11.1 Exploring Solids

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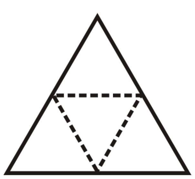
Learning Objectives

- Identify different types of solids and their parts.
- Use Euler's formula to solve problems.
- Draw and identify different views of solids.
- Draw and identify nets.

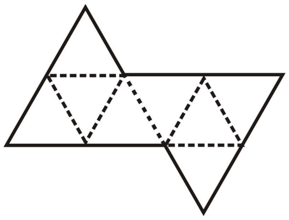
Review Queue

1. Draw an octagon and identify the edges and vertices of the octagon. How many of each are there?
2. Find the area of a square with 5 cm sides.
3. Find the area of an equilateral triangle with 10 in sides.
4. Draw the following polygons.
   a. A convex pentagon.
   b. A concave nonagon.

Know What? Until now, we have only talked about two-dimensional, or flat, shapes. In this chapter we are going to expand to 3D. Copy the equilateral triangle to the right onto a piece of paper and cut it out. Fold on the dotted lines. What shape do these four equilateral triangles make? If we place two of these equilateral triangles next to each other (like in the far right) what shape do these 8 equilateral triangles make?



[Figure 1]

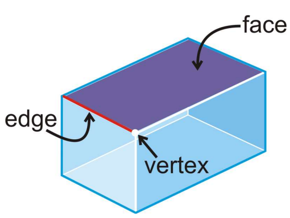


[Figure 2]

Polyhedrons

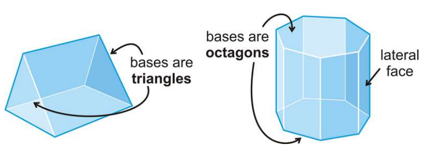
Polyhedron: A 3-dimensional figure that is formed by polygons that enclose a region in space.

Each polygon in a polyhedron is called a ***face***. The line segment where two faces intersect is called an ***edge*** and the point of intersection of two edges is a ***vertex***. There are no gaps between the edges or vertices in a polyhedron. Examples of polyhedrons include a cube, prism, or pyramid. Non-polyhedrons are cones, spheres, and cylinders because they have sides that are not polygons.



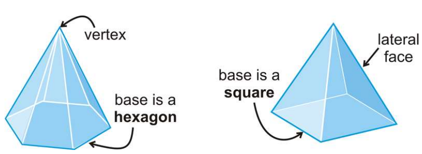
[Figure 3]

Prism: A polyhedron with two congruent bases, in parallel planes, and the lateral sides are rectangles.



[Figure 4]

Pyramid: A polyhedron with one base and all the lateral sides meet at a common vertex. The lateral sides are triangles.

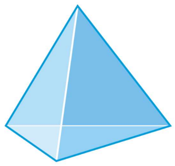


[Figure 5]

All prisms and pyramids are named by their bases. So, the first prism would be a triangular prism and the second would be an octagonal prism. The first pyramid would be a hexagonal pyramid and the second would be a square pyramid. The lateral faces of a pyramid are always triangles.

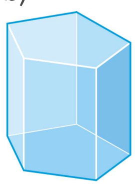
Example 1: Determine if the following solids are polyhedrons. If the solid is a polyhedron, name it and determine the number of faces, edges and vertices each has.

a)



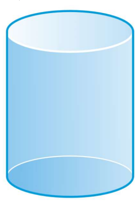
[Figure 6]

b)



[Figure 7]

c)



[Figure 8]

Solution:

a) The base is a triangle and all the sides are triangles, so this is a polyhedron, a triangular pyramid. There are 4 faces, 6 edges and 4 vertices.

b) This solid is also a polyhedron because all the faces are polygons. The bases are both pentagons, so it is a pentagonal prism. There are 7 faces, 15 edges, and 10 vertices.

c) This is a cylinder and has bases that are circles. Circles are not polygons, so it is not a polyhedron.

Euler's Theorem

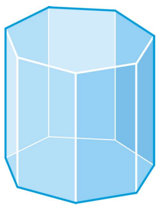
Let's put our results from Example 1 into a table.

	Faces	*Vertices*	*Edges*
Triangular Pyramid	4	4	6
Pentagonal Prism	7	10	15

Notice that the sum of the faces + vertices is two more that the number of edges. This is called Euler's Theorem, after the Swiss mathematician Leonhard Euler.

Euler's Theorem: The number of faces (F), vertices (V), and edges (E) of a polyhedron can be related such that $F+V=E+2$.

Example 2: Find the number of faces, vertices, and edges in the octagonal prism.



[Figure 9]

Solution: Because this is a polyhedron, we can use Euler's Theorem to find either the number of faces, vertices or edges. It is easiest to count the faces, there are 10 faces. If we count the vertices, there are 16. Using this, we can solve for E in Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 10+16 &= E+2 \\ 24 &= E \qquad \text{There are 24 edges.} \end{aligned}$$

Example 3: In a six-faced polyhedron, there are 10 edges. How many vertices does the polyhedron have?

Solution: Solve for V in Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 6+V &= 10+2 \\ V &= 6 \qquad \text{There are 6 vertices.} \end{aligned}$$

Example 4: A three-dimensional figure has 10 vertices, 5 faces, and 12 edges. Is it a polyhedron?

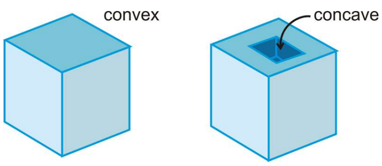
Solution: Plug in all three numbers into Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 5+10 &= 12+2 \\ 15 &\neq 14 \end{aligned}$$

Because the two sides are not equal, this figure is not a polyhedron.

Regular Polyhedra

Regular Polyhedron: A polyhedron where all the faces are congruent regular polygons.



[Figure 10]

Polyhedrons, just like polygons, can be ***convex*** or ***concave*** (also called non-convex). All regular polyhedron are convex. A concave polyhedron is similar to a concave polygon. The polyhedron "caves in," so that two non-adjacent vertices can be connected by a line segment that is outside the polyhedron.

There are five regular polyhedra called the Platonic solids, after the Greek philosopher Plato. These five solids are significant because they are the only five regular polyhedra. There are only five because the sum of the measures of the angles that meet at each vertex must be less than 360° . Therefore the only combinations are 3, 4 or 5 triangles at each vertex, 3 squares at each vertex or 3 pentagons. Each of these polyhedra have a name based on the number of sides, except the cube.

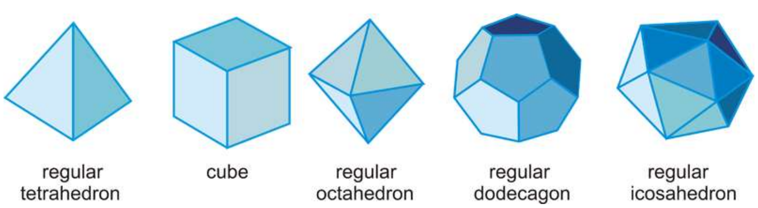
Regular Tetrahedron: A 4-faced polyhedron where all the faces are equilateral triangles.

Cube: A 6-faced polyhedron where all the faces are squares.

Regular Octahedron: An 8-faced polyhedron where all the faces are equilateral triangles.

Regular Dodecahedron: A 12-faced polyhedron where all the faces are regular pentagons.

Regular Icosahedron: A 20-faced polyhedron where all the faces are equilateral triangles.



[Figure 11]

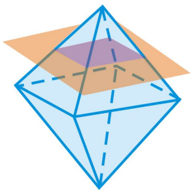
Cross-Sections

One way to "view" a three-dimensional figure in a two-dimensional plane, like this text, is to use cross-sections.

Cross-Section: The intersection of a plane with a solid.

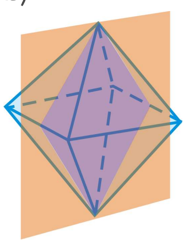
Example 5: Describe the shape formed by the intersection of the plane and the regular octahedron.

a)



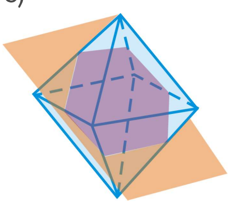
[Figure 12]

b)



[Figure 13]

c)



[Figure 14]

Solution:

a) Square

b) Rhombus

c) Hexagon

Nets

Another way to represent a three-dimensional figure in a two dimensional plane is to use a net.

Net: An unfolded, flat representation of the sides of a three-dimensional shape.

Example 6: What kind of figure does this net create?

[Figure 15]

Solution: The net creates a rectangular prism.

[Figure 16]

Example 7: Draw a net of the right triangular prism below.

[Figure 17]

Solution: This net will have two triangles and three rectangles. The rectangles are all different sizes and the two triangles are congruent.

[Figure 18]

Notice that there could be a couple different interpretations of this, or any, net. For example, this net could have the triangles anywhere along the top or bottom of the three rectangles. Most prisms have multiple nets.

See the site http://www.cs.mcgill.ca/~sqrt/unfold/unfolding.html if you would like to see a few animations of other nets, including the Platonic solids.

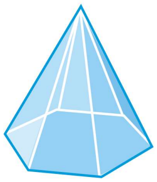
Know What? Revisited The net of the first shape is a regular tetrahedron and the second is the net of a regular octahedron.

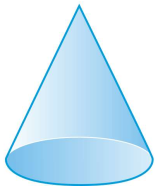
Review Questions

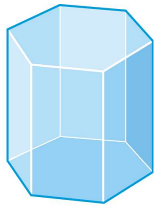
Complete the table using Euler's Theorem.

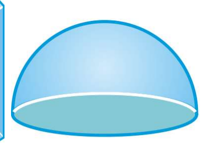
	Name	*Faces*	*Edges*	*Vertices*
1.	Rectangular Prism	6	12	
2.	Octagonal Pyramid		16	9
3.	Regular Icosahedron	20		12
4.	Cube		12	8
5.	Triangular Pyramid	4		4
6.	Octahedron	8	12	
7.	Heptagonal Prism		21	14
8.	Triangular Prism	5	9	

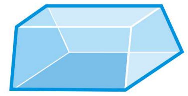
Determine if the following figures are polyhedra. If so, name the figure and find the number of faces, edges, and vertices.

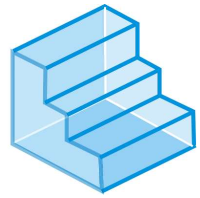

[Figure 19]


[Figure 20]

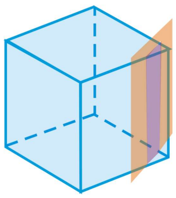

[Figure 21]

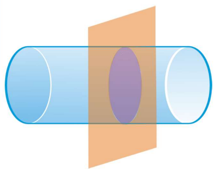

[Figure 22]

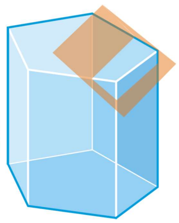

[Figure 23]


[Figure 24]

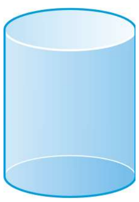
Describe the cross section formed by the intersection of the plane and the solid.

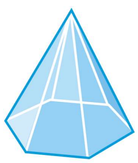

[Figure 25]

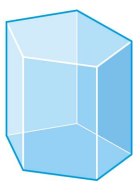

[Figure 26]


[Figure 27]

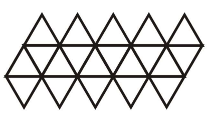
Draw the net for the following solids.

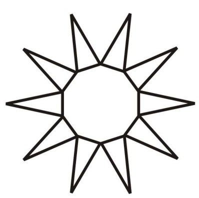

[Figure 28]

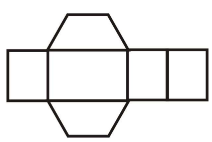

[Figure 29]


[Figure 30]

Determine what shape is formed by the following nets.


[Figure 31]


[Figure 32]


[Figure 33]

21. A cube has 11 unique nets. Draw 5 different nets of a cube.

A truncated icosahedron is a polyhedron with 12 regular pentagonal faces and 20 regular hexagonal faces and 90 edges. This icosahedron closely resembles a soccer ball. How many vertices does it have? Explain your reasoning.

[Figure 34]

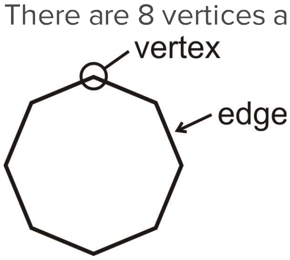
22. Use construction tools to construct a large equilateral triangle. Construct the three midsegments of the triangle. Cut out the equilateral triangle and fold along the midsegments. What net have you constructed?
23. Describe a method to construct a net for a regular octahedron.

For problems 28-30, we are going to connect the Platonic Solids to probability. A six sided die is the shape of a cube. The probability of any one side landing face up is $\frac{1}{6}$ because each of the six faces is congruent to each other.

28. What shape would we make a die with 12 faces? If we number these faces 1 to 12, and each face has the same likelihood of landing face up, what is the probability of rolling a multiple of three?
29. I have a die that is a regular octahedron. Each face is labeled with a consecutive prime number starting with 2. What is the largest prime number on my die?
30. ***Challenge*** Rebecca wants to design a new die. She wants it to have one red face. The other faces will be yellow, blue or green. How many faces should her die have and how many of each color does it need so that: the probability of rolling yellow is eight times the probability of rolling red, the probability of rolling green is half the probability of rolling yellow and the probability of rolling blue is seven times the probability of rolling red?

Review Queue Answers

There are 8 vertices and 8 edges in an octagon.

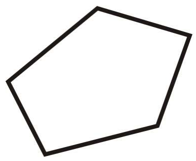


[Figure 35]

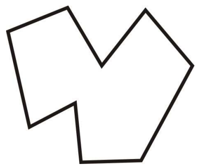
1. $5^2 = 25\ cm^2$

2. $\frac{1}{2} \cdot 10 \cdot 5\sqrt{3} = 25\sqrt{3}\, in^2$

3. Answers:



[Figure 36]



[Figure 37]