1.5 Basic Constructions

Difficulty Level: **At Grade** | Created by: CK-12
Last Modified: Feb 18, 2016

To Construct Bisector of an Angle

- http://www.mathsisfun.com/geometry/construct-anglebisect.html

To construct Perpendicular Bisector of a Line Segment

Check this link below

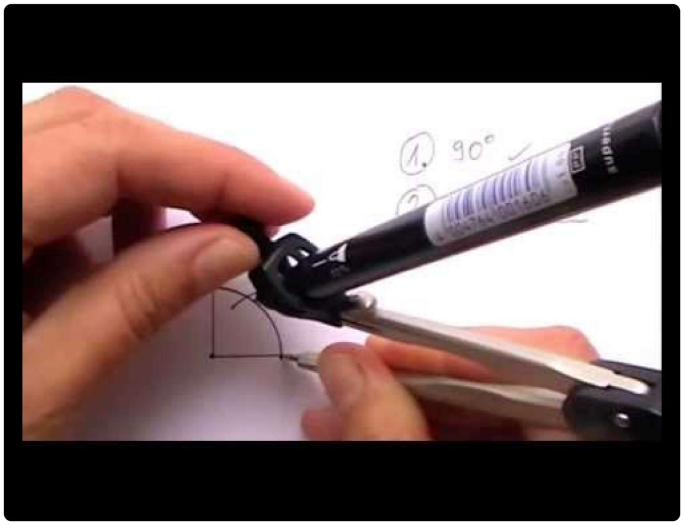
- http://www.mathopenref.com/constbisectline.html

Construction of Angles

60 Degrees: http://www.mathsisfun.com/geometry/construct-60degree.html

30 Degrees: http://www.mathsisfun.com/geometry/construct-30degree.html

45 Degrees: https://www.youtube.com/watch?v=QUtSDBQV1UA



https://www.ck12.org/flx/render/embeddedobject/162681

1.6 Angle Pairs

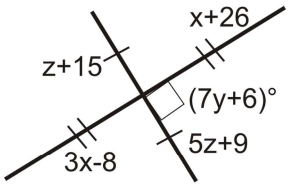
Difficulty Level: **Basic** | Created by: CK-12
Last Modified: Sep 27, 2016

Learning Objectives

- Recognize complementary angles, supplementary angles, linear pairs and vertical angles.
- Apply the Linear Pair Postulate and the Vertical Angles Theorem.

Review Queue

Use the picture below to answer questions 1-3.

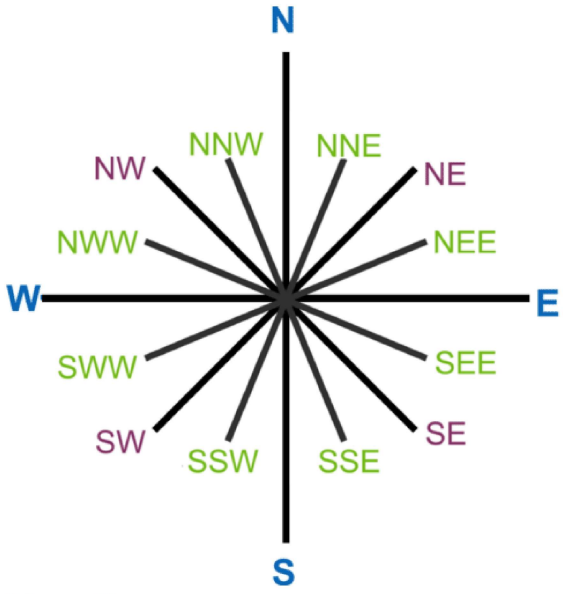


[Figure 1]

1. Find x.
2. Find y.
3. Find z.

Know What? A compass (as seen to the right) is used to determine the direction a person is traveling in. The angles between each direction are very important because they enable someone to be more specific and precise with their direction. In boating, captains use headings to determine which direction they are headed. A heading is the angle at which these compass lines intersect. So, a heading of $45^\circ NW$, would be straight out along that northwest line.

What headings have the same angle measure? What is the angle measure between each compass line?



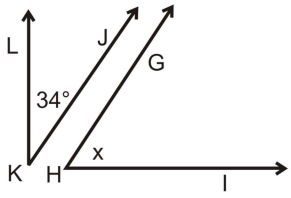
[Figure 2]

Complementary Angles

Complementary: When two angles add up to 90°.

Complementary angles do not have to be congruent to each other, nor do they have to be next to each other.

Example 1: The two angles below are complementary. $m\angle GHI = x$. What is x?

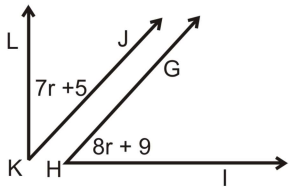


[Figure 3]

Solution: Because the two angles are complementary, they add up to 90°. Make an equation.

$$x + 34^\circ = 90^\circ$$
$$x = 56^\circ$$

Example 2: The two angles below are complementary. Find the measure of each angle.



[Figure 4]

Solution: Again, the two angles add up to 90°. Make an equation.

$$\begin{aligned} 8r + 9^\circ + 7r + 5^\circ &= 90^\circ \\ 15r + 14^\circ &= 90^\circ \\ 15r &= 76^\circ \\ r &\approx 5.1^\circ \end{aligned}$$

However, this is not what the question asks for. You need to plug r back into each expression to find each angle.

For $m\angle GHI$: $8(5.1^\circ) + 9^\circ = 49.8^\circ$, so $m\angle GHI \approx 49.8^\circ$.

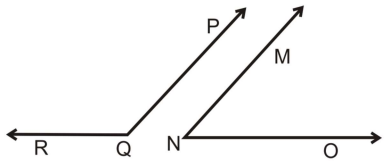
For $m\angle JKL$: $7(5.1^\circ) + 5^\circ = 40.7^\circ$, so $m\angle JKL \approx 40.7^\circ$.

Supplementary Angles

Supplementary: When two angles add up to 180°.

Just like complementary angles, supplementary angles do not have to be congruent or touching.

Example 3: The two angles below are supplementary. If $m\angle MNO = 78^\circ$ what is $m\angle PQR$?



[Figure 5]

Solution: Just like Examples 1 and 2, set up an equation. However, instead of equaling 90°, now it is 180°.

$$\begin{aligned} 78^\circ + m\angle PQR &= 180^\circ \\ m\angle PQR &= 102^\circ \end{aligned}$$

Example 4: What is the measure of two congruent, supplementary angles?

Solution: Supplementary angles add up to 180°. Congruent angles have the same measure. Divide 180° by 2, to find the measure of each angle.

$$180^\circ \div 2 = 90^\circ$$

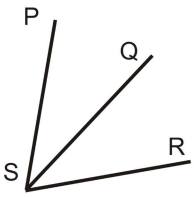
So, two congruent, supplementary angles are right angles, or 90°.

Linear Pairs

Adjacent Angles: Two angles that have the same vertex, share a side, and do not overlap.

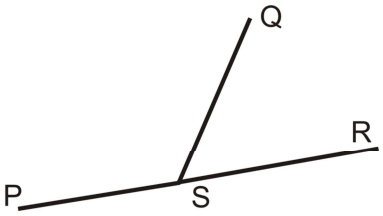
$\angle PSQ$ and $\angle QSR$ are adjacent.

$\angle PQR$ and $\angle PQS$ are NOT adjacent because they overlap.



[Figure 6]

Linear Pair: Two angles that are adjacent and whose non-common sides form a straight line.



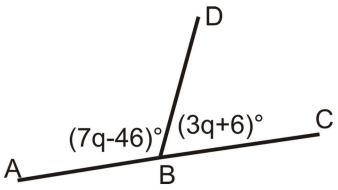
[Figure 7]

$\angle PSQ$ and $\angle QSR$ are a linear pair.

$$\begin{aligned} m\angle PSR &= 180^\circ \\ m\angle PSQ + m\angle QSR &= m\angle PSR \\ m\angle PSQ + m\angle QSR &= 180^\circ \end{aligned}$$

Linear Pair Postulate: If two angles are a linear pair, then they are supplementary.

Example 5: ***Algebra Connection*** What is the value of each angle?



[Figure 8]

Solution: These two angles are a linear pair, so they are supplementary, or add up to 180°. Write an equation.

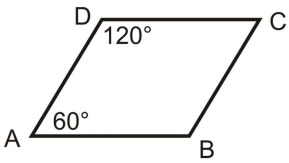
$$\begin{aligned}(7q-46)^\circ+(3q+6)^\circ&=180^\circ\\10q-40^\circ&=180^\circ\\10q&=220^\circ\\q&=22^\circ\end{aligned}$$

So, plug in q to get the measure of each angle.

$$m\angle ABD=7(22^\circ)-46^\circ=108^\circ \quad m\angle DBC=180^\circ-108^\circ=72^\circ$$

Example 6: Are $\angle CDA$ and $\angle DAB$ a linear pair? Are they supplementary?

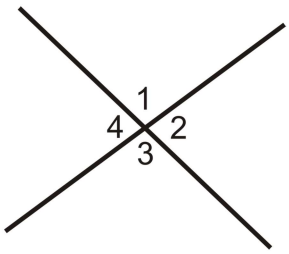
Solution: The two angles are not a linear pair because they do not have the same vertex. However, they are supplementary, $120^\circ+60^\circ=180^\circ$.



[Figure 9]

Vertical Angles

Vertical Angles: Two non-adjacent angles formed by intersecting lines.



[Figure 10]

$\angle 1$ and $\angle 3$ are vertical angles

$\angle 2$ and $\angle 4$ are vertical angles

Notice that these angles are labeled with numbers. You can tell that these are labels because they do not have a degree symbol.

Investigation 1-5: Vertical Angle Relationships

1. Draw two intersecting lines on your paper. Label the four angles created $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$. See the picture above.
2. Take your protractor and find $m\angle 1$.
3. What is the angle relationship between $\angle 1$ and $\angle 2$? Find $m\angle 2$.
4. What is the angle relationship between $\angle 1$ and $\angle 4$? Find $m\angle 4$.
5. What is the angle relationship between $\angle 2$ and $\angle 3$? Find $m\angle 3$.
6. Are any angles congruent? If so, write down the congruence statement.

From this investigation, hopefully you found out that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. This is our first theorem. That means it must be proven true in order to use it.

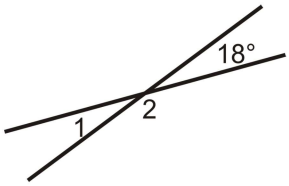
Vertical Angles Theorem: If two angles are vertical angles, then they are congruent.

We can prove the Vertical Angles Theorem using the same process we used above. However, let's not use any specific values for the angles.

From the picture above:	
$\angle 1$ and $\angle 2$ are a linear pair	$m\angle 1 + m\angle 2 = 180^\circ$
$\angle 2$ and $\angle 3$ are a linear pair	$m\angle 2 + m\angle 3 = 180^\circ$
$\angle 3$ and $\angle 4$ are a linear pair	$m\angle 3 + m\angle 4 = 180^\circ$
All of the equations $= 180^\circ$, so set the first and second equation equal to each other and the second and third.	$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ AND $m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$
Cancel out the like terms	$m\angle 1 = m\angle 3,\ m\angle 2 = m\angle 4$

Recall that anytime the measures of two angles are equal, the angles are also congruent.

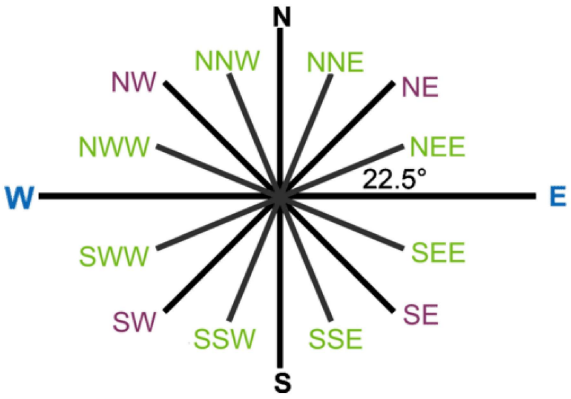
Example 7: Find $m\angle 1$ and $m\angle 2$.



[Figure 11]

Solution: $\angle 1$ is vertical angles with 18°, so $m\angle 1 = 18^\circ$. $\angle 2$ is a linear pair with $\angle 1$ or 18°, so $18^\circ + m\angle 2 = 180^\circ$. $m\angle 2 = 180^\circ - 18^\circ = 162^\circ$.

Know What? Revisited The compass has several vertical angles and all of the smaller angles are $22.5^\circ, 180^\circ \div 8$. Directions that are opposite each other, have the same angle measure, but of course, a different direction. All of the green directions have the same angle measure, 22.5°, and the purple have the same angle measure, 45°. N, S, E and W all have different measures, even though they are all 90° apart.



[Figure 12]

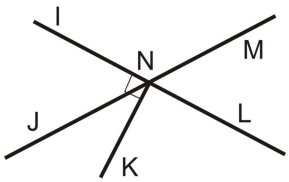
Review Questions

1. Find the measure of an angle that is complementary to $\angle ABC$ if $m\angle ABC$ is
   a. 45°
   b. 82°
   c. 19°
   d. z°

2. Find the measure of an angle that is supplementary to $\angle ABC$ if $m\angle ABC$ is
   a. 45°
   b. 118°
   c. 32°

d. x°

Use the diagram below for exercises 3-7. Note that $\overline{NK} \perp \overleftrightarrow{IL}$.



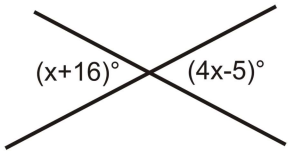
[Figure 13]

3. Name one pair of vertical angles.
4. Name one linear pair of angles.
5. Name two complementary angles.
6. Name two supplementary angles.
7. Given that $m\angle INJ = 63^\circ$, find:
   a. $m\angle JNL$
   b. $m\angle KNL$
   c. $m\angle MNL$
   d. $m\angle MNI$

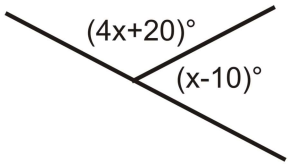
For 8-15, determine if the statement is ALWAYS true, SOMETIMES true or NEVER true.

8. Vertical angles are congruent.
9. Linear pairs are congruent.
10. Complementary angles add up to 180°.
11. Supplementary angles add up to 180°
12. Adjacent angles share a vertex.
13. Adjacent angles overlap.
14. Complementary angles are 45°.
15. The complement of x° is $(90 - x)^\circ$.

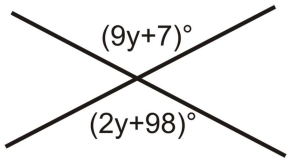
For 16-25, find the value of x or y.



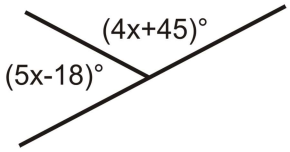
[Figure 14]



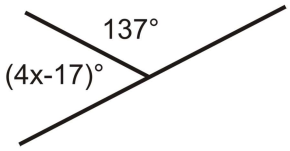
[Figure 15]



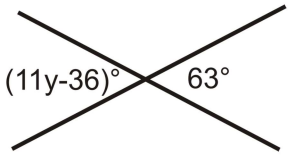
[Figure 16]



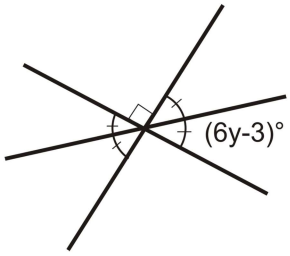
[Figure 17]



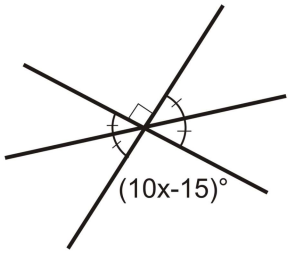
[Figure 18]



[Figure 19]



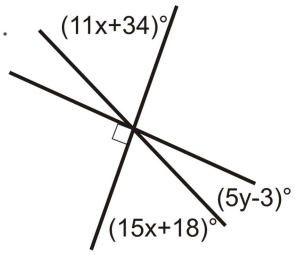
[Figure 20]



[Figure 21]

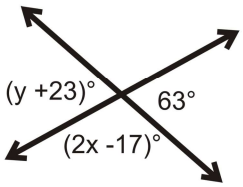
16. Find x.

Find y.

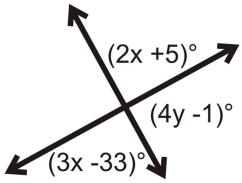


[Figure 22]

Find x and y in the following diagrams.

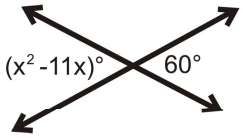


[Figure 23]

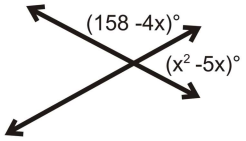


[Figure 24]

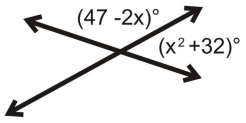
Algebra Connection. Use factoring or the quadratic formula to solve for the variables.



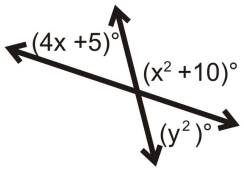
[Figure 25]



[Figure 26]



[Figure 27]



[Figure 28]

Review Queue Answers

1. $x + 26 = 3x - 8$
   $34 = 2x$
   $17 = x$
2. $(7y + 6)^\circ = 90^\circ$
   $7y = 84^\circ$
   $y = 12^\circ$
3. $z + 15 = 5z + 9$
   $6 = 4z$
   $1.5 = z$