$

$$(30)\frac{1.5}{3} + 30t + (30)\frac{1.5}{10} = 30$$

Multiply each side the least common denominator.

$$15 + 30t + 4.5 = 30$$

$$30t + 19.5 = 30$$

$$30t + 19.5 - 19.5 = 30 - 19.5$$

$$30t = 10.5$$

$$\frac{30t}{30} = \frac{10.5}{30}$$

$$t = 0.35$$

LaTanya spent 0.35 h, or 21 min at the bike shop.

Try It! **5.** LaTanya leaves her house at 12:30 P.M. and bikes at 12 mi/h to Marta's house. She stays at Marta's house for 90 min. Both girls walk back to LaTanya's house along the same route at 2.5 mi/h. They arrive at LaTanya's house at 3:30 P.M. How far is Marta's house from LaTanya's house?

CONCEPT SUMMARY Create and Solve Linear Equations

Use the following information about Kelsey's visit to the flower shop.

- Kelsey bought some roses and tulips.
- She bought twice as many tulips as roses.
- Roses cost $5 each.
- Tulips cost $2 each.
- Kelsey spent $36 total.

How many of each kind of flower did Kelsey buy?

WORDS Write an equation to represent the situation.

Cost of Roses + Cost of Tulips = Total Cost

(Cost of One Rose)(Number of Roses) + (Cost of One Tulip)(Number of Tulips) = Total Cost

ALGEBRA $\$5 \cdot x + \$2 \cdot 2x = \$36$

$5x + 4x = 36$

$9x = 36$

$x = 4$

Kelsey bought 4 roses and 8 tulips.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you create equations and use them to solve problems?

2. **Communicate and Justify** What is a first step to solving for x in the equation $9x - 7 = 10$? How would you check your solution?

3. **Use Patterns and Structure** For an equation with fractions, why is it helpful to multiply both sides of the equation by the LCD?

4. **Error Analysis** Venetta knows that 1 mi $\approx$ 1.6 km. To convert 5 mi/h to km/h, she multiplies 5 mi/h by $\frac{1 \text{ mi}}{1.6 \text{ km}}$. What error does Venetta make?

Do You KNOW HOW?

Solve each equation.

5. $4b + 14 = 22$

6. $-6k - 3 = 39$

7. $15 - 2(3 - 2x) = 46$

8. $\frac{2}{3}y - \frac{2}{5} = 5$

9. Terrence walks at a pace of 2 mi/h to the theater and watches a movie for 2 h and 15 min. He rides back home, taking the same route, on the bus that travels at a rate of 40 mi/h. The entire trip takes 3.5 h. How far along this route is Terrence's house from the theater? Explain.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

10. **Communicate and Justify** What could be a first step to solving the equation $3x + -0.5(x + 3) + 4 = 14$? Explain.

11. **Analyze and Persevere** The sum of four consecutive integers is –18. What is the greatest of these integers?

12. **Error Analysis** Describe and correct the error a student made when solving the equation $4 = -2(x - 3)$. What is the correct solution?

$$\begin{aligned} 4 &= -2(x - 3) \\ 4 &= -2x - 6 \\ 4 + 6 &= -2x - 6 + 6 \\ 10 &= -2x \\ \frac{10}{-2} &= \frac{-2x}{-2} \\ -5 &= x \end{aligned}$$

✗

13. **Communicate and Justify** Parker ran on a treadmill at a constant speed for the length of time shown. How many miles did Parker run? Explain.

14. **Use Patterns and Structure** The Division Property of Equality says that for every real number a, b, and c, if $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. Why does the property require that $c \neq 0$?

15. **Higher Order Thinking** Tonya's first step in solving the equation $\frac{1}{2}(2y + 4) = -6$ is to use the Distributive Property on the left side of the equation. Deon's first step is to multiply each side by 2. Which of these methods will result in an equivalent equation? Explain.

PRACTICE

Solve each equation. SEE EXAMPLES 1 AND 2

16. $-4x + 3x = 2$

17. $7 = 5y - 13 - y$

18. $7m - 4 - 9m - 36 = 0$

19. $-2 = -5t + 10 + 2t$

Solve each equation. SEE EXAMPLES 3 AND 4

20. $2(2x + 1) = 26$

21. $-2(2z + 1) = 26$

22. $92 = -4(2r - 5)$

23. $10(5 - n) - 1 = 29$

24. $-(7 - 2x) + 7 = -7$

25. $200 = 16(6t - 3)$

Solve each equation. SEE EXAMPLE 5

26. $\frac{1}{2}x + 2 = 1$

27. $\frac{3}{2}x - \frac{2}{3}x = 2$

28. $\frac{1}{5}(k - 3) = \frac{3}{4}$

29. $\frac{7}{60} = \frac{5}{24}w + \frac{11}{12}$

30. $\frac{3m}{4} - \frac{m}{12} = \frac{7}{8}$

31. $1{,}290 = \frac{h}{10} + \frac{h}{5}$

Solve each equation.

32. $0.1r - 1 = 0.65$

33. $1.2n + 0.68 = 5$

34. $0.025(q + 2) = 2.81$

35. $-0.07p - 0.6 = 5$

36. $1.037x + 0.02x + 25 = 30.285$

37. $-0.85t - 0.85t - 3.9 = -8.15$

38. A bee flies at 20 feet per second directly to an orange grove from its hive. The bee stays at the orange grove for 15 minutes, then flies directly back to the hive at 12 feet per second. It is away from the hive for a total of 20 minutes. SEE EXAMPLE 5

a. What equation can you use to find the distance of the orange grove from the hive?

b. How far is the orange grove from the hive?

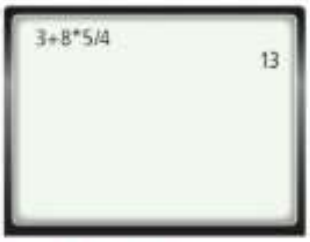
PRACTICE & PROBLEM SOLVING

APPLY

39. Apply Math Models A fastpitch softball player signs a six-year contract. Her agent expects that she will earn $1,000,000 over the next six years. If the agent is right, how many bonus payments, on average, should the pitcher expect each year? Explain.

40. Apply Math Models There are nine water bottles in Devin's refrigerator. He adds three full boxes of water bottles to the refrigerator. Then he adds two more boxes that each have 1 fewer bottle than a full box. When he is done, there are 67 bottles in the refrigerator. Write and solve an equation to find the number of bottles in a full box.

41. Check for Reasonableness Yuson used her calculator to solve the equation $\frac{4}{5}x - 8 = 3$. She entered the following on her screen and got an incorrect answer. How could she use parentheses to find the correct answer? Explain. What is the correct answer?



42. Apply Math Models A scientist makes an acid solution by adding drops of acid to 1.2 L of water. The final volume of the acid solution is 1.202 L. Assuming the volume of each drop is 0.05 mL, how many drops were added to the water? About what percent of the solution is acid? Round to the nearest hundredth of a percent.

ASSESSMENT PRACTICE

43. Anna bought 8 tetras and 2 rainbow fish for her aquarium. The rainbow fish cost $6 more than the tetras. She paid a total of $37. Which of the following are true? Select all that apply.

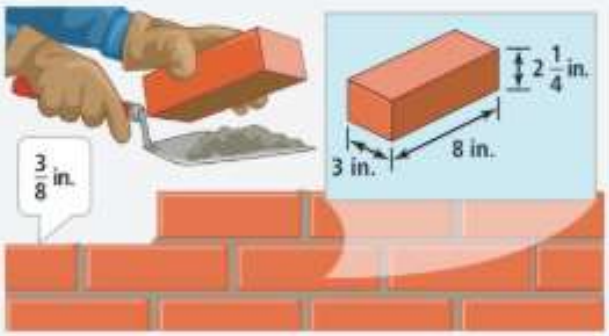
AR.2.1

- ☐ **A.** The cost of 4 tetras is the same as the cost of a rainbow fish.
- ☐ **B.** One rainbow fish plus 5 tetras cost $21.
- ☐ **C.** An equation to find the cost r, in dollars, of a rainbow fish is $8r + 2(r + 6) = 37$
- ☐ **D.** Reducing the number of rainbow fish by 1 would result in a total cost of $28.50.
- ☐ **E.** An equation to find the cost t, in dollars, of a tetra t is $8t + 2t + 6 = 37$.

44. SAT/ACT What is the solution of $1{,}200 - 5(3x + 30) = 600$?

Ⓐ 30 Ⓑ 50 Ⓒ 150 Ⓓ 200 Ⓔ 250

45. Performance Task A mason will lay rows of bricks to build a wall. The mason will spread $\frac{3}{8}$ inch of mortar on top of all but the last row of bricks. The finished wall will be $1\frac{1}{8}$ inch less than 4 feet high.



Part A The mason wants to lay the bricks so that the shortest edges of each brick are vertical. How many rows of bricks are needed? Show your work.

Part B Suppose the mason decides to lay bricks so that the 3-inch edge is vertical. If the mason lays the same number of rows of bricks that were used for the wall described in Part A, how high will this wall be?