Area and Perimeter of Triangles and Quadrilaterals

Introduction

In geometry, triangles and quadrilaterals are among the most fundamental shapes. Understanding their *area* and *perimeter* allows us to solve many practical problems, such as calculating land size or building materials. Let’s explore how to measure these properties.

1. Area and Perimeter of Triangles

Perimeter of a Triangle

The **perimeter** of a triangle is the sum of the lengths of its three sides.

- **Formula**:

$$P = a + b + c$$

where a, b, and c are the lengths of the triangle's sides.

- **Example**:
  A triangle has sides $a = 5\,\text{cm}$, $b = 6\,\text{cm}$, $c = 7\,\text{cm}$:

$$P = 5 + 6 + 7 = 18\,\text{cm}$$

.2 Area of a Triangle

he **area** of a triangle depends on its base and height.

- **Formula:**

$$A = \frac{1}{2} \times b \times h$$

  where b is the base and h is the height (the perpendicular distance from the base to the opposite vertex).

- **Example**:
  A triangle with $b = 8\,\text{m}$, $h = 5\,\text{m}$:

$$A = \frac{1}{2} \times 8 \times 5 = 20\,\text{m}^2$$

- **Special Case: Heron's Formula** (when side lengths are known):

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

  where $s = \frac{a+b+c}{2}$ is the semi-perimeter.

  - Example: For a triangle with sides $a = 6$, $b = 8$, $c = 10$:

$$s = \frac{6 + 8 + 10}{2} = 12$$

$$A = \sqrt{12(12-6)(12-8)(12-10)} = \sqrt{12 \cdot 6 \cdot 4 \cdot 2} = \sqrt{576} = 24$$

2. Area and Perimeter of Quadrilaterals

2.1 Perimeter of Quadrilaterals

The **perimeter** is the sum of the lengths of all sides.

- **Formula**:
  For any quadrilateral:

  $$P = a + b + c + d$$

  where a, b, c, and d are the lengths of the sides.

- **Example**:
  A quadrilateral with sides $a = 4\,\text{cm}$, $b = 6\,\text{cm}$, $c = 7\,\text{cm}$, $d = 5\,\text{cm}$:

  $$P = 4 + 6 + 7 + 5 = 22\,\text{cm}$$

2.2 Area of Quadrilaterals

The **area** formula varies depending on the type of quadrilateral:

a) Rectangle:

- **Formula:**

$$A = l \times w$$

  where l = length, w = width.

- **Example:**
  A rectangle with $l = 10\,\mathrm{m}, w = 6\,\mathrm{m}$:

$$A = 10 \times 6 = 60\,\mathrm{m}^2$$

---

b) Square:

- **Formula:**

$$A = s^2$$

  where s = side length.

- **Example:**
  A square with side length $s = 4\,\mathrm{cm}$:

$$A = 4^2 = 16\,\mathrm{cm}^2$$

c) Parallelogram:

- **Formula**:

$$A = b \times h$$

  where b = base, h = height (the perpendicular distance between parallel sides).

- **Example**:
  A parallelogram with $b = 8\,\text{cm}, h = 5\,\text{cm}$:

$$A = 8 \times 5 = 40\,\text{cm}^2$$

---

d) Trapezium (or Trapezoid):

- **Formula**:

$$A = \frac{1}{2} \times (a + b) \times h$$

  where a and b are the lengths of the parallel sides, and h is the height.

- **Example**:
  A trapezium with $a = 6\,\text{cm}, b = 10\,\text{cm}, h = 4\,\text{cm}$:

$$A = \frac{1}{2} \times (6 + 10) \times 4 = \frac{1}{2} \times 16 \times 4 = 32\,\text{cm}^2$$

3. Key Differences Between Area and Perimeter

Concept	Definition	Units
Perimeter	Distance around the boundary of a shape	Units (e.g., cm, m)
Area	Space enclosed within the shape	Square units (e.g., cm^2, m^2)

Summary

- Use appropriate formulas depending on the shape.

- Always include the correct units.
- Recognize whether you're measuring *around* (perimeter) or *inside* (area) the shape.

Introduction to Circles

A circle is one of the most fundamental and symmetrical shapes in geometry. It is defined as the set of all points in a plane that are at a constant distance from a fixed point called the *center*. This distance is known as the *radius*. Circles are everywhere in nature, architecture, and technology, making them a crucial topic to understand.

Parts of a Circle

1. Center

The fixed point inside the circle is called the center. All points on the circle are equidistant from this point.

2. Radius (r)

The radius is the distance from the center to any point on the circle. All radii in a circle are equal.

3. Diameter (d)

The diameter is a straight line passing through the center that connects two points on the circle.

- It is the longest chord of the circle.
- The diameter is twice the radius:

$$d = 2r$$

4. Circumference (C)

The circumference is the distance around the circle, similar to the perimeter of polygons.

- Formula:

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

  where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.

5. Chord

A chord is a line segment that connects any two points on the circle.

- The diameter is a special type of chord, the longest one.

6. Arc

An arc is a portion of the circumference.

- **Major Arc**: Larger portion of the circle.
- **Minor Arc**: Smaller portion of the circle.

7. Sector

A sector is the area enclosed between two radii and an arc. Think of it as a "pizza slice" of the circle.

8. Segment

A segment is the area enclosed between a chord and the corresponding arc.

Key Formulas for Circles

1. Circumference

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

2. Area of a Circle

$$A = \pi r^2$$

This formula tells us the space enclosed by the circle.

3. Length of an Arc

For an arc with a central angle θ (in degrees):

$$\text{Arc Length} = \frac{\theta}{360} \times 2\pi r$$

4. Area of a Sector

For a sector with a central angle θ:

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

Examples

1. Find the Circumference

A circle has a radius of $r = 7\,\mathrm{cm}$. Find its circumference.

$$C = 2\pi r = 2 \times \pi \times 7 = 14\pi \approx 43.96\,\mathrm{cm}$$

2. Find the Area

A circle has a radius of $r = 10\,\mathrm{m}$. Find its area.

$$A = \pi r^2 = \pi \times (10)^2 = 100\pi \approx 314.16\,\mathrm{m}^2$$

3. Length of an Arc

A circle has a radius of $r = 5\,\mathrm{cm}$, and the central angle is $\theta = 90^\circ$.

$$\text{Arc Length} = \frac{90}{360} \times 2\pi r = \frac{1}{4} \times 2\pi \times 5 = \frac{10\pi}{4} \approx 7.85\,\mathrm{cm}$$

4. Area of a Sector

A circle has a radius of $r = 6\,\mathrm{m}$, and the central angle is $\theta = 120^\circ$.

$$\text{Area of Sector} = \frac{120}{360} \times \pi r^2 = \frac{1}{3} \times \pi \times (6)^2 = 12\pi \approx 37.7\,\mathrm{m}^2$$

Real-World Applications

1. **Measuring Circular Objects**: Wheels, clocks, coins, plates, etc.
2. **Construction**: Calculating round structures like silos, pools, or circular parks.
3. **Navigation**: Using circular motion and arcs for calculating paths.
4. **Engineering**: Designing gears, pipelines, or other mechanical parts.

Summary

Concept	Formula	Description
Circumference	$C = 2\pi r$ or $C = \pi d$	Distance around the circle
Area	$A = \pi r^2$	Space enclosed by the circle
Arc Length	$\frac{\theta}{360} \times 2\pi r$	Distance along an arc
Area of a Sector	$\frac{\theta}{360} \times \pi r^2$	Space enclosed by a sector

Understanding circles is essential not only for geometry but also for solving real-world problems. With these tools, you're equipped to analyze and apply the properties of circles effectively!