4.2 Congruent Figures

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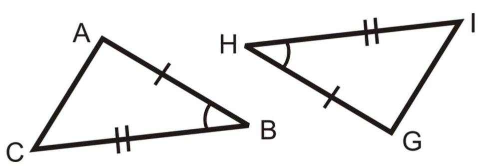
Learning Objectives

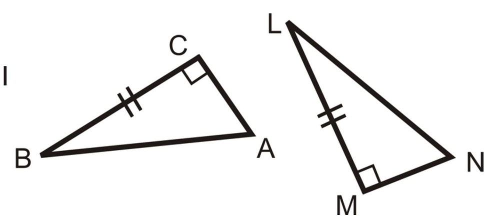
- Define congruent triangles and use congruence statements.
- Understand the Third Angle Theorem.
- Use properties of triangle congruence.

Review Queue

Which corresponding parts of each pair of triangles are congruent? Write all congruence statements for Questions 1 and 2.

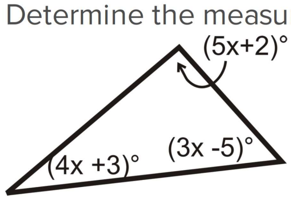


[Figure 1]



[Figure 2]

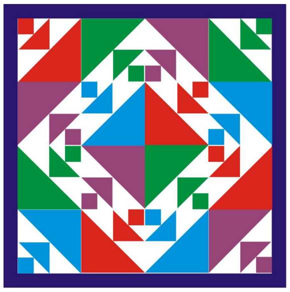
Determine the measure of x.



[Figure 3]

Know What? Quilt patterns are very geometrical. The pattern to the right is made up of several congruent figures. In order for these patterns to come together, the quilter rotates and flips each block (in this case, a large triangle, smaller triangle, and a smaller square) to get new patterns and arrangements.

How many different sets of colored congruent triangles are there? How many triangles are in each set? How do you know these triangles are congruent?

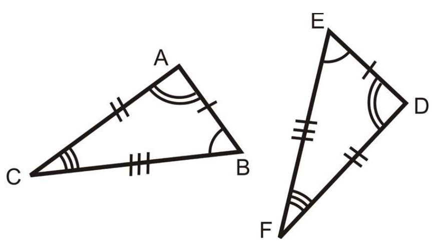


[Figure 4]

Congruent Triangles

Recall that two figures are congruent if and only if they have exactly the same size and shape.

Congruent Triangles: Two triangles are congruent if the three corresponding angles and sides are congruent.



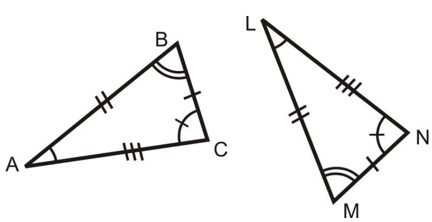
[Figure 5]

$\triangle ABC$ and $\triangle DEF$ are congruent because

$$\begin{array}{lcl} AB \cong DE & & \angle A \cong \angle D \\ BC \cong EF & \text{and} & \angle B \cong \angle E \\ AC \cong DF & & \angle C \cong \angle F \end{array}$$

When referring to corresponding congruent parts of triangles it is called **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent, or **CPCTC**.

Example 1: Are the two triangles below congruent?



[Figure 6]

Solution: To determine if the triangles are congruent, each pair of corresponding sides and angles must be congruent.

Start with the sides and match up sides with the same number of tic marks. Using the tic marks: $BC \cong MN, AB \cong LM, AC \cong LN$.

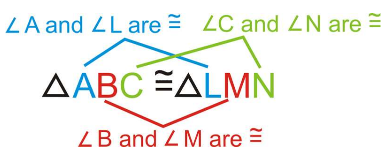
Next match the angles with the same markings; $\angle A \cong \angle L, \angle B \cong \angle M$, and $\angle C \cong \angle N$. Because all six parts are congruent, the two triangles are congruent.

We will learn, later in this chapter that it is impossible for two triangles to have all six parts be congruent and the triangles are not congruent, *when they are drawn to scale*.

Creating Congruence Statements

Looking at Example 1, we know that the two triangles are congruent because the three angles and three sides are congruent to the three angles and three sides in the other triangle.

When stating that two triangles are congruent, the order of the letters is very important. Corresponding parts must be written in the same order. Using Example 1, we would have:



[Figure 7]

Notice that the congruent sides also line up within the congruence statement.

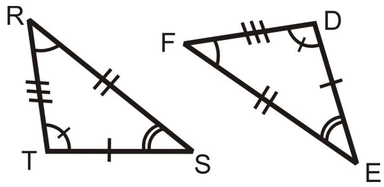
$$AB \cong LM, BC \cong MN, AC \cong LN$$

We can also write this congruence statement several other ways, as long as the congruent angles match up. For example, we can also write $\triangle ABC \cong \triangle LMN$ as:

$$\begin{array}{ll} \triangle ACB \cong \triangle LNM & \triangle BCA \cong \triangle MNL \\ \triangle BAC \cong \triangle MLN & \triangle CBA \cong \triangle NML \\ \triangle CAB \cong \triangle NLM & \end{array}$$

One congruence statement can always be written six ways. Any of the six ways above would be correct when stating that the two triangles in Example 1 are congruent.

Example 2: Write a congruence statement for the two triangles below.



[Figure 8]

Solution: To write the congruence statement, you need to line up the corresponding parts in the triangles: $\angle R \cong \angle F, \angle S \cong \angle E,$ and $\angle T \cong \angle D$. Therefore, the triangles are $\triangle RST \cong \triangle FED$.

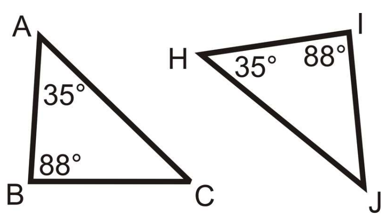
Example 3: If $\triangle CAT \cong \triangle DOG$, what else do you know?

Solution: From this congruence statement, we can conclude three pairs of angles and three pairs of sides are congruent.

$$\begin{array}{lll} \angle C \cong \angle D & \angle A \cong \angle O & \angle T \cong \angle G \\ CA \cong DO & AT \cong OG & CT \cong DG \end{array}$$

The Third Angle Theorem

Example 4: Find $m\angle C$ and $m\angle J$.



[Figure 9]

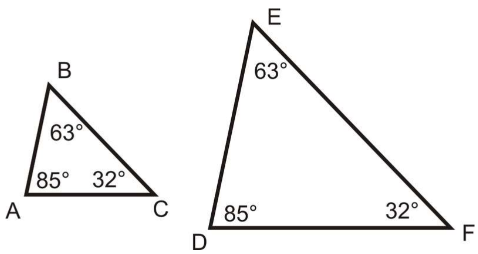
Solution: The sum of the angles in each triangle is 180° . So, for $\triangle ABC, 35^\circ + 88^\circ + m\angle C = 180^\circ$ and $m\angle C = 57^\circ$. For $\triangle HIJ$, $35^\circ + 88^\circ + m\angle J = 180^\circ$ and $m\angle J$ is also 57° .

Notice that we were given that $m\angle A = m\angle H$ and $m\angle B = m\angle I$ and we found out that $m\angle C = m\angle J$. This can be generalized into the Third Angle Theorem.

Third Angle Theorem: If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles must also congruent.

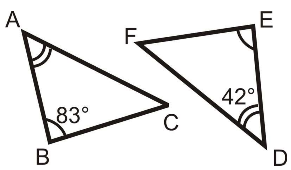
In other words, for triangles $\triangle ABC$ and $\triangle DEF, \angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Notice that this theorem does not state that the triangles are congruent. That is because if two sets of angles are congruent, the sides could be different lengths. See the picture to the left.



[Figure 10]

Example 5: Determine the measure of the missing angles.



[Figure 11]

Solution: From the markings, we know that $\angle A \cong D$ and $\angle E \cong \angle B$. Therefore, the Third Angle Theorem tells us that $\angle C \cong \angle F$. So,

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ m\angle D + m\angle B + m\angle C &= 180^\circ \\ 42^\circ + 83^\circ + m\angle C &= 180^\circ \\ m\angle C &= 55^\circ = m\angle F \end{aligned}$$

Congruence Properties

Recall the Properties of Congruence from Chapter 2. They will be very useful in the upcoming sections.

Reflexive Property of Congruence: Any shape is congruent to itself.

$AB \cong AB$ or $\triangle ABC \cong \triangle ABC$

Symmetric Property of Congruence: If two shapes are congruent, the statement can be written with either shape on either side of the $\cong$ sign.

$\angle EFG \cong \angle XYZ$ and $\angle XYZ \cong \angle EFG$ or $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle ABC$

Transitive Property of Congruence: If two shapes are congruent and one of those is congruent to a third, the first and third shapes are also congruent.

$\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$, then $\triangle ABC \cong \triangle GHI$

These three properties will be very important when you begin to prove that two triangles are congruent.

Example 6: In order to say that $\triangle ABD \cong \triangle ABC$, you must determine that the three corresponding angles and sides are congruent. Which pair of sides is congruent by the Reflexive Property?

[Figure 12]

Solution: The side AB is shared by both triangles. So, in a geometric proof, $AB \cong AB$ by the Reflexive Property of Congruence.

Know What? Revisited There are 16 " A " triangles and they are all congruent. There are 16 " B " triangles and they are all congruent. The quilt pattern is made from dividing up the square into smaller squares. The " A " triangles are all $\frac{1}{32}$ of the overall square and the " B " triangles are each $\frac{1}{128}$ of the large square. Both the " A " and " B " triangles are right triangles.

[Figure 13]

Review Questions

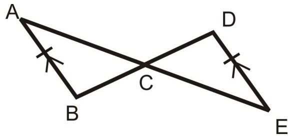
1. If $\triangle RAT \cong \triangle UGH$, what is also congruent?
2. If $\triangle BIG \cong \triangle TOP$, what is also congruent?

For questions 3-7, use the picture to the right.

[Figure 14]

3. What theorem tells us that $\angle FGH \cong \angle FGI$?
4. What is $m\angle FGI$ and $m\angle FGH$? How do you know?
5. What property tells us that the third side of each triangle is congruent?
6. How does FG relate to $\angle IFH$?
7. Write the congruence statement for these two triangles.

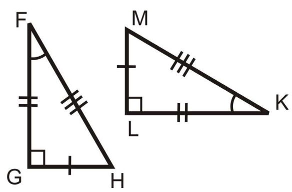
For questions 8-12, use the picture to the right.



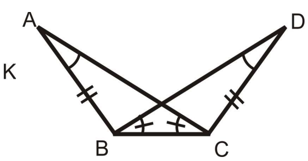
[Figure 15]

8. If $AB \mid\mid DE$, what angles are congruent? How do you know?
9. Why is $\angle ACB \cong \angle ECD$? It is not the same reason as #8.
10. Are the two triangles congruent with the information you currently have? Why or why not?
11. If you are told that C is the midpoint of AE and BD, what segments are congruent?
12. Write a congruence statement for the two triangles.

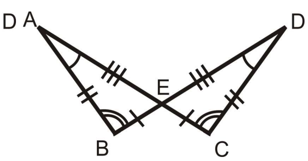
For questions 13-16, determine if the triangles are congruent. If they are, write the congruence statement.



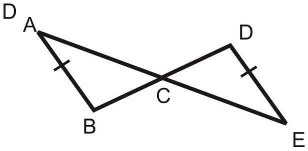
[Figure 16]



[Figure 17]

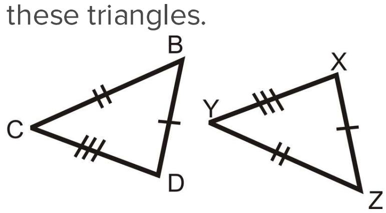


[Figure 18]



[Figure 19]

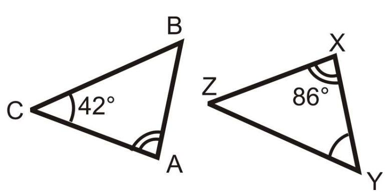
Suppose the two triangles to the right are congruent. Write a congruence statement for these triangles.



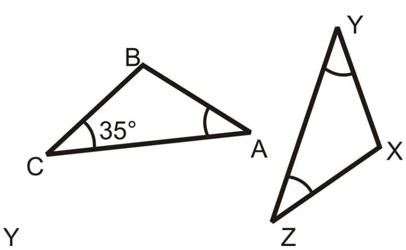
[Figure 20]

13. Explain how we know that if the two triangles are congruent, then $\angle B \cong \angle Z$.

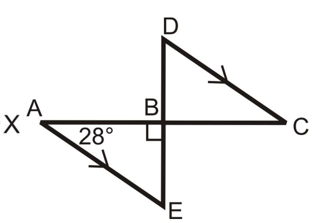
For questions 19-22, determine the measure of all the angles in the each triangle.



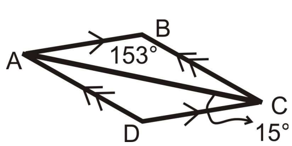
[Figure 21]



[Figure 22]

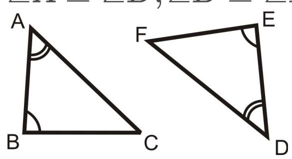


[Figure 23]



[Figure 24]

Fill in the blanks in the Third Angle Theorem proof below. Given: $\angle A \cong \angle D, \angle B \cong \angle E$ Prove: $\angle C \cong \angle F$



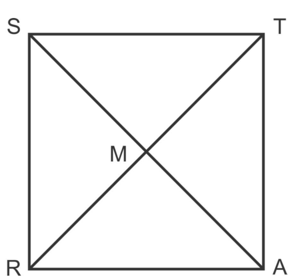
[Figure 25]

Statement	Reason
1. $A \cong \angle D, \angle B \cong \angle E$	
2.	$\cong$ angles have = measures
3. $m\angle A + m\angle B + m\angle C = 180^\circ$ $m\angle D + m\angle E + m\angle F = 180^\circ$	
4.	Substitution PoE
5.	Substitution PoE
6. $m\angle C = m\angle F$	
7. $\angle C \cong \angle F$	

For each of the following questions, determine if the Reflexive, Symmetric or Transitive Properties of Congruence is used.

24. $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$
25. $AB \cong AB$
26. $\triangle XYZ \cong \triangle LMN$ and $\triangle LMN \cong \triangle XYZ$
27. $\triangle ABC \cong \triangle BAC$
28. What type of triangle is $\triangle ABC$ in #27? How do you know?

Use the following diagram for questions 29 and 30.



[Figure 26]

29. Mark the diagram with the following information. $ST \parallel RA; SR \parallel TA; ST \perp TA$ and $SR; SA$ and RT are perpendicularly bisect each other.

30. Using the given information and your markings, name all of the congruent triangles in the diagram.

Review Queue Answers

1. $\angle B \cong \angle H, AB \cong GH, BC \cong HI$

2. $\angle C \cong \angle M, BC \cong LM$

3. The angles add up to 180°

$$\begin{aligned}(5x+2)^\circ + (4x+3)^\circ + (3x-5)^\circ &= 180^\circ \\ 12x &= 180^\circ \\ x &= 15^\circ\end{aligned}$$