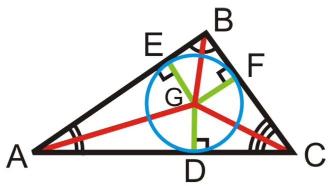
Notice that the circle touches all three sides of the triangle. We say that this circle is ***inscribed*** in the triangle because it touches all three sides. The incenter is on all three angle bisectors, so ***the incenter is equidistant from all three sides of the triangle.***

Concurrency of Angle Bisectors Theorem: The angle bisectors of a triangle intersect in a point that is equidistant from the three sides of the triangle.

If $\overline{AG}, \overline{BG}$, and $\overline{GC}$ are the angle bisectors of the angles in the triangle, then $EG = GF = GD$.

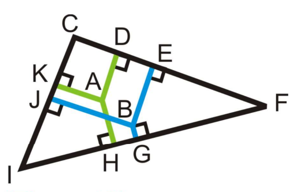


[Figure 11]

In other words, $\overline{EG}, \overline{FG}$, and $\overline{DG}$ are the radii of the inscribed circle.

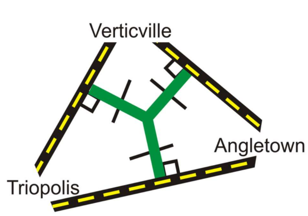
Example 3: If J, E, and G are midpoints and $KA = AD = AH$ what are points A and B called?

Solution: A is the incenter because $KA = AD = AH$, which means that it is equidistant to the sides. B is the circumcenter because $\overline{JB}, \overline{BE}$, and $\overline{BG}$ are the perpendicular bisectors to the sides.



[Figure 12]

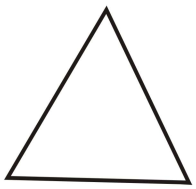
Know What? Revisited The airport needs to be equidistant to the three highways between the three cities. Therefore, the roads are all perpendicular to each side and congruent. The airport should be located at the incenter of the triangle.



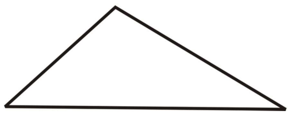
[Figure 13]

Review Questions

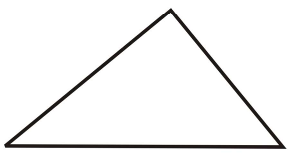
Construction Construct the incenter for the following triangles by tracing each triangle onto a piece of paper and using Investigation 5-2. Draw the inscribed circle too.



[Figure 14]



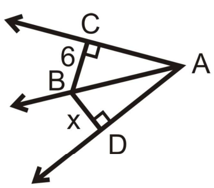
[Figure 15]



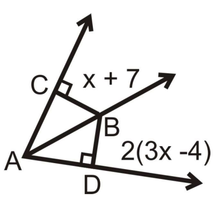
[Figure 16]

1. Is the incenter always going to be inside of the triangle? Why?
2. For an equilateral triangle, what can you conclude about the circumcenter and the incenter?

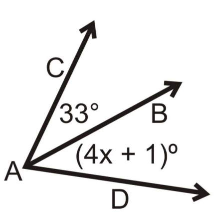
For questions 6-11, $\overrightarrow{AB}$ is the angle bisector of $\angle CAD$. Solve for the missing variable.



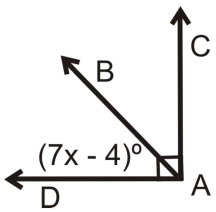
[Figure 17]



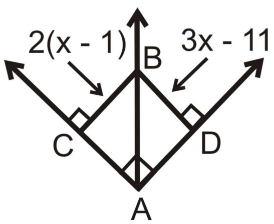
[Figure 18]



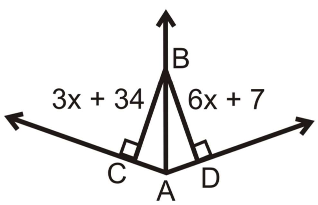
[Figure 19]



[Figure 20]

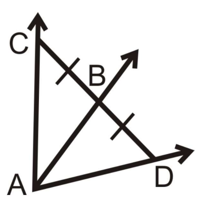


[Figure 21]

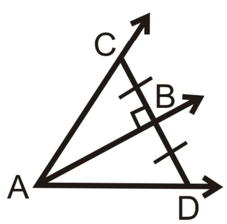


[Figure 22]

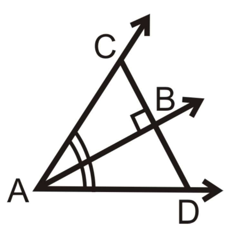
Is there enough information to determine if $\overrightarrow{AB}$ is the angle bisector of $\angle CAD$? Why or why not?



[Figure 23]



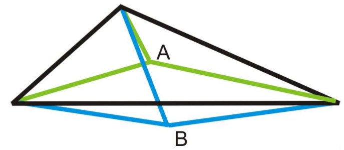
[Figure 24]



[Figure 25]

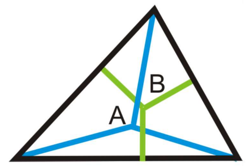
What are points A and B? How do you know?

The blue lines are congruent The green lines are angle bisectors



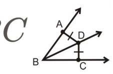
[Figure 26]

Both sets of lines are congruent The green lines are perpendicular to the sides



[Figure 27]

Fill in the blanks in the Angle Bisector Theorem Converse. Given: $\overline{AD} \cong \overline{DC}$, such that AD and DC are the shortest distances to $\overrightarrow{BA}$ and $\overrightarrow{BC}$ Prove: $\overrightarrow{BD}$ bisects $\angle ABC$



[Figure 28]

Statement	Reason
1.	
2.	The shortest distance from a point to a line is perpendicular.
3. $\angle DAB$ and $\angle DCB$ are right angles	
4. $\angle DAB \cong \angle DCB$	
5. $\overline{BD} \cong \overline{BD}$	
6. $\triangle ABD \cong \triangle CBD$	
7.	CPCTC
8. $\overrightarrow{BD}$ bisects $\angle ABC$	

Determine if the following descriptions refer to the incenter or circumcenter of the triangle.

18. A lighthouse on a triangular island is equidistant to the three coastlines.
19. A hospital is equidistant to three cities.
20. A circular walking path passes through three historical landmarks.
21. A circular walking path connects three other straight paths.

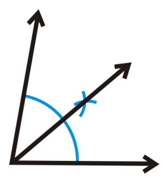
Constructions

22. Construct an equilateral triangle.
23. Construct the angle bisectors of two of the angles to locate the incenter.
24. Construct the perpendicular bisectors of two sides to locate the circumcenter.
25. What do you notice? Use these points to construct an inscribed circle inside the triangle and a circumscribed circle about the triangle.

Multi- Step Problem

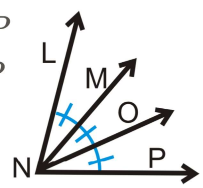
26. Draw $\angle ABC$ through $A(1,3), B(3,-1)$ and $C(7,1)$.
27. Use slopes to show that $\angle ABC$ is a right angle.
28. Use the distance formula to find AB and BC.
29. Construct a line perpendicular to AB through A.
30. Construct a line perpendicular to BC through C.
31. These lines intersect in the interior of $\angle ABC$. Label this point D and draw $\overrightarrow{BD}$.
32. Is $\overrightarrow{BD}$ the angle bisector of $\angle ABC$? Justify your answer.

Review Queue Answers



[Figure 29]

$$\angle LNM \cong \angle MNO \cong \angle ONP$$
$$\angle LNO \cong \angle MNP$$



[Figure 30]

1. Answers:

a.
$$\begin{aligned} 5x + 11 &= 26 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

b.
$$\begin{aligned} 9x - 1 &= 2(4x+5) \\ 9x - 1 &= 8x + 10 \\ x &= 11^\circ \end{aligned}$$

5.4 Medians and Altitudes in Triangles

Difficulty Level: **At Grade** | Created by: CK-12
Last Modified: Feb 03, 2020

Learning Objectives

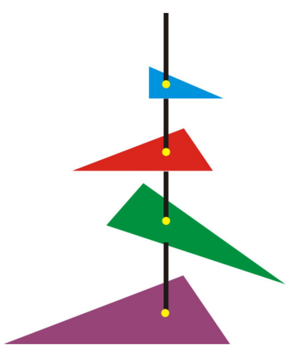
- Define median and find their point of concurrency in a triangle.
- Apply medians to the coordinate plane.
- Construct the altitude of a triangle and find their point of concurrency in a triangle.

Review Queue

1. Find the midpoint between (9, -1) and (1, 15).
2. Find the equation of the line between the two points from #1.
3. Find the equation of the line that is perpendicular to the line from #2 through (-6, 2).

Know What? Triangles are frequently used in art. Your art teacher assigns an art project involving triangles. You decide to make a series of hanging triangles of all different sizes from one long piece of wire. Where should you hang the triangles from so that they balance horizontally?

You decide to plot one triangle on the coordinate plane to find the location of this point. The coordinates of the vertices are (0, 0), (6, 12) and (18, 0). What is the coordinate of this point?

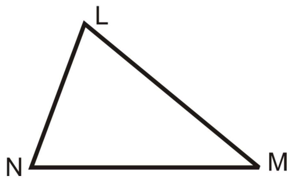


[Figure 1]

Medians

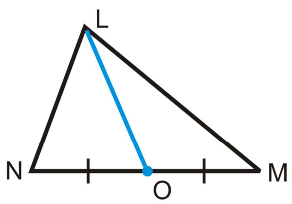
Median: The line segment that joins a vertex and the midpoint of the opposite side (of a triangle).

Example 1: Draw the median $\overline{LO}$ for $\triangle LMN$ below.



[Figure 2]

Solution: From the definition, we need to locate the midpoint of $\overline{NM}$. We were told that the median is $\overline{LO}$, which means that it will connect the vertex L and the midpoint of $\overline{NM}$, to be labeled O. Measure NM and make a point halfway between N and M. Then, connect O to L.

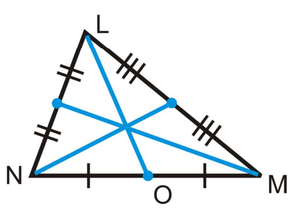


[Figure 3]

Notice that a median is very different from a perpendicular bisector or an angle bisector. A perpendicular bisector also goes through the midpoint, but it does not necessarily go through the vertex of the opposite side. And, unlike an angle bisector, a median does not necessarily bisect the angle.

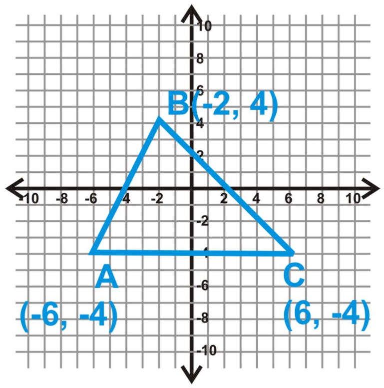
Example 2: Find the other two medians of $\triangle LMN$.

Solution: Repeat the process from Example 1 for sides $\overline{LN}$ and $\overline{LM}$. Be sure to always include the appropriate tick marks to indicate midpoints.



[Figure 4]

Example 3: Find the equation of the median from B to the midpoint of $\overline{AC}$ for the triangle in the $x-y$ plane below.



[Figure 5]

Solution: To find the equation of the median, first we need to find the midpoint of $\overline{AC}$, using the Midpoint Formula.

$$\left(\frac{-6+6}{2}, \frac{-4+(-4)}{2}\right) = \left(\frac{0}{2}, \frac{-8}{2}\right) = (0, -4)$$

Now, we have two points that make a line, B and the midpoint. Find the slope and $y-$ intercept.

$$\begin{aligned} m &= \frac{-4-4}{0-(-2)} = \frac{-8}{2} = -4 \\ y &= -4x+b \\ -4 &= -4(0)+b \\ -4 &= b \end{aligned}$$

The equation of the median is $y = -4x - 4$

Point of Concurrency for Medians

From Example 2, we saw that the three medians of a triangle intersect at one point, just like the perpendicular bisectors and angle bisectors. This point is called the centroid.

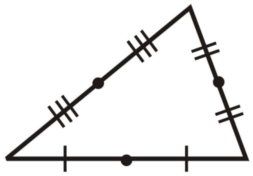
Centroid: The point of concurrency for the medians of a triangle.

Unlike the circumcenter and incenter, the centroid does not have anything to do with circles. It has a different property.

Investigation 5-3: Properties of the Centroid

Tools Needed: pencil, paper, ruler, compass

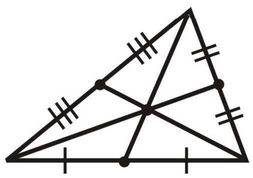
1. Construct a scalene triangle with sides of length 6 cm, 10 cm, and 12 cm (Investigation 4-2). Use the ruler to measure each side and mark the midpoint.



[Figure 6]

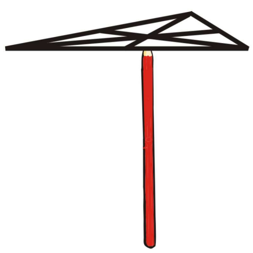
2. Draw in the medians and mark the centroid.

Measure the length of each median. Then, measure the length from each vertex to the centroid and from the centroid to the midpoint. Do you notice anything?



[Figure 7]

3. Cut out the triangle. Place the centroid on either the tip of the pencil or the pointer of the compass. What happens?



[Figure 8]

From this investigation, we have discovered the properties of the centroid. They are summarized below.

Concurrency of Medians Theorem: The medians of a triangle intersect in a point that is two-thirds of the distance from the vertices to the midpoint of the opposite side. The centroid is also the "balancing point" of a triangle.

If G is the centroid, then we can conclude: