9.7 Extension: Writing and Graphing the Equations of Circles

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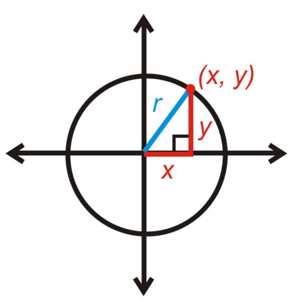
Last Modified: Dec 25, 2014

Learning Objectives

- Graph a circle.
- Find the equation of a circle in the coordinate plane.
- Find the radius and center, given the equation of a circle and vice versa.
- Find the equation of a circle, given the center and a point on the circle.

Graphing a Circle in the Coordinate Plane

Recall that the definition of a circle is the set of all points that are the same distance from a point, called the center. This definition can be used to find an equation of a circle in the coordinate plane.

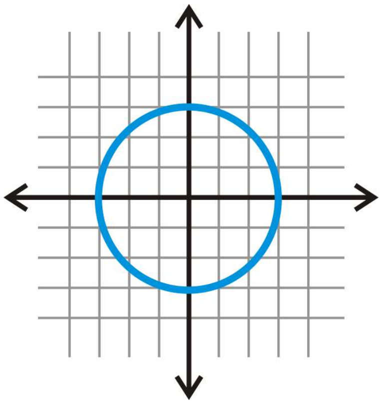


[Figure 1]

Let's start with the circle centered at the origin, (0, 0). If (x, y) is a point on the circle, then the distance from the center to this point would be the radius, r . x is the horizontal distance of the coordinate and y is the vertical distance. Drawing those in, we form a right triangle. Therefore, the equation of a circle, *centered at the origin* is $x^2 + y^2 = r^2$, by the Pythagorean Theorem.

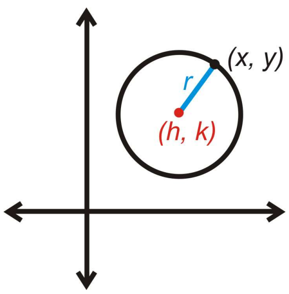
Example 1: Graph $x^2 + y^2 = 9$.

Solution: This circle is centered at the origin. It's radius is the square root of 9, or 3. The easiest way to graph a circle is to plot the center, and then go out 3 units in every direction and connect them to form a circle.



[Figure 2]

The center does not always have to be on (0, 0). If it is not, then we label the center (h, k) and would use the distance formula to find the length of the radius.



[Figure 3]

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

If you square both sides of this equation, then we would have the standard equation of a circle.

Standard Equation of a Circle: The standard equation of a circle with center (h, k) and radius r is $r^2 = (x-h)^2 + (y-k)^2$.

Example 2: Find the center and radius of the following circles.

a) $(x-3)^2 + (y-1)^2 = 25$

b) $(x+2)^2 + (y-5)^2 = 49$

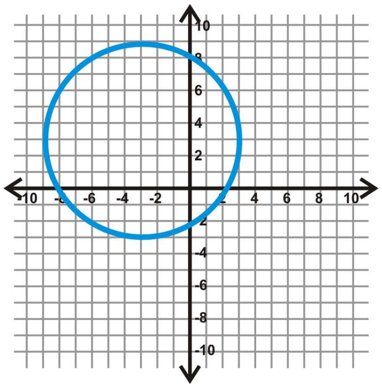
Solution:

a) Rewrite the equation as $(x-3)^2 + (y-1)^2 = 5^2$. Therefore, the center is (3, 1) and the radius is 5.

b) Rewrite the equation as $(x-(-2))^2 + (y-5)^2 = 7^2$. From this, the center is (-2, 5) and the radius is 7.

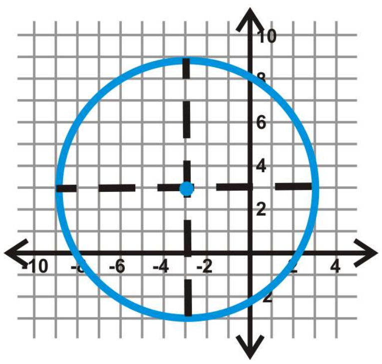
When finding the center of a circle always take the ***opposite sign*** of what the value is in the equation.

Example 3: Find the equation of the circle below.



[Figure 4]

Solution: First locate the center. Draw in a couple diameters. It is easiest to use the horizontal and vertical diameters.



[Figure 5]

From the intersecting diameters, we see that the center is (-3, 3). If we count the units from the center to the circle on either of these diameters, we find that the radius is 6. Plugging this information into the equation of a circle, we get $(x-(-3))^2+(y-3)^2=6^2$ or $(x+3)^2+(y-3)^2=36$.

Finding the Equation of a Circle

Example 4: Find the equation of the circle with center (4, -1) and passes through (-1, 2).

Solution: To find the equation, first plug in the center to the standard equation.

$(x-4)^2+(y-(-1))^2=r^2$ or $(x-4)^2+(y+1)^2=r^2$

Now, plug in (-1, 2) for x and y and solve for r .

$$
\begin{aligned}
(-1-4)^2+(2+1)^2 &= r^2 \\
(-5)^2+(3)^2 &= r^2 \\
25+9 &= r^2 \\
34 &= r^2
\end{aligned}
$$

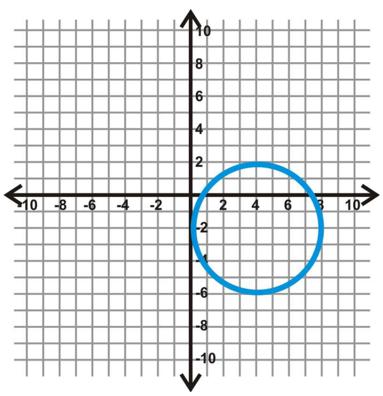
At this point, we don't need to solve for r because r^2 is what is in the equation. Substituting in 34 for r^2, we have $(x-4)^2+(y+1)^2=34$.

Review Questions

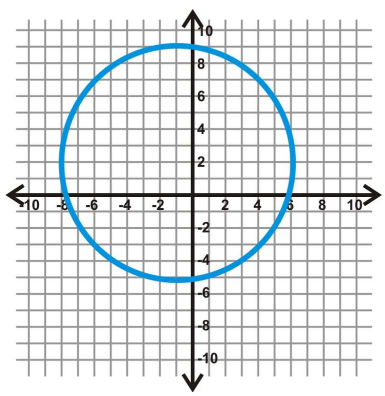
Find the center and radius of each circle. Then, graph each circle.

1. $(x+5)^2+(y-3)^2=16$
2. $x^2+(y+8)^2=4$
3. $(x-7)^2+(y-10)^2=20$
4. $(x+2)^2+y^2=8$

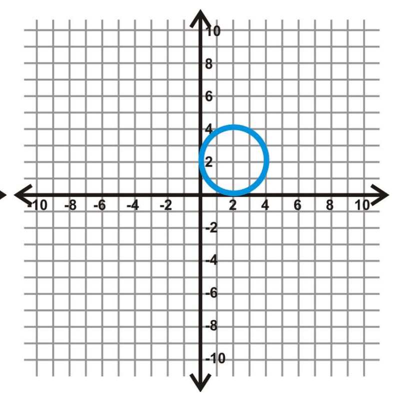
Find the equation of the circles below.



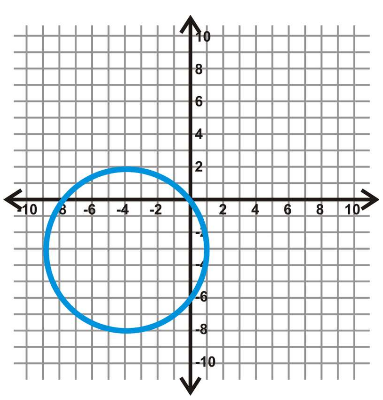
[Figure 6]



[Figure 7]



[Figure 8]



[Figure 9]

5. Determine if the following points are on $(x+1)^2+(y-6)^2=45$.

   a. (2, 0)

   b. (-3, 4)

   c. (-7, 3)

Find the equation of the circle with the given center and point on the circle.

10. center: (2, 3), point: (-4, -1)
11. center: (10, 0), point: (5, 2)
12. center: (-3, 8), point: (7, -2)
13. center: (6, -6), point: (-9, 4)
14. Now let's find the equation of a circle using three points on the circle. Do you remember how we found the center and radius of a circle given three points on the circle in problem 30 of Section 9-3? We used the fact that the perpendicular bisector of any chord in the circle will pass through the center. By finding the perpendicular bisectors of two different chords and their intersection we can find the center of the circle. Then we can use the distance formula with the center and a point on the circle to find the radius. Finally, we will write the equation. Given the points $A(-12,-21), B(2,27)$ and $C(19,10)$ on the circle (an arc could be drawn through these points from A to C), follow the steps below.

    a. Since the perpendicular bisector passes through the midpoint of a segment we must first find the midpoint between A and B .

    b. Now the perpendicular line must have a slope that is the opposite reciprocal of the slope of $\overleftrightarrow{AB}$. Find the slope of $\overleftrightarrow{AB}$ and then its opposite reciprocal.

    c. Finally, you can write the equation of the perpendicular bisector of AB using the point you found in part a and the slope you found in part b.

    d. Repeat steps a-c for chord BC .

    e. Now that we have the two perpendicular bisectors of the chord we can find their intersection. Solve the system of linear equations to find the center of the circle.

    f. Find the radius of the circle by finding the distance from the center (point found in part e) to any of the three given points on the circle.

    g. Now, use the center and radius to write the equation of the circle.

Find the equations of the circles which contain three points in problems 15 and 16.

15. $A(-2,5), B(5,6)$ and $C(6,-1)$

16. $A(-11,-14), B(5,16)$ and $C(12,9)$