2.3 Deductive Reasoning

Difficulty Level: **Basic** | Created by: CK-12

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Learning Objectives

- Apply some basic rules of logic.
- Compare inductive reasoning and deductive reasoning.
- Use truth tables to analyze patterns of logic.

Review Queue

1. Write the converse, inverse, and contrapositive of the following statement:

Football players wear shoulder pads.

2. Is the converse, inverse or contrapositive of #1 true? If not, find a counterexample.

3. If flowers are in bloom, then it is spring.

If it is spring, then the weather is nice.

So, if flowers are blooming, what can we conclude?

Know What? In a fictitious far-away land, a poor peasant is awaiting his fate from the king. He is standing in a stadium, filled with spectators pointing and wondering what is going to happen. Finally, the king directs everyone's attention to two doors, at the floor level with the peasant. Both doors have signs on them, which are below:

Door A	Door B
IN THIS ROOM THERE IS A LADY, AND IN THE OTHER ROOM THERE IS A TIGER.	IN ONE OF THESE ROOMS THERE IS A LADY, AND IN ONE OF THE OTHER ROOMS THERE IS A TIGER.

The king states, "Only one of these statements is true. If you pick correctly, you will find the lady. If not, the tiger will be waiting for you." Which door should the peasant pick?

Deductive Reasoning

Logic: The study of reasoning.

In the first section, you learned about inductive reasoning, which is to make conclusions based upon patterns and observations. Now, we will learn about deductive reasoning. Deductive reasoning draws conclusions from facts.

Deductive Reasoning: When a conclusion is drawn from facts. Typically, conclusions are drawn from general statements about something more specific.

Example 1: Suppose Bea makes the following statements, which are known to be true.

If Central High School wins today, they will go to the regional tournament.

Central High School won today.

What is the logical conclusion?

Solution: This is an example of deductive reasoning. There is one logical conclusion if these two statements are true: *Central High School will go to the regional tournament.*

Example 2: Here are two true statements.

Every odd number is the sum of an even and an odd number.

5 is an odd number.

What can you conclude?

Solution: Based on only these two true statements, there is one conclusion: *5 is the sum of an even and an odd number.* (This is true, $5 = 3 + 2$ or $4 + 1$).

Law of Detachment

Let's look at Example 2 and change it into symbolic form.

$$p: \text{ A number is odd} \qquad q: \text{ It is the sum of an even and odd number}$$

So, the first statement is $p \rightarrow q$.

- The second statement in Example 2, "5 is an odd number," is a specific example of p. "A number" is 5.
- The conclusion is q. Again it is a specific example, such as $4 + 1$ or $2 + 3$.

The symbolic form of Example 2 is:

$$\begin{aligned} & p \rightarrow q \\ & p \\ & \therefore q \qquad\qquad \therefore \text{ symbol for ``therefore''} \end{aligned}$$

All deductive arguments that follow this pattern have a special name, the Law of Detachment.

Law of Detachment: Suppose that $p \rightarrow q$ is a true statement and given p. Then, you can conclude q.

Another way to say the Law of Detachment is: "If $p \rightarrow q$ is true, and p is true, then q is true."

Example 3: Here are two true statements.

If $\angle A$ *and* $\angle B$ *are a linear pair, then* $m\angle A + m\angle B = 180^\circ$.

$\angle ABC$ *and* $\angle CBD$ *are a linear pair.*

What conclusion can you draw from this?

Solution: This is an example of the Law of Detachment, therefore:

$$m\angle ABC + m\angle CBD = 180^\circ$$

Example 4: Here are two true statements. *Be careful!*

If $\angle A$ *and* $\angle B$ *are a linear pair, then* $m\angle A + m\angle B = 180^\circ$.

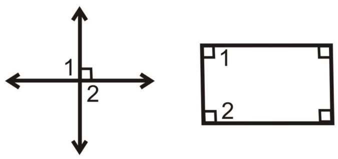
$m\angle 1 = 90^\circ$ *and* $m\angle 2 = 90^\circ$.

What conclusion can you draw from these two statements?

Solution: Here there is NO conclusion. These statements are in the form:

$$\begin{aligned} &p \rightarrow q \\ &q \end{aligned}$$

We ***cannot*** conclude that $\angle 1$ and $\angle 2$ are a linear pair. We are told that $m\angle 1 = 90^\circ$ and $m\angle 2 = 90^\circ$ and while $90^\circ + 90^\circ = 180^\circ$, this does not mean they are a linear pair. Here are two counterexamples.



[Figure 1]

In both of these counterexamples, $\angle 1$ and $\angle 2$ are right angles. In the first, they are vertical angles and in the second, they are two angles in a rectangle.

This is called the *Converse Error* because the second statement is the conclusion of the first, like the converse of a statement.

Law of Contrapositive

Example 5: The following two statements are true.

If a student is in Geometry, then he or she has passed Algebra I.

Daniel has not passed Algebra I.

What can you conclude from these two statements?

Solution: These statements are in the form:

$$p \to q$$
$$\sim q$$

Not q is the beginning of the contrapositive $(\sim q \to \sim p)$, therefore the logical conclusion is *not* p*: Daniel is not in Geometry.*

This example is called the Law of Contrapositive.

Law of Contrapositive: Suppose that $p \to q$ is a true statement and given $\sim q$. Then, you can conclude $\sim p$.

Recall that the logical equivalent to a conditional statement is its contrapositive. Therefore, the Law of Contrapositive is a logical argument.

Example 6: Determine the conclusion from the true statements below.

Babies wear diapers.

My little brother does not wear diapers.

Solution: The second statement is the equivalent of $\sim q$. Therefore, the conclusion is $\sim p$, or: *My little brother is not a baby.*

Example 7a: Determine the conclusion from the true statements below.

If you are not in Chicago, then you can't be on the L.

Bill is in Chicago.

Solution: If we were to rewrite this symbolically, it would look like:

$$\sim p \rightarrow \sim q$$
$$p$$

This is not in the form of the Law of Contrapositive or the Law of Detachment, so there is no logical conclusion. You cannot conclude that Bill is on the L because he could be anywhere in Chicago. This is an example of the *Inverse Error* because the second statement is the negation of the hypothesis, like the beginning of the inverse of a statement.

Example 7b: Determine the conclusion from the true statements below.

If you are not in Chicago, then you can't be on the L.

Sally is on the L.

Solution: If we were to rewrite this symbolically, it would look like:

$$\sim p \rightarrow \sim q$$
$$q$$

Even though it looks a little different, this is an example of the Law of Contrapositive. Therefore, the logical conclusion is: *Sally is in Chicago.*

Law of Syllogism

Example 8: Determine the conclusion from the following true statements.

If Pete is late, Mark will be late.

If Mark is late, Karl will be late.

So, if Pete is late, what will happen?

Solution: If Pete is late, this starts a domino effect of lateness. Mark will be late and Karl will be late too. So, if Pete is late, then *Karl will be late*, is the logical conclusion.

Each "then" becomes the next "if" in a chain of statements. The chain can consist of any number of connected statements. This is called the Law of Syllogism

Law of Syllogism: If $p \to q$ and $q \to r$ are true, then $p \to r$ is the logical conclusion.

Typically, when there are more than two linked statements, we continue to use the next letter(s) in the alphabet to represent the next statement(s); $r \to s, s \to t$, and so on.

Example 9: Look back at the **Know What? Revisited** from the previous section. There were 12 linked if-then statements, making one LARGE Law of Syllogism. Write the conclusion from these statements.

Solution: Symbolically, the statements look like this:

$$\begin{array}{llllll} A \to B & B \to C & C \to D & D \to E & E \to F & F \to G \\ G \to H & H \to I & I \to J & J \to K & K \to L & L \to M \\ \therefore A \to M & & & & & \end{array}$$

So, *If the man raises his spoon, then his face is wiped with the napkin.*

Inductive vs. Deductive Reasoning

You have now worked with both inductive and deductive reasoning. They are different but not opposites. Inductive reasoning means reasoning from examples or patterns. Enough examples might make you suspect that a relationship is always true. But, until you go beyond the inductive stage, you can't be absolutely sure that it is always true. That is, you cannot ***prove*** something is true with inductive reasoning.

That's where deductive reasoning takes over. Let's say we have a conjecture that was arrived at inductively, but is not proven. We can use the Law of Detachment, Law of Contrapositive, Law of Syllogism, and other logic rules to prove this conjecture.

Example 10: Determine if the following statements are examples of inductive or deductive reasoning.

a) Solving an equation for x.

b) 1, 10, 100, 1000,...

c) Doing an experiment and writing a hypothesis.

Solution: Inductive Reasoning = Patterns, Deductive Reasoning = Logic from Facts

a) Deductive Reasoning: Each step follows from the next.

b) Inductive Reasoning: This is a pattern.

c) Inductive Reasoning: You make a hypothesis or conjecture comes from the patterns that you found in the experiment (not facts). If you were to *prove* your hypothesis, then you would have to use deductive reasoning.

Truth Tables

So far we know these symbols for logic:

$\sim$ not (negation)

$\rightarrow$ if-then

$\therefore$ therefore

Two more symbols are:

$\wedge$ and

$\vee$ or

We would write "p and q" as $p \wedge q$ and "p or q" as $p \vee q$.

Truth tables use these symbols and are another way to analyze logic.

First, let's relate p and $\sim p$. To make it easier, set p as: *An even number.*

Therefore, $\sim p$ is *An odd number*. Make a truth table to find out if they are both true. Begin with all the "truths" of p, true (T) or false (F).

p
T
F

Next we write the corresponding truth values for $\sim p$. $\sim p$ has the opposite truth values of p. So, if p is true, then $\sim p$ is false and vice versa.

p	$\sim p$
T	F
F	T

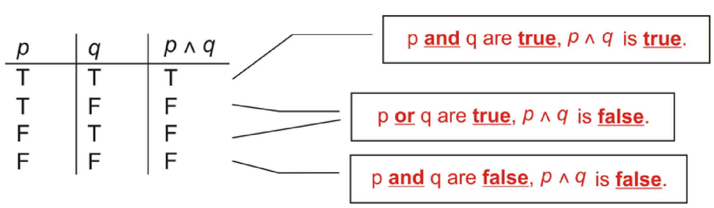
Example 11: Draw a truth table for p, q and $p \wedge q$.

Solution: First, make columns for p and q. Fill the columns with all the possible true and false combinations for the two.

p	q
T	T
T	F
F	T
F	F

Notice all the combinations of p and q. **Anytime we have truth tables with two variables, this is always how we fill out the first two columns.**

Next, we need to figure out when $p \wedge q$ is true, based upon the first two columns. $p \wedge q$ **can only be true if BOTH p and q are true.** So, the completed table looks like this:



[Figure 2]

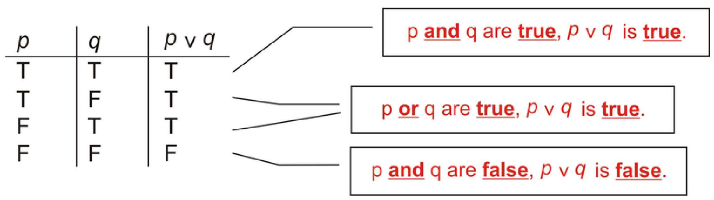
This is how a truth table with two variables and their "and" column is always filled out.

Example 12: Draw a truth table for p, q and $p \vee q$.

Solution: First, make columns for p and q, just like Example 11.

p	q
T	T
T	F
F	T
F	F

Next, we need to figure out when $p \vee q$ is true, based upon the first two columns. $p \vee q$ **is true if** p **OR** q **are true, or both are true.** So, the completed table looks like this:



[Figure 3]

The difference between $p \wedge q$ and $p \vee q$ is the second and third rows. For "and" both p and q have to be true, but for "or" only one has to be true.

Example 13: Determine the truths for $p \wedge (\sim q \vee r)$.

Solution: First, there are three variables, so we are going to need all the combinations of their truths. **For three variables, there are always 8 possible combinations.**

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Next, address the $\sim q$. It will just be the opposites of the q column.

p	q	r	$\sim q$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

Now, let's do what's in the parenthesis, $\sim q \vee r$. Remember, for "or" only $\sim q$ OR r has to be true. Only use the $\sim q$ and r columns to determine the values in this column.

p	q	r	$\sim q$	$\sim q \vee r$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

Finally, we can address the entire problem, $p \wedge (\sim q \vee r)$. Use the p and $\sim q \vee r$ to determine the values. Remember, for "and" both p and $\sim q \vee r$ must be true.

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

To Recap:

- Start truth tables with all the possible combinations of truths. For 2 variables there are 4 combinations for 3 variables there are 8. **You always start a truth table this way.**
- Do any negations on the any of the variables.
- Do any combinations in parenthesis.
- Finish with completing what the problem was asking for.

Know What? Revisited Analyze the two statements on the doors.

Door A: IN THIS ROOM THERE IS A LADY, AND IN THE OTHER ROOM THERE IS A TIGER.

Door B: IN ONE OF THESE ROOMS THERE IS A LADY, AND IN ONE OF THE OTHER ROOMS THERE IS A TIGER.

We know that one door is true, so the other one must be false. Let's assume that Door A is true. That means the lady is behind Door A and the tiger is behind Door B. However, if we read Door B carefully, it says "in one of these rooms," which means the lady could be behind either door, which is actually the true statement. So, because Door B is the true statement, Door A is false and the tiger is actually behind it. Therefore, the peasant should pick Door B.

Review Questions

Determine the logical conclusion and state which law you used (Law of Detachment, Law of Contrapositive, or Law of Syllogism). If no conclusion can be drawn, write "no conclusion."

1. People who vote for Jane Wannabe are smart people. I voted for Jane Wannabe.

2. If Rae is the driver today then Maria is the driver tomorrow. Ann is the driver today.
3. If a shape is a circle, then it never ends. If it never ends, then it never starts. If it never starts, then it doesn't exist. If it doesn't exist, then we don't need to study it.
4. If you text while driving, then you are unsafe. You are a safe driver.
5. If you wear sunglasses, then it is sunny outside. You are wearing sunglasses.
6. If you wear sunglasses, then it is sunny outside. It is cloudy.
7. I will clean my room if my mom asks me to. I am not cleaning my room.
8. If I go to the park, I bring my dog. If I bring my dog, we play fetch with a stick. If we play fetch, my dog gets dirty. If my dog gets dirty, I give him a bath.
9. Write the symbolic representation of #3. Include your conclusion. Is this a sound argument? Does it make sense?
10. Write the symbolic representation of #1. Include your conclusion.
11. Write the symbolic representation of #7. Include your conclusion.

For questions 12 and 13, rearrange the order of the statements (you may need to use the Law of Contrapositive too) to discover the logical conclusion.

12. If I shop, then I will buy shoes. If I don't shop, then I didn't go to the mall. If I need a new watch battery, then I go to the mall.
13. If Anna's parents don't buy her ice cream, then she didn't get an A on her test. If Anna's teacher gives notes, Anna writes them down. If Anna didn't get an A on her test, then she couldn't do the homework. If Anna writes down the notes, she can do the homework.

Determine if the problems below represent inductive or deductive reasoning. Briefly explain your answer.

14. John is watching the weather. As the day goes on it gets more and more cloudy and cold. He concludes that it is going to rain.
15. Beth's 2-year-old sister only eats hot dogs, blueberries and yogurt. Beth decides to give her sister some yogurt because she is hungry.
16. Nolan Ryan has the most strikeouts of any pitcher in Major League Baseball. Jeff debates that he is the best pitcher of all-time for this reason.
17. Ocean currents and waves are dictated by the weather and the phase of the moon. Surfers use this information to determine when it is a good time to hit the water.
18. As Rich is driving along the 405, he notices that as he gets closer to LAX the traffic slows down. As he passes it, it speeds back up. He concludes that anytime he drives past an airport, the traffic will slow down.

19. Amani notices that the milk was left out on the counter. Amani remembers that she put it away after breakfast so it couldn't be her who left it out. She also remembers hearing her mother tell her brother on several occasions to put the milk back in the refrigerator. She concludes that he must have left it out.
20. At a crime scene, the DNA of four suspects is found to be present. However, three of them have an alibi for the time of the crime. The detectives conclude that the fourth possible suspect must have committed the crime.

Write a truth table for the following variables.

21. $p \wedge \sim p$
22. $\sim p \vee \sim q$
23. $p \wedge (q \vee \sim q)$
24. $(p \wedge q) \vee \sim r$
25. $p \vee (\sim q \vee r)$
26. $p \wedge (q \vee \sim r)$
27. The only difference between 24 and 26 is the placement of the parenthesis. How does the truth table differ?
28. When is $p \vee q \vee r$ true?

Is the following a valid argument? If so, what law is being used? HINT: Statements could be out of order.

29. $$\begin{array}{r} p \rightarrow q \\ r \rightarrow p \\ \therefore r \rightarrow q \end{array}$$
30. $$\begin{array}{r} p \rightarrow q \\ r \rightarrow q \\ \therefore p \rightarrow r \end{array}$$
31. $$\begin{array}{r} p \rightarrow \sim r \\ r \\ \therefore \sim p \end{array}$$
32. $$\begin{array}{r} \sim q \rightarrow r \\ q \\ \therefore \sim r \end{array}$$

33. $$\begin{aligned} &p \rightarrow (r \rightarrow s) \\ &p \\ &\therefore r \rightarrow s \end{aligned}$$

34. $$\begin{aligned} &r \rightarrow q \\ &r \rightarrow s \\ &\therefore q \rightarrow s \end{aligned}$$

Review Queue Answers

1. Converse: If you wear shoulder pads, then you are a football player.

Inverse: If you are not a football player, then you do not wear shoulder pads.

Contrapositive: If you do not wear shoulder pads, then you are not a football player.

2. The converse and inverse are both false. A counterexample for both could be a woman from the 80's. They definitely wore shoulder pads!

3. You could conclude that the weather is nice.