10.4 Circumference and Arc Length

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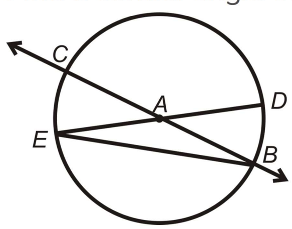
Learning Objectives

- Find the circumference of a circle.
- Define the length of an arc and find arc length.

Review Queue

Find a central angle in that intercepts $\widehat{CE}$



[Figure 1]

1. Find an inscribed angle that intercepts $\widehat{CE}$.
2. How many degrees are in a circle? Find $m\widehat{ECD}$.
3. If $m\widehat{CE} = 26^\circ$, find $m\widehat{CD}$ and $m\angle CBE$.

Know What? A typical large pizza has a diameter of 14 inches and is cut into 8 or 10 pieces. Think of the crust as the circumference of the pizza. Find the *length* of the crust for the entire pizza. Then, find the length of the crust for one piece of pizza if the entire pizza is cut into a) 8 pieces or b) 10 pieces.

[Figure 2]

Circumference of a Circle

Circumference: The distance around a circle.

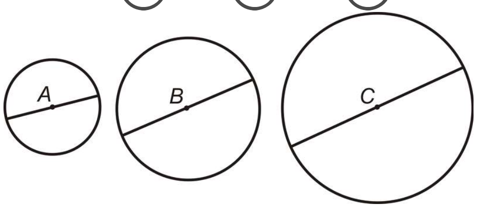
The circumference can also be called the perimeter of a circle. However, we use the term circumference for circles because they are round. The term perimeter is reserved for figures with straight sides. In order to find the formula for the circumference of a circle, we first need to determine the ratio between the circumference and diameter of a circle.

Investigation 10-1: Finding π (pi)

Tools Needed: paper, pencil, compass, ruler, string, and scissors

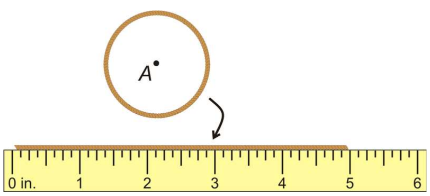
1. Draw three circles with radii of 2 in, 3 in, and 4 in. Label the centers of each A, B , and C .

   Draw in the diameters and determine their lengths. Are all the diameter lengths the same in $\odot A$? $\odot B$? $\odot C$?



[Figure 3]

   Take the string and outline each circle with it. The string represents the circumference of the circle. Cut the string so that it perfectly outlines the circle. Then, lay it out straight and measure, in inches. Round your answer to the nearest $\frac{1}{8}$ -inch. Repeat this for the other two circles.



[Figure 4]

2. Find $\frac{circumference}{diameter}$ for each circle. Record your answers to the nearest thousandth. What do you notice?

From this investigation, you should see that $\frac{circumference}{diameter}$ approaches 3.14159... The bigger the diameter, the closer the ratio was to this number. We call this number π , the Greek letter "pi." It is an irrational number because the decimal never repeats itself. Pi has been calculated out to the millionth place and there is still no pattern in the sequence of numbers. When finding the circumference and area of circles, we must use π .

π , or **"pi":** The ratio of the circumference of a circle to its diameter. It is approximately equal to 3.14159265358979323846...

To see more digits of π , go to http://www.eveandersson.com/pi/digits/.

You are probably familiar with the formula for circumference. From Investigation 10-1, we found that $\frac{circumference}{diameter} = \pi$. Let's shorten this up and solve for the circumference.

$\frac{C}{d} = \pi$, multiplying both sides by d, we have $C = \pi d$. We can also say $C = 2\pi r$ because $d = 2r$.

Circumference Formula: If d is the diameter or r is the radius of a circle, then $C = \pi d$ or $C = 2\pi r$.

Example 1: Find the circumference of a circle with a radius of 7 cm.

Solution: Plug the radius into the formula.

$$C = 2\pi(7) = 14\pi \approx 44\ cm$$

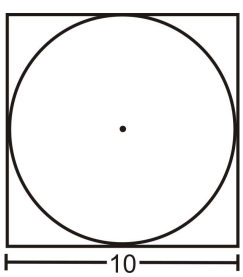
Depending on the directions in a given problem, you can either leave the answer in terms of π or multiply it out and get an approximation. Make sure you read the directions.

Example 2: The circumference of a circle is 64π. Find the diameter.

Solution: Again, you can plug in what you know into the circumference formula and solve for d.

$$64\pi = \pi d = 14\pi$$

Example 3: A circle is inscribed in a square with 10 in. sides. What is the circumference of the circle? Leave your answer in terms of π.



[Figure 5]

Solution: From the picture, we can see that the diameter of the circle is equal to the length of a side. Use the circumference formula.

$$C = 10\pi\ in.$$

Example 4: Find the perimeter of the square. Is it more or less than the circumference of the circle? Why?

Solution: The perimeter is $P = 4(10) = 40\ in$. In order to compare the perimeter with the circumference we should change the circumference into a decimal.

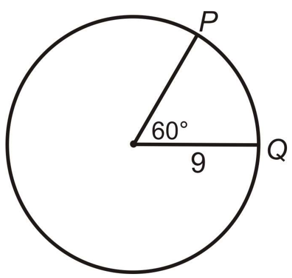
$C = 10\pi \approx 31.42\ in$. This is less than the perimeter of the square, which makes sense because the circle is smaller than the square.

Arc Length

In Chapter 9, we measured arcs in degrees. This was called the "arc measure" or "degree measure." Arcs can also be measured in length, as a portion of the circumference.

Arc Length: The length of an arc or a portion of a circle's circumference.

The arc length is directly related to the degree arc measure. Let's look at an example.



[Figure 6]

Example 5: Find the length of $\widehat{PQ}$. Leave your answer in terms of π .

Solution: In the picture, the central angle that corresponds with $\widehat{PQ}$ is 60° . This means that $m\widehat{PQ} = 60^\circ$ as well. So, think of the arc length as a portion of the circumference. There are 360° in a circle, so 60° would be $\frac{1}{6}$ of that $\left(\frac{60^\circ}{360^\circ} = \frac{1}{6}\right)$. Therefore, the length of $\widehat{PQ}$ is $\frac{1}{6}$ of the circumference.

$$length\ of\ \widehat{PQ} = \frac{1}{6} \cdot 2\pi(9) = 3\pi$$

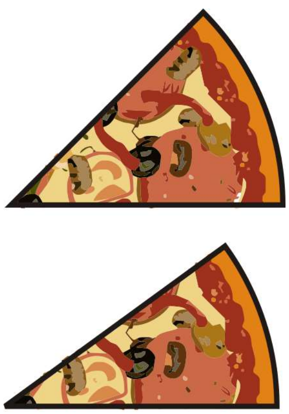
Arc Length Formula: If d is the diameter or r is the radius, the length of $\widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot \pi d$ or $\frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$.

Example 6: The arc length of $\widehat{AB} = 6\pi$ and is $\frac{1}{4}$ the circumference. Find the radius of the circle.

Solution: If 6π is $\frac{1}{4}$ the circumference, then the total circumference is $4(6\pi) = 24\pi$. To find the radius, plug this into the circumference formula and solve for r.

$$24\pi = 2\pi r$$
$$12 = r$$

Know What? Revisited The entire length of the crust, or the circumference of the pizza is $14\pi \approx 44\ in$. In the picture to the right, the top piece of pizza is if it is cut into 8 pieces. Therefore, for $\frac{1}{8}$ of the pizza, one piece would have $\frac{44}{8} \approx 5.5\ inches$ of crust. The bottom piece of pizza is if the pizza is cut into 10 pieces. For $\frac{1}{10}$ of the crust, one piece would have $\frac{44}{10} \approx 4.4\ inches$ of crust.



[Figure 7]

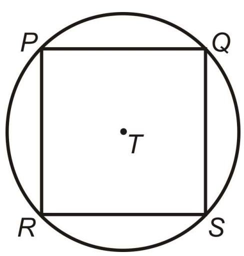
Review Questions

Fill in the following table. Leave all answers in terms of π.

	diameter	radius	circumference
1.	15		
2.		4	
3.	6		
4.			84π
5.		9	
6.			25π
7.			2π
8.	36		

9. Find the radius of circle with circumference 88 in.
10. Find the circumference of a circle with $d = \frac{20}{\pi} cm$.

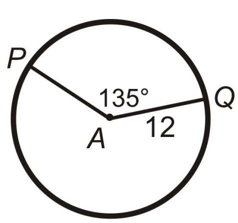
Square $PQSR$ is inscribed in $\odot T$. $RS = 8\sqrt{2}$.



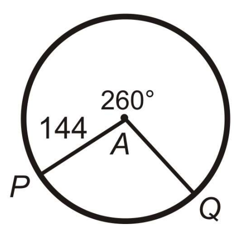
[Figure 8]

11. Find the length of the diameter of $\odot T$.
12. How does the diameter relate to $PQSR$?
13. Find the perimeter of $PQSR$.
14. Find the circumference of $\odot T$.

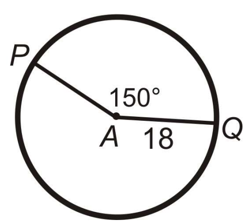
Find the arc length of $\widehat{PQ}$ in $\odot A$. Leave your answers in terms of π .



[Figure 9]

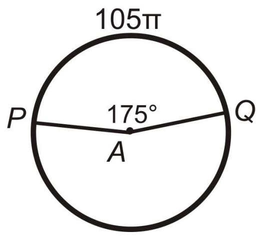


[Figure 10]

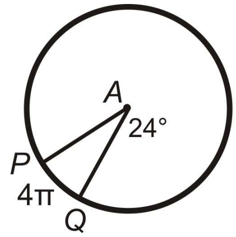


[Figure 11]

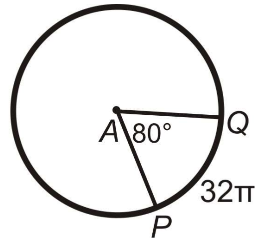
Find PA (the radius) in $\odot A$. Leave your answer in terms of π.



[Figure 12]

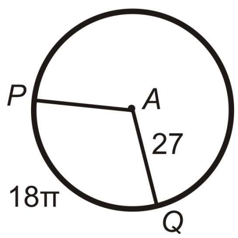


[Figure 13]

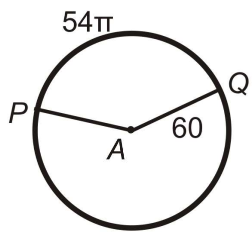


[Figure 14]

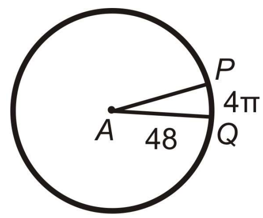
Find the central angle or $m\widehat{PQ}$ in $\odot A$. Round any decimal answers to the nearest tenth.



[Figure 15]

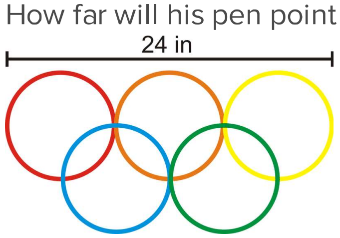


[Figure 16]



[Figure 17]

The Olympics symbol is five congruent circles arranged as shown below. Assume the top three circles are tangent to each other. Brad is tracing the entire symbol for a poster. How far will his pen point travel?



[Figure 18]

21. A truck has tires with a 26 in diameter.

   a. How far does the truck travel every time a tire turns exactly once?

   b. How many times will the tire turn after the truck travels 1 mile? (1 mile = 5280 feet)

22. Mario's Pizza Palace offers a stuffed crust pizza in three sizes (diameter length) for the indicated prices: The Little Cheese, 8 in, $7.00 The Big Cheese, 10 in, $9.00 The Cheese Monster, 12 in, $12.00 What is the crust (in) to price ($) ratio for each of these pizzas? Michael thinks the cheesy crust is the best part of the pizza and wants to get the most crust for his money. Which pizza should he buy?

23. Jay is decorating a cake for a friend's birthday. They want to put gumdrops around the edge of the cake which has a 12 in diameter. Each gumdrop is has a diameter of 1.25 cm. To the nearest gumdrop, how many will they need?

24. A speedometer in a car measures the distance travelled by counting the rotations of the tires. The number of rotations required to travel one tenth of a mile in a particular vehicle is approximately 9.34. To the nearest inch, find the diameter of the wheel. (1 mile = 5280 feet)
25. Bob wants to put new weather stripping around a semicircular window above his door. The base of the window (diameter) is 36 inches. How much weather stripping does he need?
26. Each car on a Ferris wheel travels 942.5 ft during the 10 rotations of each ride. How high is each car at the highest point of each rotation?

Review Queue Answers

1. $\angle CAE$
2. $\angle CBE$
3. $360^\circ, 180^\circ$
4. $m\widehat{CD} = 180^\circ - 26^\circ = 154^\circ, m\angle CBE = 13^\circ$