1.3 Angles and Measurement

Difficulty Level: **At Grade** | Created by: CK-12
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Learning Objectives

- Define and classify angles.
- Apply the Protractor Postulate and the Angle Addition Postulate.

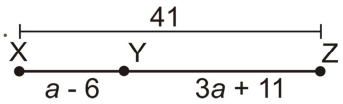
Review Queue

Answer the following questions.

Label the following geometric figure. What is it called?

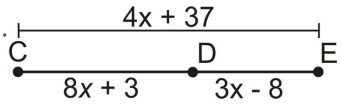
[Figure 1]

Find a, XY and YZ.



[Figure 2]

Find x, CD and DE.



[Figure 3]

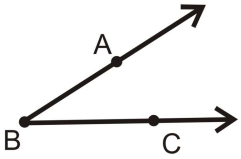
1. B is between A and C on $\overline{AC}$. If $AB = 4$ and $BC = 9$, what is AC? What postulate do you use to solve this problem?

Know What? Back to the building blocks. Every block has its own dimensions, angles and measurements. Using a protractor, find the measure of the three outlined angles in the "castle" below. Also, determine which other angles are equal to these measurements. Use appropriate angle markings. Do not measure any arcs.

[Figure 4]

Two Rays = One Angle

In #1 above, the figure was a ray. It is labeled $\overrightarrow{AB}$, with the arrow over the point that is NOT the endpoint. When two rays have the same endpoint, an angle is created.



[Figure 5]

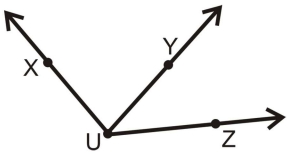
Here, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ meet to form an angle. An angle is labeled with an "$\angle$" symbol in front of the three letters used to label it. This angle can be labeled $\angle ABC$ or $\angle CBA$. **Always put the vertex in the middle of the three points.** It doesn't matter which side point is written first.

Angle: When two rays have the same endpoint.

Vertex: The common endpoint of the two rays that form an angle.

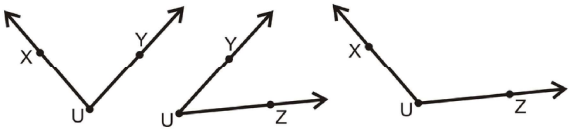
Sides: The two rays that form an angle.

Example 1: How many angles are in the picture below? Label each one two different ways.



[Figure 6]

Solution: There are three angles with vertex U. It might be easier to see them all if we separate them out.

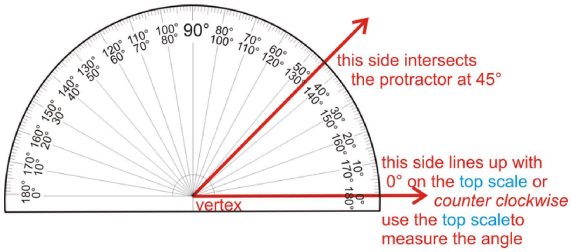


[Figure 7]

So, the three angles can be labeled, $\angle XUY$ or $\angle YUX$, $\angle YUZ$ or $\angle ZUY$, and $\angle XUZ$ or $\angle ZUX$.

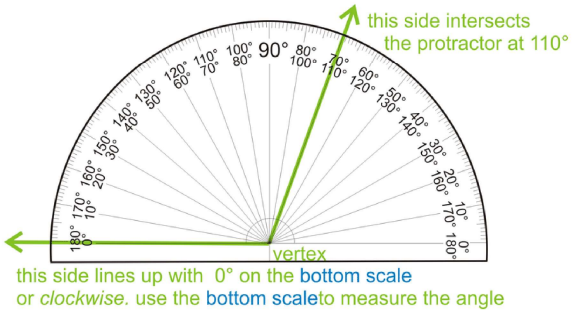
Protractor Postulate

We measure a line segment's *length* with a ruler. Angles are measured with something called a ***protractor***. A protractor is a measuring device that measures how "open" an angle is. Angles are measured in degrees, and labeled with a $^\circ$ symbol.



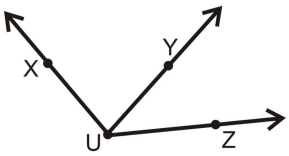
[Figure 8]

Notice that there are two sets of measurements, one opening clockwise and one opening counter-clockwise, from 0° to 180°. When measuring angles, always line up one side with 0°, and see where the other side hits the protractor. The vertex lines up in the middle of the bottom line, where all the degree lines meet.



[Figure 9]

Example 2: Measure the three angles from Example 1, using a protractor.



[Figure 10]

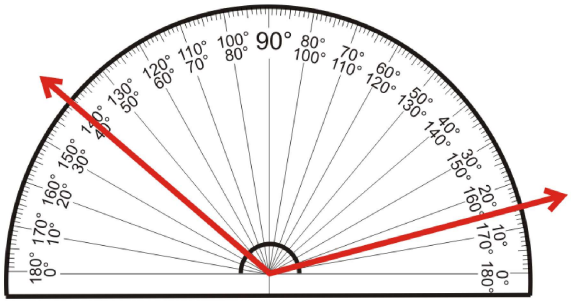
Solution: Just like in Example 1, it might be easier to measure these three angles if you separate them. With measurement, we put an m in front of the $\angle$ sign to indicate measure. So, $m\angle XUY = 84^\circ$, $m\angle YUZ = 42^\circ$ and $m\angle XUZ = 126^\circ$.

In the last lesson, we introduced the Ruler Postulate. Here we introduce the Protractor Postulate.

Protractor Postulate: For every angle there is a number between 0° and 180° that is the measure of the angle in degrees. The angle's measure is then the absolute value of the difference of the numbers shown on the protractor where the sides of the angle intersect the protractor.

In other words, you do not have to start measuring an angle at 0°, as long as you subtract one measurement from the other.

Example 3: What is the measure of the angle shown below?



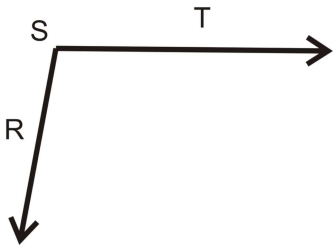
[Figure 11]

Solution: This angle is not lined up with 0°, so use subtraction to find its measure. It does not matter which scale you use.

Using the inner scale, $|140 - 25| = 125^\circ$

Using the outer scale, $|165 - 40| = 125^\circ$

Example 4: Use a protractor to measure $\angle RST$ below.



[Figure 12]

Solution: The easiest way to measure any angle is to line one side up with 0°. This angle measures 100°.

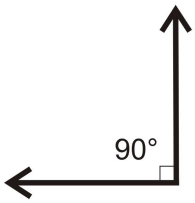
Classifying Angles

By looking at the protractor we measure angles from 0° to 180°. Angles can be classified, or grouped, into four different categories.

Straight Angle: When an angle measures 180°. The angle measure of a straight line. The rays that form this angle are called opposite rays.

[Figure 13]

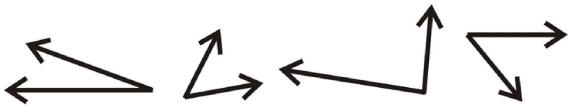
Right Angle: When an angle measures 90°.



[Figure 14]

Notice the half-square, marking the angle. This marking is always used to mark right, or 90°, angles.

Acute Angles: Angles that measure between 0° and 90°.


[Figure 15]

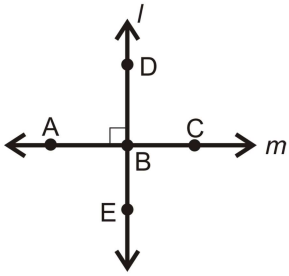
Obtuse Angles: Angles that measure between 90° and 180°.


[Figure 16]

It is important to note that 90° is NOT an acute angle and 180° is NOT an obtuse angle.

Additionally, any two lines or line segments can intersect to form four angles. If the two lines intersect to form right angles, we say the lines are perpendicular.

Perpendicular: When two lines intersect to form four right angles.



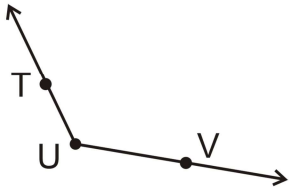
[Figure 17]

Even though all four angles are 90°, only one needs to be marked. It can be assumed thatall four are 90°.

The symbol for perpendicular is $\perp$, so these two lines would be labeled $l \perp m$ or $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$.

There are several other ways to label these two intersecting lines. This picture shows **two perpendicular lines, four right angles, four 90° angles**, and even **two straight angles,** $\angle ABC$ and $\angle DBE$.

Example 5: Name the angle and determine what type of angle it is.



[Figure 18]

Solution: The vertex is U. So, the angle can be $\angle TUV$ or $\angle VUT$. To determine what type of angle it is, compare it to a right angle. Because it opens wider than a right angle, and less than a straight angle it is **obtuse**.

Example 6: What type of angle is 84°? What about 165°?

Solution: 84° is less than 90°, so it is **acute**. 165° is greater than 90°, but less than 180°, so it is **obtuse**.

Drawing an Angle

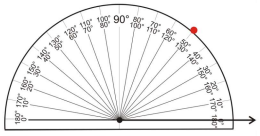
Investigation 1-1: Drawing a 50° Angle with a Protractor

1. Start by drawing a horizontal line across the page, about 2 in long.

   Place an endpoint at the left side of your line.

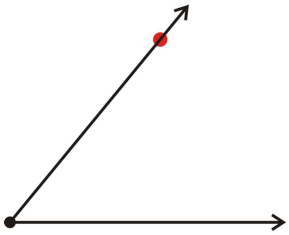
[Figure 19]

   Place the protractor on this point. Make sure to put the 50° on the center point on the bottom line of the protractor on the vertex. Mark appropriate scale.



[Figure 20]

   Remove the protractor and connect the vertex and the 50° mark.

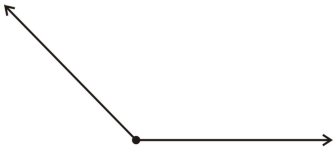


[Figure 21]

This process can be used to draw any angle between 0° and 180°. See http://www.mathsisfun.com/geometry/protractor-using.html for an **animation** of this investigation.

Example 7: Draw a 135° angle.

Solution: Following the steps from above, your angle should look like this:



[Figure 22]

Now that we know how to draw an angle, we can also copy that angle with a compass and a straightedge, usually a ruler. Anytime we use a compass and ruler to draw different geometric figures, it called a **construction**.

[Figure 23]

Compass: A tool used to draw circles and arcs.

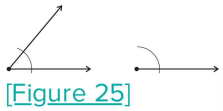
Investigation 1-2: Copying an Angle with a Compass and Straightedge

We are going to copy the 50° angle created in the previous investigation, a

1. Draw a straight line, about 2 inches long, and place an endpoint at one end.

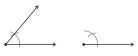
[Figure 24]

2. With the point (non-pencil side) of the compass on the vertex, draw an arc that passes through both sides of the angle. Repeat this arc with the line we drew in #1.



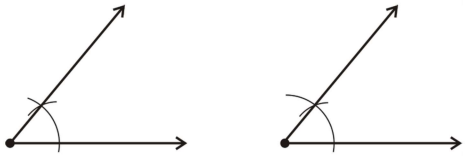
[Figure 25]

3. Move the point of the compass to the horizontal side of the angle we are



copying. Place the point where the arc intersects this side. Open (or close) the "mouth" of the compass so you can draw an arc that intersects the other side of the arc drawn in #2. Repeat this on the line we drew in #1. [Figure 26]

Draw a line from the new vertex to the arc intersections.



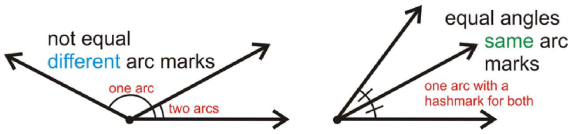
[Figure 27]

To watch an **animation** of this construction, see http://www.mathsisfun.com/geometry/construct-anglesame.html

Marking Angles and Segments in a Diagram

With all these segments and angles, we need to have different ways to label equal angles and segments.

Angle Markings

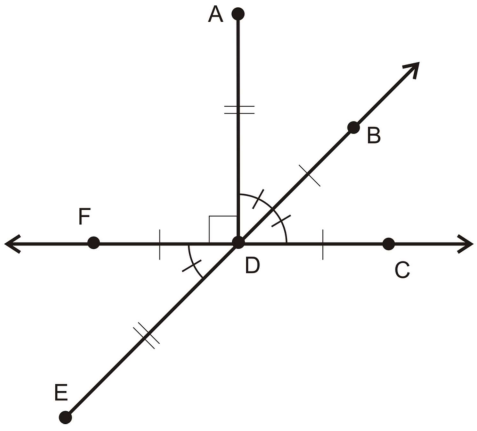


[Figure 28]

Segment Markings

[Figure 29]

Example 8: Interpret the picture below. Write all equal angle and segment statements.



[Figure 30]

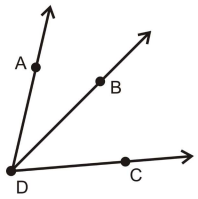
Solution:

$$\overline{AD} \perp \overleftrightarrow{FC}$$
$$m\angle ADB = m\angle BDC = m\angle FDE = 45^\circ$$
$$AD = DE$$
$$FD = DB = DC$$
$$m\angle ADF = m\angle ADC = 90^\circ$$

Angle Addition Postulate

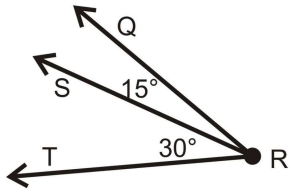
Much like the Segment Addition Postulate, there is an Angle Addition Postulate.

Angle Addition Postulate: If B is on the interior of $\angle ADC$, then $m\angle ADC = m\angle ADB + m\angle BDC$. See the picture below.



[Figure 31]

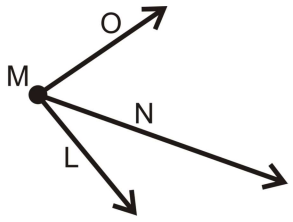
Example 9: What is $m\angle QRT$ in the diagram below?



[Figure 32]

Solution: Using the Angle Addition Postulate, $m\angle QRT = 15^\circ + 30^\circ = 45^\circ$.

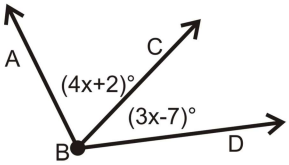
Example 10: What is $m\angle LMN$ if $m\angle LMO = 85^\circ$ and $m\angle NMO = 53^\circ$?



[Figure 33]

Solution: From the Angle Addition Postulate, $m\angle LMO = m\angle NMO + m\angle LMN$. Substituting in what we know, $85^\circ = 53^\circ + m\angle LMN$, so $85^\circ - 53^\circ = m\angle LMN$ or $m\angle LMN = 32^\circ$.

Example 11: ***Algebra Connection*** If $m\angle ABD = 100^\circ$, find x and $m\angle ABC$ and $m\angle CBD$?



[Figure 34]

Solution: From the Angle Addition Postulate, $m\angle ABD = m\angle ABC + m\angle CBD$. Substitute in what you know and solve the equation.

$$\begin{aligned} 100^\circ &= (4x+2)^\circ + (3x-7)^\circ \\ 100^\circ &= 7x - 5^\circ \\ 105^\circ &= 7x \\ 15^\circ &= x \end{aligned}$$

So, $m\angle ABC = 4(15^\circ) + 2^\circ = 62^\circ$ and $m\angle CBD = 3(15^\circ) - 7^\circ = 38^\circ$.

Know What? Revisited Using a protractor, the measurement marked in the red triangle is 90°, the measurement in the green triangle is 45° and the measurement in the blue square is 90°.

All of the equal angles are marked in the picture below. All of the acute angles in the triangles are equal and all the other angles are right, or 90°.

[Figure 35]

Review Questions

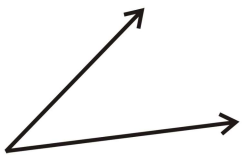
For questions 1-10, determine if the statement is true or false. If you answered FALSE for any question, state why.

1. Two angles always add up to be greater than 90°.
2. 180° is an obtuse angle.
3. 180° is a straight angle.
4. Two perpendicular lines intersect to form four right angles.
5. A construction uses a protractor and a ruler.
6. For an angle $\angle ABC, C$ is the vertex.
7. For an angle $\angle ABC, \overline{AB}$ and $\overline{BC}$ are the sides.
8. The m in front of $m\angle ABC$ means measure.
9. Angles are always measured in degrees.
10. The Angle Addition Postulate says that an angle is equal to the sum of the smaller angles around it.

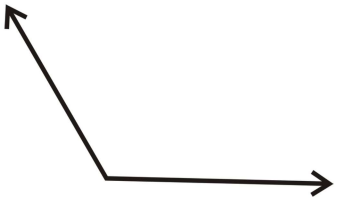
For 11-16, draw the angle with the given degree, using a protractor and a ruler. Also, state what type of angle it is.

11. 55°
12. 92°
13. 178°
14. 5°
15. 120°
16. 73°
17. ***Construction*** Copy the angle you made from #12, using a compass and a straightedge.
18. ***Construction*** Copy the angle you made from #16, using a compass and a straightedge.

For 19-22, use a protractor to determine the measure of each angle.

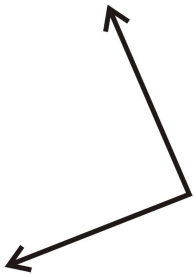


[Figure 36]



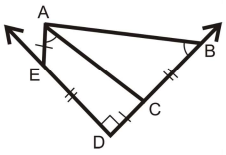
[Figure 37]

[Figure 38]



[Figure 39]

Interpret the picture. Write down all equal angles, segments and if any lines are perpendicular.



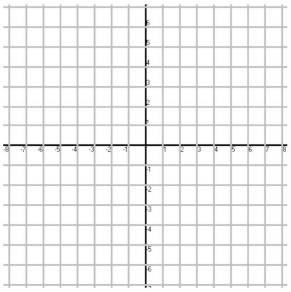
[Figure 40]

19. Draw a picture with the following requirements.

$AB = BC = BD$ $\quad$ $m\angle ABD = 90^\circ$
$m\angle ABC = m\angle CBD$ $\quad$ A, B, C and D are coplanar

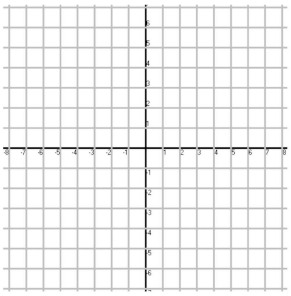
In 25 and 26, plot and sketch $\angle ABC$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle.

$A(5, -3)$
$B(-3, -1)$
$C(2, 2)$



[Figure 41]

$A(-3, 0)$
$B(3, 1)$
$C(5, 0)$



[Figure 42]

In Exercises 27-31, use the following information: Q is in the interior of $\angle ROS$. S is in the interior of $\angle QOP$. P is in the interior of $\angle SOT$. S is in the interior of $\angle ROT$ and $m\angle ROT = 160^\circ$, $m\angle SOT = 100^\circ$, and $m\angle ROQ = m\angle QOS = m\angle POT$.

27. Make a sketch.

28. Find $m\angle QOP$

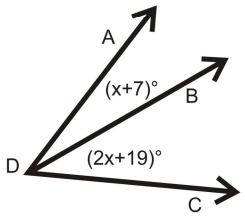
29. Find $m\angle QOT$

30. Find $m\angle ROQ$

31. Find $m\angle SOP$

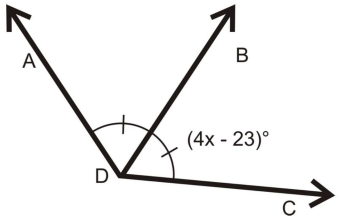
Algebra Connection Solve for x.

$m\angle ADC = 56^\circ$



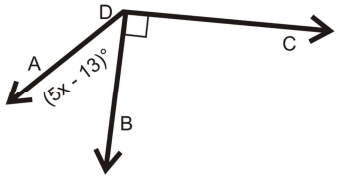
[Figure 43]

$m\angle ADC = 130^\circ$



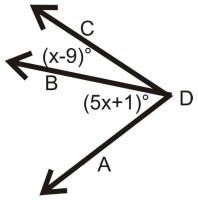
[Figure 44]

$m\angle ADC = (16x - 55)^\circ$



[Figure 45]

$m\angle ADC = (9x - 80)^\circ$



[Figure 46]

32. ***Writing*** Write a paragraph about why the degree measure of a straight line is 180, the degree measure of a right angle is 90, etc. In other words, answer the question, "Why is the straight line divided into exactly 180 degrees and not some other number of degrees?"

Review Queue Answers

1. $\overrightarrow{AB}$, a ray

2.
$$\begin{aligned} XY = 3,\ YZ &= 38 \\ a - 6 + 3a + 11 &= 41 \\ 4a + 5 &= 41 \\ 4a &= 36 \\ a &= 9 \end{aligned}$$

3.
$$\begin{aligned} CD = 51,\ DE &= 10 \\ 8x + 3 + 3x - 8 &= 4x + 37 \\ 11x - 5 &= 4x + 37 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$

4. Use the Segment Addition Postulate, $AC = 13$.