3.1 Lines and Angles

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Learning Objectives

- Identify parallel lines, skew lines, and parallel planes.
- Use the Parallel Line Postulate and the Perpendicular Line Postulate.
- Identify angles made by transversals.

Review Queue

1. What is the equation of a line with slope -2 and passes through the point (0, 3)?
2. What is the equation of the line that passes through (3, 2) and (5, -6).
3. Change $4x - 3y = 12$ into slope-intercept form.
4. Are $y = \frac{1}{3}x$ and $y = -3x$ perpendicular? How do you know?

Know What? Below is a partial map of Washington DC. The streets are designed on a grid system, where lettered streets, A through Z run east to west and numbered streets 1^{st} to 30^{th} run north to south. Just to mix things up a little, every state has its own street that runs diagonally through the city. There are, of course other street names, but we will focus on these three groups for this chapter. Can you explain which streets are parallel and perpendicular? Are any skew? How do you know these streets are parallel or perpendicular?

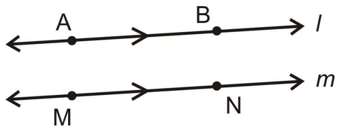
[Figure 1]

If you are having trouble viewing this map, check out the interactive map here: http://www.travelguide.tv/washington/map.html

Defining Parallel and Skew

Parallel: When two or more lines lie in the same plane and never intersect.

The symbol for parallel is $||$. To mark lines parallel, draw arrows $(>)$ on each parallel line. If there are more than one pair of parallel lines, use two arrows $(>>)$ for the second pair. The two lines to the right would be labeled $\overleftrightarrow{AB} \,||\, \overleftrightarrow{MN}$ or $l \,||\, m$.



[Figure 2]

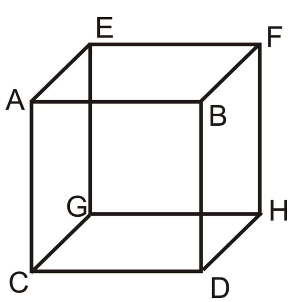
Planes can also be parallel or perpendicular. The image to the left shows two parallel planes, with a third blue plane that is perpendicular to both of them.

[Figure 3]

An example of parallel planes could be the top of a table and the floor. The legs would be in perpendicular planes to the table top and the floor.

Skew lines: Lines that are in different planes and never intersect.

Example 1: In the cube above, list:



[Figure 4]

a) 3 pairs of parallel planes

b) 2 pairs of perpendicular planes

c) 3 pairs of skew line segments

Solution:

a) Planes ABC and EFG, Planes AEG and FBH, Planes AEB and CDH

b) Planes ABC and CDH, Planes AEB and FBH (there are others, too)

c) BD and CG, BF and EG, GH and AE (there are others, too)

Parallel Line Postulate

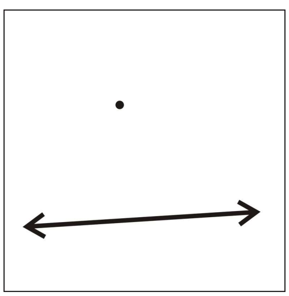
Parallel Line Postulate: Fora line and a point not on the line, there is exactly one line parallel to this line through the point.

There are infinitely many lines that pass through A, but only one is parallel to l.

Investigation 3-1: Patty Paper and Parallel Lines

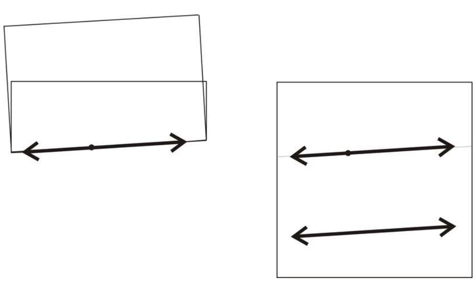
1. Get a piece of patty paper (a translucent square piece of paper).

Draw a line and a point above the line.



[Figure 5]

2. Fold up the paper so that the line is over the point. Crease the paper and unfold.



[Figure 6]

3. Are the lines parallel? Yes, by design, this investigation replicates the line we drew in #1 over the point. Therefore, there is only one line parallel through this point to this line.

Perpendicular Line Postulate

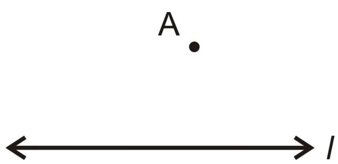
Perpendicular Line Postulate: For a line and a point not on the line, there is exactly one line perpendicular to the line that passes through the point.

There are infinitely many lines that pass through A, but only one that is perpendicular to l.

Investigation 3-2: Perpendicular Line Construction; through a Point NOT on the Line

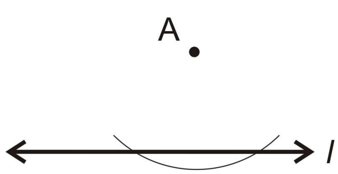
1. Draw a horizontal line and a point above that line.

Label the line l and the point A.



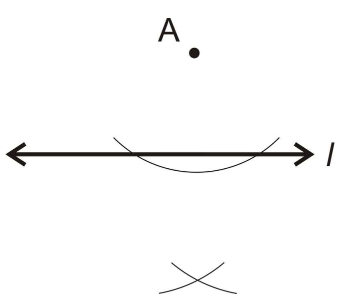
[Figure 7]

2. Take the compass and put the pointer on A. Open the compass so that it reaches beyond line l. Draw an arc that intersects the line twice.



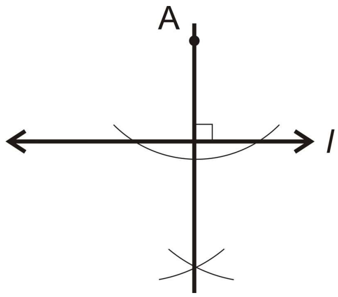
[Figure 8]

3. Move the pointer to one of the arc intersections. Widen the compass a little and draw an arc below the line. Repeat this on the other side so that the two arc marks intersect.



[Figure 9]

4. Take your straightedge and draw a line from point A to the arc intersections below the line. This line is perpendicular to l and passes through A.



[Figure 10]

Notice that this is a different construction from a perpendicular bisector.

To see a demonstration of this construction, go to:
http://www.mathsisfun.com/geometry/construct-perpnotline.html

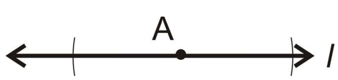
Investigation 3-3: Perpendicular Line Construction; through a Point on the Line

1. Draw a horizontal line and a point on that line.

Label the line l and the point A.

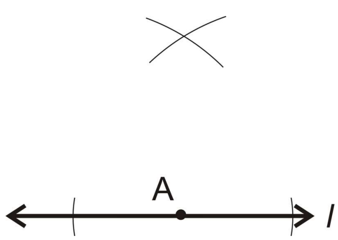
[Figure 11]

2. Take the compass and put the pointer on A. Open the compass so that it reaches out horizontally along the line. Draw two arcs that intersect the line on either side of the point.



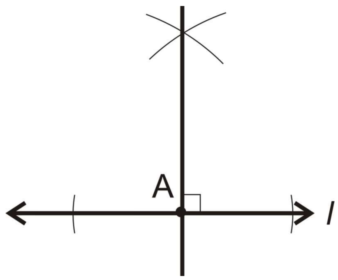
[Figure 12]

3. Move the pointer to one of the arc intersections. Widen the compass a little and draw an arc above or below the line. Repeat this on the other side so that the two arc marks intersect.



[Figure 13]

4. Take your straightedge and draw a line from point A to the arc intersections above the line. This line is perpendicular to l and passes through A.



[Figure 14]

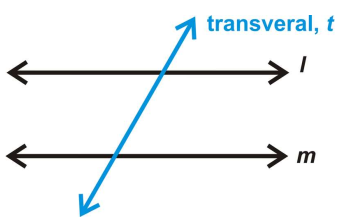
Notice that this is a different construction from a perpendicular bisector.

To see a demonstration of this construction, go to:
http://www.mathsisfun.com/geometry/construct-perponline.html

Angles and Transversals

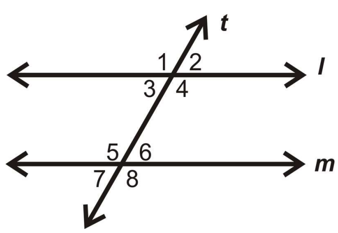
Transversal: A line that intersects two distinct lines. These two lines may or may not be parallel.

The area *between* l and m is the called the *interior*. The area *outside* l and m is called the *exterior*.



[Figure 15]

Looking at t, l and m, there are 8 angles formed and several linear pairs vertical angle pairs. There are also 4 new angle relationships, defined here:



[Figure 16]

Corresponding Angles: Two angles that are in the "same place" with respect to the transversal, but on different lines. Imagine sliding the four angles formed with line l down to line m. The angles which match up are corresponding. ***$\angle 2$ and $\angle 6$ are corresponding angles.***

Alternate Interior Angles: Two angles that are on the <u>interior</u> of l and m, but on opposite sides of the transversal. ***$\angle 3$ and $\angle 6$ are alternate interior angles.***

Alternate Exterior Angles: Two angles that are on the exterior of l and m, but on opposite sides of the transversal. $\angle 1$ ***and*** $\angle 8$ ***are alternate exterior angles.***

Same Side Interior Angles: Two angles that are on the same side of the transversal and on the interior of the two lines. $\angle 3$ ***and*** $\angle 5$ ***are same side interior angles.***

Example 2: Using the picture above, list all the other pairs of each of the newly defined angle relationships.

Solution:

Corresponding Angles: $\angle 3$ and $\angle 7$, $\angle 1$ and $\angle 5,$ $\angle 4$ and $\angle 8$

Alternate Interior Angles: $\angle 4$ and $\angle 5$

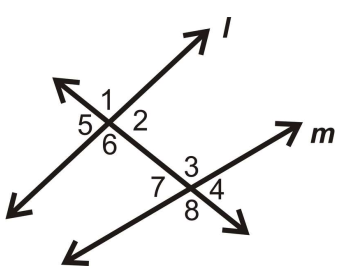
Alternate Exterior Angles: $\angle 2$ and $\angle 7$

Same Side Interior Angles: $\angle 4$ and $\angle 6$

Example 3: If $\angle 2 = 48^\circ$ (in the picture above), what other angles do you know?

Solution: $\angle 2 \cong \angle 3$ by the Vertical Angles Theorem, so $m\angle 3 = 48^\circ$. $\angle 2$ is also a linear pair with $\angle 1$ and $\angle 4$, so it is supplementary to those two. They are both 132°. We do not know the measures of $\angle 5,$ $\angle 6,$ $\angle 7$, or $\angle 8$ because we do not have enough information.

Example 4: For the picture to the right, determine:



[Figure 17]

a) A corresponding angle to $\angle 3$?

b) An alternate interior angle to $\angle 7$?

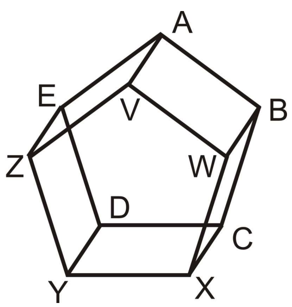
c) An alternate exterior angle to $\angle 4$?

Solution: The corresponding angle to $\angle 3$ is $\angle 1$. The alternate interior angle to $\angle 7$ is $\angle 2$. And, the alternate exterior angle to $\angle 4$ is $\angle 5$.

Know What? Revisited For Washington DC, all of the lettered streets are parallel, as are all of the numbered streets. The lettered streets are perpendicular to the numbered streets. There are no skew streets because all of the streets are in the same plane. We also do not know if any of the state-named streets are parallel or perpendicular.

Review Questions

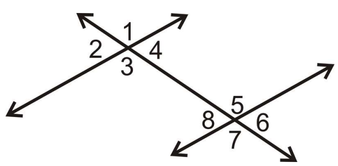
Use the figure below to answer questions 1-5. The two pentagons are parallel and all of the rectangular sides are perpendicular to both of them.



[Figure 18]

1. Find two pairs of skew lines.
2. List a pair of parallel lines.
3. List a pair of perpendicular lines.
4. For AB, how many perpendicular lines pass through point V? What line is this?
5. For XY, how many parallel lines passes through point D? What line is this?

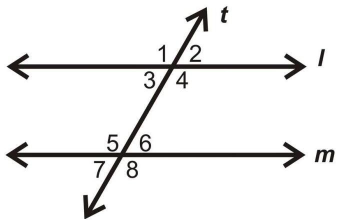
For questions 6-12, use the picture below.



[Figure 19]

6. What is the corresponding angle to $\angle 4$?
7. What is the alternate interior angle with $\angle 5$?
8. What is the corresponding angle to $\angle 8$?
9. What is the alternate exterior angle with $\angle 7$?
10. What is the alternate interior angle with $\angle 4$?
11. What is the same side interior angle with $\angle 3$?
12. What is the corresponding angle to $\angle 1$?

Use the picture below for questions 13-16.



[Figure 20]

13. If $m\angle 2 = 55^\circ$, what other angles do you know?
14. If $m\angle 5 = 123^\circ$, what other angles do you know?
15. If $t \perp l$, is $t \perp m$? Why or why not?
16. Is $l \, || \, m$? Why or why not?
17. ***Construction*** Draw a line and a point not on the line. Construct a perpendicular line to your original line through your point.
18. ***Construction*** Construct a perpendicular line to the line you constructed in #12. Use the point you originally drew, so that you will be constructing a perpendicular line through a point on the line.
19. Can you use patty paper to do the construction in number 17? Draw a line and a point not on the line on a piece of patty paper (or any thin white paper or tracing paper). Think about how you could make a crease in the paper that would be a line perpendicular to your original line through your point.
20. Using what you discovered in number 19, use patty paper to construct a line perpendicular to a given line through a point on the given line.
21. Draw a pair of parallel lines using your ruler. Describe how you did this.
22. Draw a pair of perpendicular lines using your ruler. Describe your method.

Geometry is often apparent in nature. Think of examples of each of the following in nature.

23. Parallel Lines or Planes
24. Perpendicular Lines or Planes
25. Skew Lines

Algebra Connection In questions 26-35 we will begin to explore the concepts of parallel and perpendicular lines in the coordinate plane.

26. Write the equations of two lines parallel to $y = 3$.
27. Write the equations of two lines perpendicular to $y = 5$.
28. What is the relationship between the two lines you found for number 27?

29. Plot the points $A(2,-5),\ B(-3,1),\ C(0,4),\ D(-5,10)$. Draw the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. What are the slopes of these lines? What is the geometric relationship between these lines?
30. Plot the points $A(2,1),\ B(7,-2),\ C(2,-2),\ D(5,3)$. Draw the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. What are the slopes of these lines? What is the geometric relationship between these lines?
31. Based on what you discovered in numbers 29 and 30, can you make a conjecture about the slopes of parallel and perpendicular lines?

Find the equation of the line that is parallel to the given line and passes through (5, -1).

32. $y=2x-7$
33. $y=-\frac{3}{5}x+1$

Find the equation of the line that is *perpendicular* to the given line and passes through (2, 3).

34. $y=\frac{2}{3}x-5$
35. $y=-\frac{1}{4}x+9$

Review Queue Answers

1. $y=-2x+3$
2. $y=-4x+14$
3. $y=\frac{4}{3}x-4$
4. Yes, the lines are perpendicular. The slopes are reciprocals and opposite signs.