5.1 Midsegments of a Triangle

FlexBooks® 2.0 > American HS Geometry > Midsegments of a Triangle

Last Modified: Dec 25, 2014

Learning Objectives

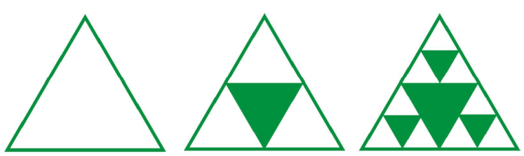
- Identify the midsegments of a triangle.
- Use the Midsegment Theorem to solve problems involving side lengths, midsegments, and algebra.

Review Queue

Find the midpoint between the given points.

1. (-4, 1) and (6, 7)
2. (5, -3) and (11, 5)
3. (0, -2) and (-4, 14)
4. Find the equation of the line between (-2, -3) and (-1, 1).
5. Find the equation of the line that is parallel to the line from #4 through (2, -7).

Know What? A fractal is a repeated design using the same shape (or shapes) of different sizes. Below, is an example of the first few steps of a fractal. What does the next figure look like? How many triangles are in each figure (green and white triangles)? Is there a pattern?

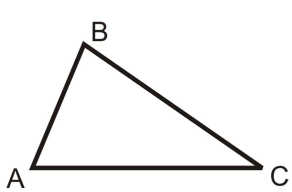


[Figure 1]

Defining Midsegment

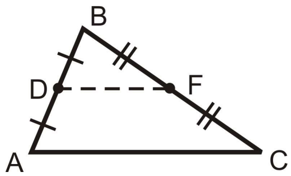
Midsegment: A line segment that connects two midpoints of adjacent sides of a triangle.

Example 1: Draw the midsegment $\overline{DF}$ between $\overline{AB}$ and $\overline{BC}$. Use appropriate tic marks.



[Figure 2]

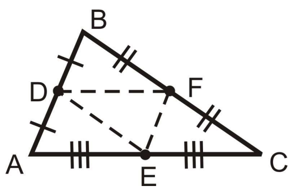
Solution: Find the midpoints of AB and BC using your ruler. Label these points D and F . Connect them to create the midsegment.



[Figure 3]

Don't forget to put the tic marks, indicating that D and F are midpoints, $AD \cong DB$ and $BF \cong FC$.

Example 2: Find the midpoint of AC from $\triangle ABC$. Label it E and find the other two midsegments of the triangle.



[Figure 4]

Solution:

For every triangle there are three midsegments.

Let's transfer what we know about midpoints in the coordinate plane to midsegments in the coordinate plane. We will need to use the midpoint formula, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Example 3: The vertices of $\triangle LMN$ are $L(4,5), M(-2,-7)$ and $N(-8,3)$. Find the midpoints of all three sides, label them O, P and Q . Then, graph the triangle, it's midpoints and draw in the midsegments.

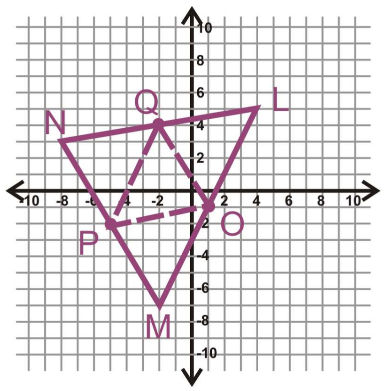
Solution: Use the midpoint formula 3 times to find all the midpoints.

L and $M = \left(\frac{4+(-2)}{2}, \frac{5+(-7)}{2}\right) = (1,-1)$, point O

L and $N = \left(\frac{4+(-8)}{2}, \frac{5+3}{2}\right) = (-2,4)$, point Q

M and $N = \left(\frac{-2+(-8)}{2}, \frac{-7+3}{2}\right) = (-5,-2)$, point P

The graph would look like the graph to the right. We will use this graph to explore the properties of midsegments.



[Figure 5]

Example 4: Find the slopes of NM and QO.

Solution: The slope of NM is $\frac{-7-3}{-2-(-8)} = \frac{-10}{6} = -\frac{5}{3}$.

The slope of QO is $\frac{-1-4}{1-(-2)} = -\frac{5}{3}$.

From this we can conclude that $NM \,||\, QO$. If we were to find the slopes of the other sides and midsegments, we would find $LM \,||\, QP$ and $NL \,||\, PO$. ***This is a property of all midsegments.***

Example 5: Find NM and QO.

Solution: Now, we need to find the lengths of NM and QO. Use the distance formula.

$$NM = \sqrt{(-7-3)^2 + (-2-(-8))^2} = \sqrt{(-10)^2 + 6^2} = \sqrt{100+36} = \sqrt{136} \approx 11.6$$
$$QO = \sqrt{(1-(-2))^2 + (-1-4)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34} \approx 5.83$$

From this we can conclude that QO is **half** of NM. If we were to find the lengths of the other sides and midsegments, we would find that OP is **half** of NL and QP is **half** of LM. ***This is a property of all midsegments.***

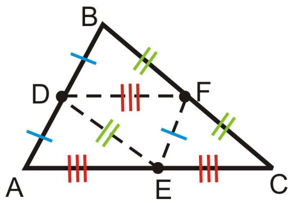
The Midsegment Theorem

The conclusions drawn in Examples 4 and 5 can be generalized into the Midsegment Theorem.

Midsegment Theorem: The midsegment of a triangle is half the length of the side it is parallel to.

Example 6: Mark everything you have learned from the Midsegment Theorem on $\triangle ABC$ above.

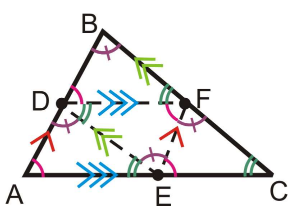
Solution: Let's draw two different triangles, one for the congruent sides, and one for the parallel lines.



[Figure 6]

Because the midsegments are half the length of the sides they are parallel to, they are congruent to half of each of those sides (as marked). Also, this means that all four of the triangles in $\triangle ABC$, created by the midsegments are congruent by SSS.

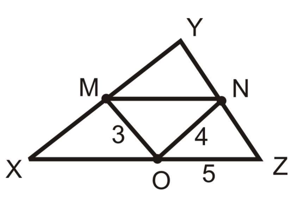
As for the parallel midsegments and sides, several congruent angles are formed. In the picture to the right, the pink and teal angles are congruent because they are corresponding or alternate interior angles. Then, the purple angles are congruent by the 3^{rd} Angle Theorem.



[Figure 7]

To play with the properties of midsegments, go to http://www.mathopenref.com/trianglemidsegment.html.

Example 7: $M, N,$ and O are the midpoints of the sides of the triangle.



[Figure 8]

Find

a) MN

b) XY

c) The perimeter of $\triangle XYZ$

Solution: Use the Midsegment Theorem.

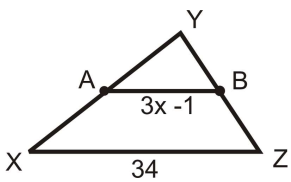
a) $MN = OZ = 5$

b) $XY = 2(ON) = 2 \cdot 4 = 8$

c) The perimeter is the sum of the three sides of $\triangle XYZ$.

$$XY + YZ + XZ = 2 \cdot 4 + 2 \cdot 3 + 2 \cdot 5 = 8 + 6 + 10 = 24$$

Example 8: ***Algebra Connection*** Find the value of x and AB.



[Figure 9]

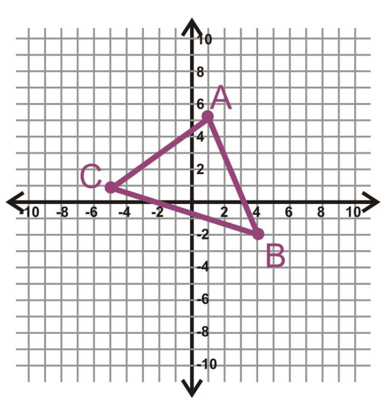
Solution: First, AB is half of 34, or 17. To find x, set $3x - 1$ equal to 17.

$$\begin{aligned} 3x - 1 &= 17 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

Let's go back to the coordinate plane.

Example 9: If the midpoints of the sides of a triangle are $A(1,5), B(4,-2)$, and $C(-5,1)$, find the vertices of the triangle.

Solution: The easiest way to solve this problem is to graph the midpoints and then apply what we know from the Midpoint Theorem.



[Figure 10]

Now that the points are plotted, find the slopes between all three.

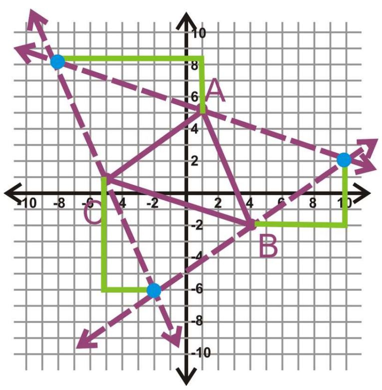
slope $AB = \frac{5+2}{1-4} = -\frac{7}{3}$

slope $BC = \frac{-2-1}{4+5} = \frac{-3}{9} = -\frac{1}{3}$

slope $AC = \frac{5-1}{1+5} = \frac{4}{6} = \frac{2}{3}$

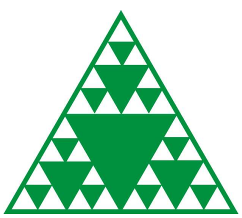
Using the slope between two of the points and the third, plot the slope triangle on either side of the third point and extend the line. Repeat this process for all three midpoints. For example, use the slope of AB with point C.

The green lines in the graph to the left represent the slope triangles of each midsegment. The three dotted lines represent the sides of the triangle. Where they intersect are the vertices of the triangle (the blue points), which are (-8, 8), (10, 2) and (-2, 6).

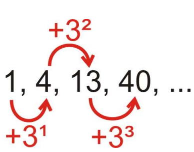


[Figure 11]

Know What? Revisited To the left is a picture of the 4^{th} figure in the fractal pattern. The number of triangles in each figure is 1, 4, 13, and 40. The pattern is that each term increase by the next power of 3.



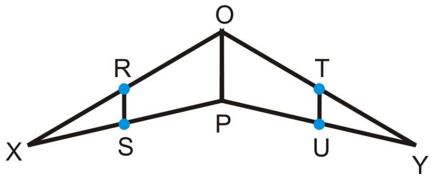
[Figure 12]



[Figure 13]

Review Questions

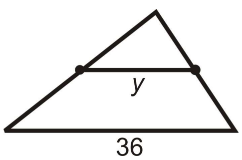
$R, S, T,$ and U are midpoints of the sides of $\triangle XPO$ and $\triangle YPO$.



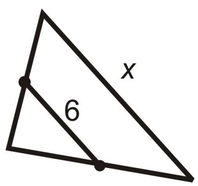
[Figure 14]

1. If $OP = 12$, find RS and TU.
2. If $RS = 8$, find TU.
3. If $RS = 2x$, and $OP = 20$, find x and TU.
4. If $OP = 4x$ and $RS = 6x - 8$, find x.
5. Is $\triangle XOP \cong \triangle YOP$? Why or why not?

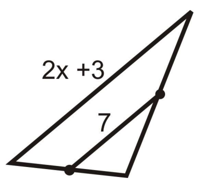
For questions 6-13, find the indicated variable(s). You may assume that all line segments within a triangle are midsegments.



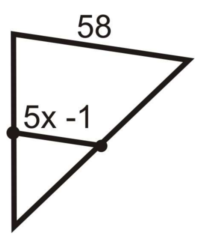
[Figure 15]



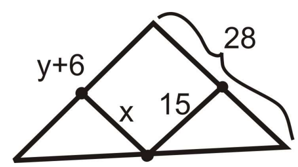
[Figure 16]



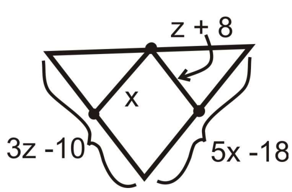
[Figure 17]



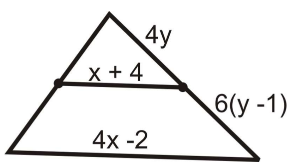
[Figure 18]



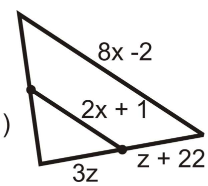
[Figure 19]



[Figure 20]



[Figure 21]



[Figure 22]

6. The sides of $\triangle XYZ$ are 26, 38, and 42. $\triangle ABC$ is formed by joining the midpoints of $\triangle XYZ$.
   a. Find the perimeter of $\triangle ABC$.
   b. Find the perimeter of $\triangle XYZ$.
   c. What is the relationship between the perimeter of a triangle and the perimeter of the triangle formed by connecting its midpoints?

Coordinate Geometry Given the vertices of $\triangle ABC$ below, find the midpoints of each side.

15. $A(5,-2), B(9,4)$ and $C(-3,8)$
16. $A(-10,1), B(4,11)$ and $C(0,-7)$
17. $A(0,5), B(4,-1)$ and $C(-2,-3)$
18. $A(2,4), B(8,-4)$ and $C(2,-4)$

Multi-Step Problem The midpoints of the sides of $\triangle RST$ are $G(0,-2), H(9,1)$, and $I(6,-5)$. Answer the following questions.

19. Find the slope of GH, HI , and GI .
20. Plot the three midpoints and connect them to form midsegment triangle, $\triangle GHI$.
21. Using the slopes, find the coordinates of the vertices of $\triangle RST$.
22. Find GH using the distance formula. Then, find the length of the sides it is parallel to. What should happen?

More Coordinate Geometry Given the midpoints of the sides of a triangle, find the vertices of the triangle. Refer back to problems 19-21 for help.

23. (-2, 1), (0, -1) and (-2, -3)
24. (1, 4), (4, 1) and (2, 1)

Given the vertices of $\triangle ABC$, find:

a) the midpoints of M, N and O where M is the midpoint of AB , N is the midpoint of BC and C is the midpoint of AC .

b) Show that midsegments MN, NO and OM are parallel to sides AC, AB and BC respectively.

c) Show that midsegments MN, NO and OM are half the length of sides AC, AB and BC respectively.

25. $A(-3,5), B(3,1)$ and $C(-5,-5)$
26. $A(-2,2), B(4,4)$ and $C(6,0)$

For questions 27-30, $\triangle CAT$ has vertices $C(x_1,y_1), A(x_2,y_2)$ and $T(x_3,y_3)$.

27. Find the midpoints of sides CA and CT . Label them L and M respectively.
28. Find the slopes of LM and AT .
29. Find the lengths of LM and AT .
30. What have you just proven algebraically?

Review Queue Answers

1. $\left(\frac{-4+6}{2}, \frac{1+7}{2}\right) = (1,4)$
2. $\left(\frac{5+11}{2}, \frac{-3+5}{2}\right) = (8,1)$
3. $\left(\frac{0-4}{2}, \frac{-2+14}{2}\right) = (-2,6)$
4. $$\begin{aligned} m = \frac{-3-1}{-2-(-1)} = \frac{-4}{-1} &= 4 \\ y &= mx+b \\ -3 &= 4(-2)+b \\ b = 5,\ y &= 4x+5 \end{aligned}$$
5. $$\begin{aligned} -7 &= 4(2)+b \\ b = -15,\ y &= 4x-15 \end{aligned}$$