3.2 Properties of Parallel Lines

FlexBooks® 2.0 > American HS Geometry > Properties of Parallel Lines

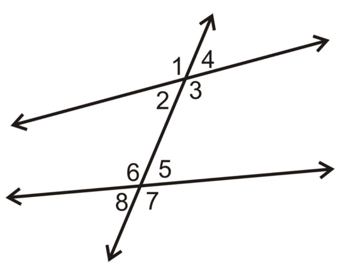
Last Modified: Dec 25, 2014

Learning Objectives

- Use the Corresponding Angles Postulate.
- Use the Alternate Interior Angles Theorem.
- Use the Alternate Exterior Angles Theorem.
- Use Same Side Interior Angles Theorem.

Review Queue

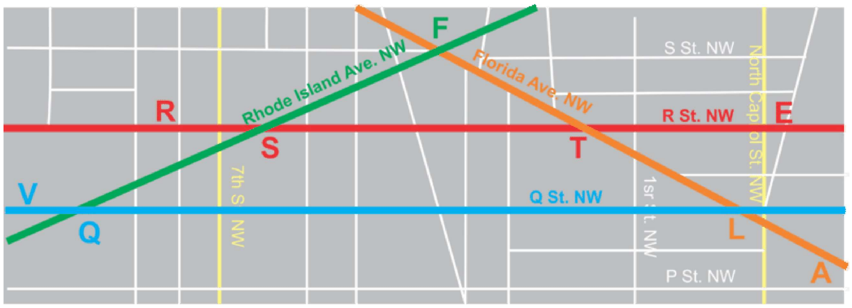
Use the picture below to determine:



[Figure 1]

1. A pair of corresponding angles.
2. A pair of alternate interior angles.
3. A pair of same side interior angles.
4. If $m\angle 4 = 37^\circ$, what other angles do you know?

Know What? The streets below are in Washington DC. The red street is R St. and the blue street is Q St. These two streets are parallel. The transversals are: Rhode Island Ave. (green) and Florida Ave. (orange).



[Figure 2]

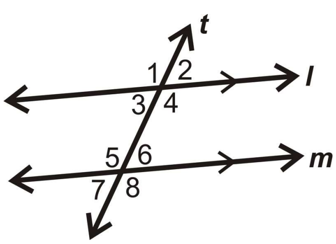
1. If $m\angle FTS = 35^\circ$, determine the other angles that are 35° .
2. If $m\angle SQV = 160^\circ$, determine the other angles that are 160° .
3. Why do you think the "State Streets" exists? Why aren't all the streets parallel or perpendicular?

In this section, we are going to discuss a specific case of two lines cut by a transversal. The two lines are now going to be parallel. If the two lines are parallel, all of the angles, corresponding, alternate interior, alternate exterior and same side interior have new properties. We will begin with corresponding angles.

Corresponding Angles Postulate

Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

If $l || m$ and both are cut by t , then $\angle 1 \cong \angle 5, \angle 2 \cong \angle 6, \angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.



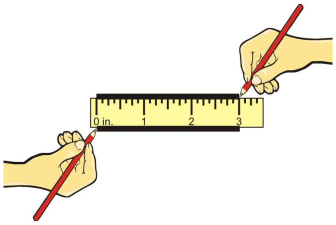
[Figure 3]

l **must be parallel** to m in order to use this postulate. Recall that a postulate is just like a theorem, but does not need to be proven. We can take it as true and use it just like a theorem from this point.

Investigation 3-4: Corresponding Angles Exploration

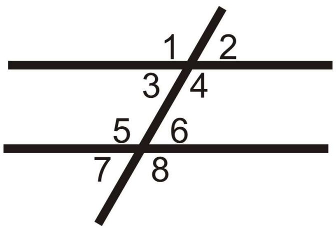
You will need: paper, ruler, protractor

Place your ruler on the paper. On either side of the ruler, draw lines, 3 inches long. This is the easiest way to ensure that the lines are parallel.



[Figure 4]

Remove the ruler and draw a transversal. Label the eight angles as shown.

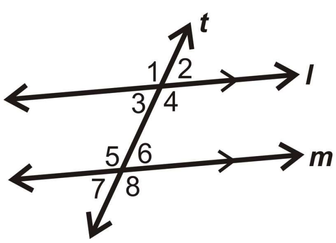


[Figure 5]

1. Using your protractor, measure all of the angles. What do you notice?

In this investigation, you should see that $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$ and $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$. $\angle 1 \cong \angle 4, \ \angle 5 \cong \angle 8$ by the Vertical Angles Theorem. By the Corresponding Angles Postulate, we can say $\angle 1 \cong \angle 5$ and therefore $\angle 1 \cong \angle 8$ by the Transitive Property. You can use this reasoning for the other set of congruent angles as well.

Example 1: If $m\angle 2 = 76^\circ$, what is $m\angle 6$?



[Figure 6]

Solution: $\angle 2$ and $\angle 6$ are corresponding angles and $l \ || \ m$, from the markings in the picture. By the Corresponding Angles Postulate the two angles are equal, so $m\angle 6 = 76^\circ$.

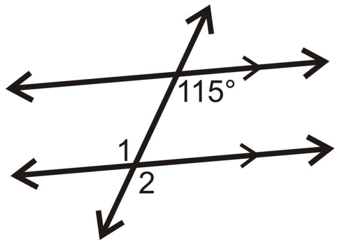
Example 2: Using the measures of $\angle 2$ and $\angle 6$ from Example 2, find all the other angle measures.

Solution: If $m\angle 2 = 76^\circ$, then $m\angle 1 = 180^\circ - 76^\circ = 104^\circ$ because they are a linear pair. $\angle 3$ is a vertical angle with $\angle 2$, so $m\angle 3 = 76^\circ$. $\angle 1$ and $\angle 4$ are vertical angles, so $m\angle 4 = 104^\circ$. By the Corresponding Angles Postulate, we know

$\angle 1 \cong \angle 5, \angle 2 \cong \angle 6, \angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$, so $m\angle 5 = 104^\circ, m\angle 6 = 76^\circ, m\angle 7 = 76^\circ$, and $m\angle 104^\circ$.

Alternate Interior Angles Theorem

Example 3: Find $m\angle 1$.

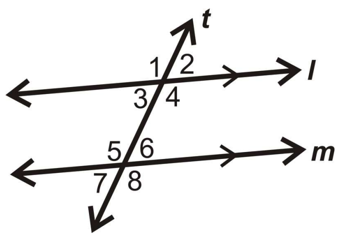


[Figure 7]

Solution: $m\angle 2 = 115^\circ$ because they are corresponding angles and the lines are parallel. $\angle 1$ and $\angle 2$ are vertical angles, so $m\angle 1 = 115^\circ$ also.

$\angle 1$ and the 115° angle are alternate interior angles.

Alternate Interior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.



[Figure 8]

Proof of Alternate Interior Angles Theorem

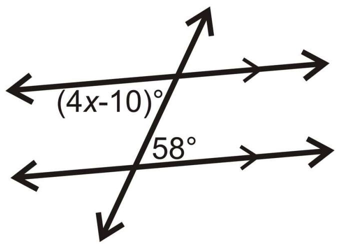
Given: $l \,||\, m$

Prove: $\angle 3 \cong \angle 6$

Statement	***Reason***
1. $l \,||\, m$	Given
2. $\angle 3 \cong \angle 7$	Corresponding Angles Postulate
3. $\angle 7 \cong \angle 6$	Vertical Angles Theorem
4. $\angle 3 \cong \angle 6$	Transitive PoC

There are several ways we could have done this proof. For example, Step 2 could have been $\angle 2 \cong \angle 6$ for the same reason, followed by $\angle 2 \cong \angle 3$. We could have also proved that $\angle 4 \cong \angle 5$.

Example 4: ***Algebra Connection*** Find the measure of the angle and x .



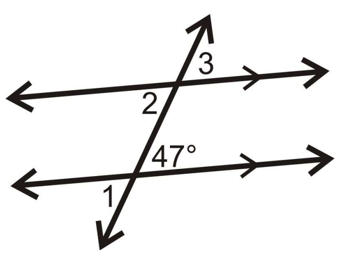
[Figure 9]

Solution: The two given angles are alternate interior angles so, they are equal. Set the two expressions equal to each other and solve for x .

$$\begin{aligned}(4x-10)^\circ &= 58^\circ \\ 4x &= 68^\circ \\ x &= 17^\circ\end{aligned}$$

Alternate Exterior Angles Theorem

Example 5: Find $m\angle 1$ and $m\angle 3$.



[Figure 10]

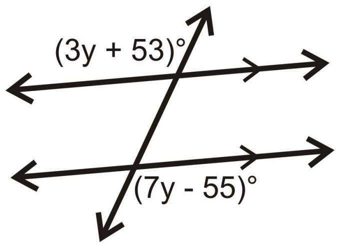
Solution: $m\angle 1 = 47^\circ$ because they are vertical angles. Because the lines are parallel, $m\angle 3 = 47^\circ$ by the Corresponding Angles Theorem. Therefore, $m\angle 2 = 47^\circ$.

$\angle 1$ and $\angle 3$ are alternate exterior angles.

Alternate Exterior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

The proof of this theorem is very similar to that of the Alternate Interior Angles Theorem and you will be asked to do in the exercises at the end of this section.

Example 6: ***Algebra Connection*** Find the measure of each angle and the value of y .



[Figure 11]

Solution: The given angles are alternate exterior angles. Because the lines are parallel, we can set the expressions equal to each other to solve the problem.

$$\begin{aligned}(3y+53)^\circ &= (7y-55)^\circ \\ 108^\circ &= 4y \\ 27^\circ &= y\end{aligned}$$

If $y = 27^\circ$, then each angle is $3(27^\circ)+53^\circ$, or 134° .

Same Side Interior Angles Theorem

Same side interior angles have a different relationship that the previously discussed angle pairs.

Example 7: Find $m\angle 2$.

[Figure 12]

Solution: Here, $m\angle 1 = 66^\circ$ because they are alternate interior angles. $\angle 1$ and $\angle 2$ are a linear pair, so they are supplementary.

$$\begin{aligned}m\angle 1 + m\angle 2 &= 180^\circ \\ 66^\circ + m\angle 2 &= 180^\circ \\ m\angle 2 &= 114^\circ\end{aligned}$$

This example shows that if two parallel lines are cut by a transversal, the same side interior angles are supplementary.

Same Side Interior Angles Theorem: If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.

If $l || m$ and both are cut by t , then

$m\angle 3 + m\angle 5 = 180^\circ$ and $m\angle 4 + m\angle 6 = 180^\circ$.

[Figure 13]

You will be asked to do the proof of this theorem in the review questions.

Example 8: ***Algebra Connection*** Find the measure of x .

[Figure 14]

Solution: The given angles are same side interior angles. The lines are parallel, therefore the angles add up to 180° . Write an equation.

$$\begin{aligned}(2x+43)^\circ+(2x-3)^\circ &= 180^\circ\\(4x+40)^\circ &= 180^\circ\\4x &= 140^\circ\\x &= 35^\circ\end{aligned}$$

While you might notice other angle relationships, there are no more theorems to worry about. However, we will continue to explore these other angle relationships. For example, same side exterior angles are also supplementary. You will prove this in the review questions.

Example 9: $l \,||\, m$ and $s \,||\, t$. Prove $\angle 1 \cong \angle 16$.

[Figure 15]

Solution:

Statement	***Reason***
1. $l \,\|\|\, m$ and $s \,\|\|\, t$	Given
2. $\angle 1 \cong \angle 3$	Corresponding Angles Postulate
3. $\angle 3 \cong \angle 16$	Alternate Exterior Angles Theorem
4. $\angle 1 \cong \angle 16$	Transitive PoC

Know What? Revisited Using what we have learned in this lesson, the other angles that are 35° are $\angle TLQ, \angle ETL$, and the vertical angle with $\angle TLQ$. The other angles that are 160° are $\angle FSR, \angle TSQ$, and the vertical angle with $\angle SQV$. You could argue that the "State Streets" exist to help traffic move faster and more efficiently through the city.

Review Questions

For questions 1-7, determine if each angle pair below is congruent, supplementary or neither.

[Figure 16]

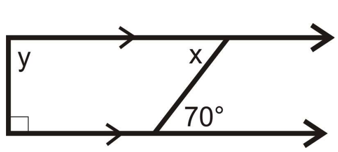
1. $\angle 1$ and $\angle 7$
2. $\angle 4$ and $\angle 2$
3. $\angle 6$ and $\angle 3$
4. $\angle 5$ and $\angle 8$
5. $\angle 1$ and $\angle 6$
6. $\angle 4$ and $\angle 6$
7. $\angle 2$ and $\angle 3$

For questions 8-16, determine if the angle pairs below are: Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Vertical Angles, Linear Pair or None.

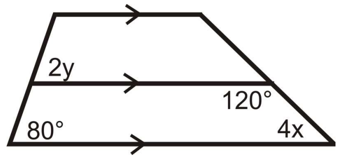
[Figure 17]

8. $\angle 2$ and $\angle 13$
9. $\angle 7$ and $\angle 12$
10. $\angle 1$ and $\angle 11$
11. $\angle 6$ and $\angle 10$
12. $\angle 14$ and $\angle 9$
13. $\angle 3$ and $\angle 11$
14. $\angle 4$ and $\angle 15$
15. $\angle 5$ and $\angle 16$
16. List all angles congruent to $\angle 8$.

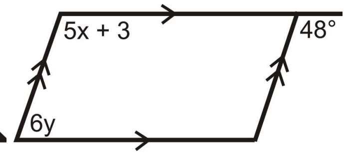
For 17-20, find the values of x and y .



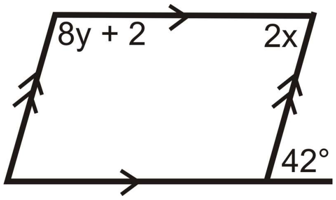
[Figure 18]



[Figure 19]

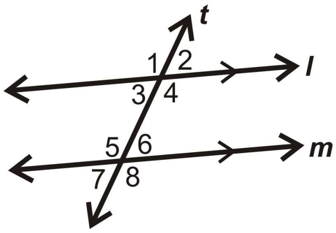


[Figure 20]



[Figure 21]

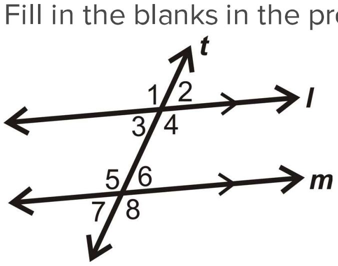
Algebra Connection For questions 21-25, use the picture below. Find the value of x and/or y.



[Figure 22]

21. $m\angle 1 = (4x+35)^\circ$, $m\angle 8 = (7x-40)^\circ$
22. $m\angle 2 = (3y+14)^\circ$, $m\angle 6 = (8x-76)^\circ$
23. $m\angle 3 = (3x+12)^\circ$, $m\angle 5 = (5x+8)^\circ$
24. $m\angle 4 = (5x-33)^\circ$, $m\angle 5 = (2x+60)^\circ$
25. $m\angle 1 = (11y-15)^\circ$, $m\angle 7 = (5y+3)^\circ$

Fill in the blanks in the proof below.

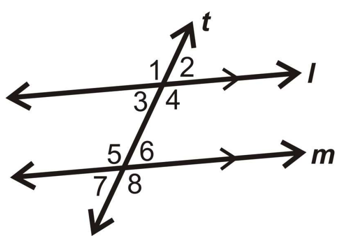


[Figure 23]

Given: $l \,||\, m$ Prove: $\angle 3$ and $\angle 5$ are supplementary (Same Side Interior Angles Theorem)

Statement	*Reason*
1.	Given
2. $\angle 1 \cong \angle 5$	
3.	$\cong$ angles have = measures
4.	Linear Pair Postulate
5.	Definition of Supplementary Angles
6. $m\angle 3 + m\angle 5 = 180^\circ$	
7. $\angle 3$ and $\angle 5$ are supplementary	

For 27 and 28, use the picture to the right to complete each proof.

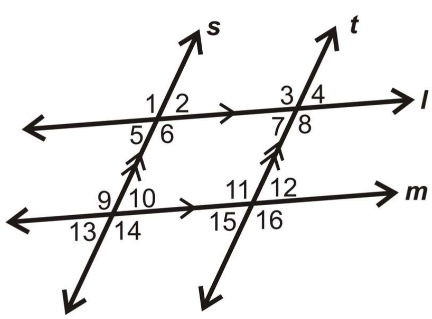


[Figure 24]

27. Given: $l \,||\, m$ Prove: $\angle 1 \cong \angle 8$ (Alternate Exterior Angles Theorem)

28. Given: $l \,||\, m$ Prove: $\angle 2$ and $\angle 8$ are supplementary

For 29-31, use the picture to the right to complete each proof.



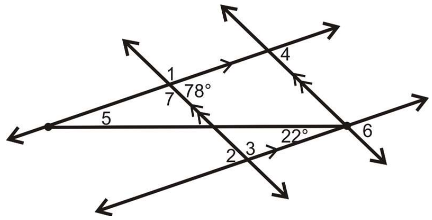
[Figure 25]

29. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 4 \cong \angle 10$

30. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 2 \cong \angle 15$

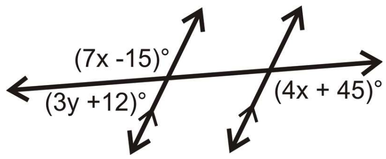
31. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 4$ and $\angle 9$ are supplementary

Find the measures of all the numbered angles in the figure below.

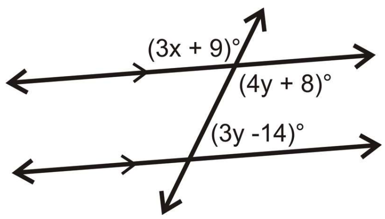


[Figure 26]

Algebra Connection For 33 and 34, find the values of x and y.



[Figure 27]



[Figure 28]

33. ***Error Analysis*** Nadia is working on Problem 31. Here is her proof:

Statement	***Reason***
1. $l \parallel m,\ s \parallel t$	Given
2. $\angle 4 \cong \angle 15$	Alternate Exterior Angles Theorem
3. $\angle 15 \cong \angle 14$	Same Side Interior Angles Theorem
4. $\angle 14 \cong \angle 9$	Vertical Angles Theorem
5. $\angle 4 \cong \angle 9$	Transitive PoC

What happened? Explain what is needed to be done to make the proof correct.

Review Queue Answers

1. $\angle 1$ and $\angle 6, \angle 2$ and $\angle 8, \angle 3$ and $\angle 7$, or $\angle 4$ and $\angle 5$
2. $\angle 2$ and $\angle 5$ or $\angle 3$ and $\angle 6$
3. $\angle 1$ and $\angle 7$ or $\angle 4$ and $\angle 8$
4. $\angle 3$ and $\angle 5$ or $\angle 2$ and $\angle 6$

3.3 Proving Lines Parallel

FlexBooks® 2.0 > American HS Geometry > Proving Lines Parallel

Last Modified: Dec 25, 2014

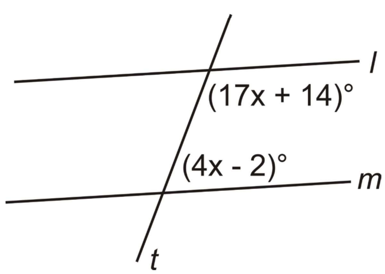
Learning Objectives

- Use the *converses* of the Corresponding Angles Postulate, Alternate Interior Angles Theorem, Alternate Exterior Angles Theorem, and the Same Side Interior Angles Theorem to show that lines are parallel.
- Construct parallel lines using the above converses.
- Use the Parallel Lines Property.

Review Queue

Answer the following questions.

1. Write the converse of the following statements:
   a. *If it is summer, then I am out of school.*
   b. *I will go to the mall when I am done with my homework.*
   c. *If two parallel lines are cut by a transversal, then the corresponding angles are congruent.*
2. Are any of the three converses from #1 true? Why or why not? Give a counterexample.
3. Determine the value of x if $l || m$.



[Figure 1]

Know What? Here is a picture of the support beams for the Coronado Bridge in San Diego. This particular bridge, called a girder bridge, is usually used in straight, horizontal situations. The Coronado Bridge is diagonal, so the beams are subject to twisting forces (called torque). This can be fixed by building a curved bridge deck. To aid the curved bridge deck, the support beams should not be parallel. If they are, the bridge would be too fragile and susceptible to damage.

[Figure 2]

This bridge was designed so that $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$. Are the support beams parallel?

Corresponding Angles Converse

Recall that the converse of a statement switches the conclusion and the hypothesis. So, if a , then b becomes if b , then a . We will find the converse of all the theorems from the last section and will determine if they are true.

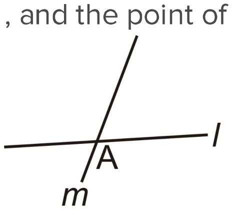
The Corresponding Angles Postulate says: *If two lines are parallel, then the corresponding angles are congruent.* The converse is:

Converse of Corresponding Angles Postulate: If corresponding angles are congruent when two lines are cut by a transversal, then the lines are parallel.

Is this true? For example, if the corresponding angles both measured 60° , would the lines be parallel? YES. All eight angles created by l, m and the transversal are either 60° or 120° , making the slopes of l and m the same which makes them parallel. This can also be seen by using a construction.

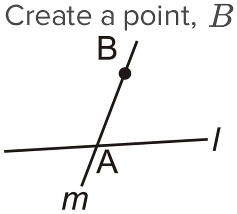
Investigation 3-5: Creating Parallel Lines using Corresponding Angles

Draw two intersecting lines. Make sure they are not perpendicular. Label them l and m , and the point of intersection, A , as shown.



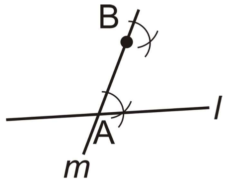
[Figure 3]

Create a point, B , on line m , above A .



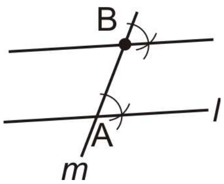
[Figure 4]

Copy the acute angle at A (the angle to the right of m) at point B. See Investigation 2-2 in Chapter 2 for the directions on how to copy an angle.



[Figure 5]

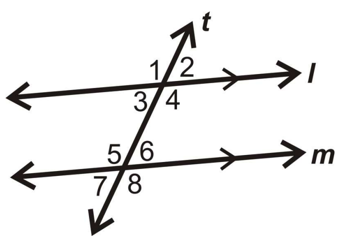
Draw the line from the arc intersections to point B.



[Figure 6]

From this construction, we can see that the lines are parallel.

Example 1: If $m\angle 8 = 110^\circ$ and $m\angle 4 = 110^\circ$, then what do we know about lines l and m?



[Figure 7]

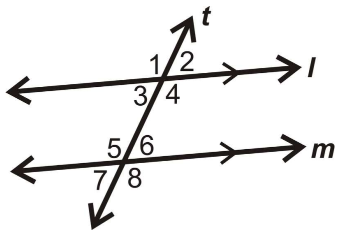
Solution: $\angle 8$ and $\angle 4$ are corresponding angles. Since $m\angle 8 = m\angle 4$, we can conclude that $l \,||\, m$.

Alternate Interior Angles Converse

We also know, from the last lesson, that when parallel lines are cut by a transversal, the alternate interior angles are congruent. The converse of this theorem is also true:

Converse of Alternate Interior Angles Theorem: If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Example 3: Prove the Converse of the Alternate Interior Angles Theorem.



[Figure 8]

Given: l and m and transversal t

$\angle 3 \cong \angle 6$

Prove: $l \,||\, m$

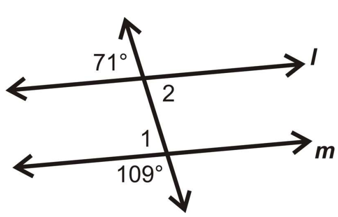
Solution:

Statement	*Reason*
1. l and m and transversal $t \angle 3 \cong \angle 6$	Given
2. $\angle 3 \cong \angle 2$	Vertical Angles Theorem
3. $\angle 2 \cong \angle 6$	Transitive PoC
4. $l \,||\, m$	Converse of the Corresponding Angles Postulate

Prove Move: Shorten the names of these theorems. Discuss with your teacher an appropriate abbreviations. For example, the Converse of the Corresponding Angles Theorem could be "Converse CA Thm" or "ConvCA."

Notice that the Corresponding Angles Postulate was not used in this proof. The Transitive Property is the reason for Step 3 because we do not know if l is parallel to m until we are done with the proof. You could conclude that if we are trying to prove two lines are parallel, the converse theorems will be used. And, if we are proving two angles are congruent, we must be given that the two lines are parallel.

Example 4: Is $l \,||\, m$?

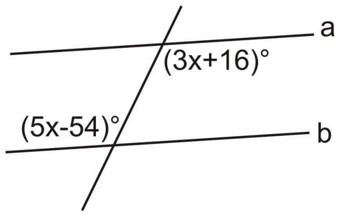


[Figure 9]

Solution: First, find $m\angle 1$. We know its linear pair is 109° . By the Linear Pair Postulate, these two angles add up to 180° , so $m\angle 1 = 180^\circ - 109^\circ = 71^\circ$. This means that $l \,||\, m$, by the Converse of the Corresponding Angles Postulate.

Example 5: ***Algebra Connection*** What does x have to be to make $a \,||\, b$?

Solution: Because these are alternate interior angles, they must be equal for $a \,||\, b$. Set the expressions equal to each other and solve.



[Figure 10]

$$\begin{aligned} 3x+16^\circ &= 5x-54^\circ \\ 70^\circ &= 2x \\ 35^\circ &= x \qquad \text{To make } a \,||\, b,\ x=35^\circ. \end{aligned}$$

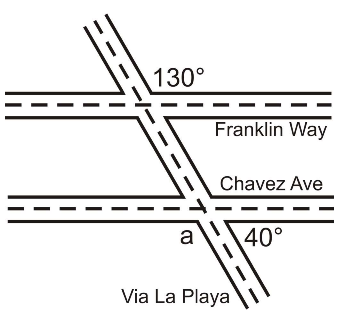
Converse of Alternate Exterior Angles & Consecutive Interior Angles

You have probably guessed that the converse of the Alternate Exterior Angles Theorem and the Consecutive Interior Angles Theorem areal so true.

Converse of the Alternate Exterior Angles Theorem: If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Example 6: ***Real-World Situation*** The map below shows three roads in Julio's town.

Julio used a surveying tool to measure two angles at the intersections in this picture he drew (NOT to scale). ***Julio wants to know if Franklin Way is parallel to Chavez Avenue.***



[Figure 11]

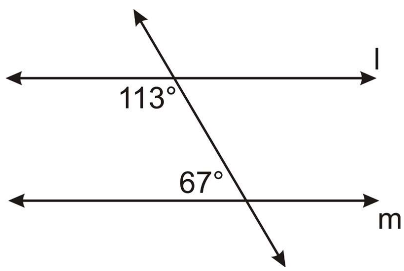
Solution: The labeled 130° angle and $\angle a$ are alternate exterior angles. If $m\angle a = 130^\circ$, then the lines are parallel. To find $m\angle a$, use the other labeled angle which is 40° , and its linear pair. Therefore, $\angle a + 40^\circ = 180^\circ$ and $\angle a = 140^\circ$. $140^\circ \neq 130^\circ$, so Franklin Way and Chavez Avenue are <u>not</u> parallel streets.

The final converse theorem is of the Same Side Interior Angles Theorem. Remember that these angles are not congruent when lines are parallel, they are **supplementary**.

Converse of the Same Side Interior Angles Theorem: If two lines are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Example 7: Is $l \,||\, m$? How do you know?

Solution: These are Same Side Interior Angles. So, if they add up to 180° , then $l \,||\, m$. $113^\circ + 67^\circ = 180^\circ$, therefore $l \,||\, m$.



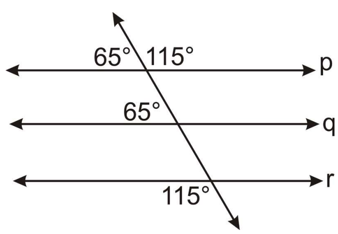
[Figure 12]

Parallel Lines Property

The Parallel Lines Property is a transitive property that can be applied to parallel lines. Remember the Transitive Property of Equality is: If $a = b$ and $b = c$, then $a = c$. The Parallel Lines Property changes = to $||$.

Parallel Lines Property: If lines $l \,||\, m$ and $m \,||\, n$, then $l \,||\, n$.

Example 8: Are lines q and r parallel?



[Figure 13]

Solution: First find if $p \,||\, q$, followed by $p \,||\, r$. If so, then $q \,||\, r$.

$p \,||\, q$ by the Converse of the Corresponding Angles Postulate, the corresponding angles are 65° . $p \,||\, r$ by the Converse of the Alternate Exterior Angles Theorem, the alternate exterior angles are 115° . Therefore, by the Parallel Lines Property, $q \,||\, r$.

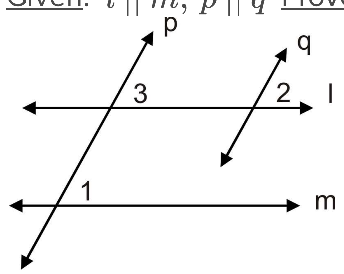
Know What? Revisited: The CoronadoBridge has $\angle 1$ and $\angle 2$, which are corresponding angles. These angles must be equal for the beams to be parallel. $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$ and $92^\circ \neq 88^\circ$, so the beams are not parallel, therefore a sturdy and safe girder bridge.

Review Questions

1. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing congruent alternate interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy the angle in a different location.
2. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing supplementary consecutive interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy a different angle.

For Questions 3-5, fill in the blanks in the proofs below.

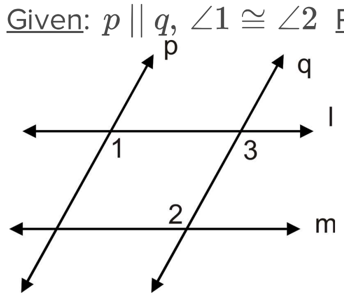
Given: $l \,||\, m$, $p \,||\, q$ Prove: $\angle 1 \cong \angle 2$



[Figure 14]

Statement	***Reason***
1. $l \,\|\, m$	1.
2.	2. Corresponding Angles Postulate
3. $p \,\|\, q$	3.
4.	4.
5. $\angle 1 \cong \angle 2$	5.

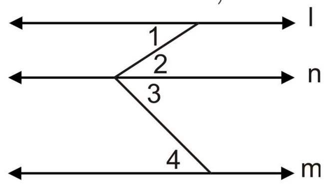
Given: $p \,||\, q$, $\angle 1 \cong \angle 2$ Prove: $l \,||\, m$



[Figure 15]

Statement	Reason
1. $p \parallel q$	1.
2.	2. Corresponding Angles Postulate
3. $\angle 1 \cong \angle 2$	3.
4.	4. Transitive PoC
5.	5. Converse of Alternate Interior Angles Theorem

Given: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$ Prove: $l \parallel m$

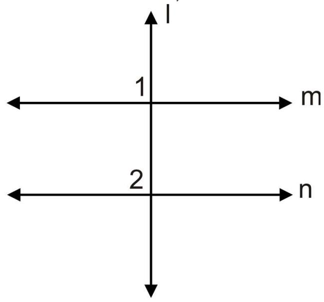


[Figure 16]

Statement	Reason
1. $\angle 1 \cong \angle 2$	1.
2. $l \parallel n$	2.
3. $\angle 3 \cong \angle 4$	3.
4.	4. Converse of Alternate Interior Angles Theorem
5. $l \parallel m$	5.

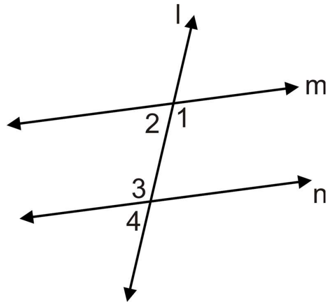
For Questions 6-9, create your own two column proof.

Given: $m \perp l, n \perp l$ Prove: $m || n$



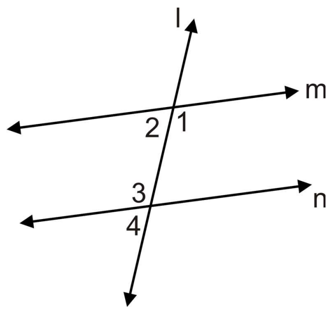
[Figure 17]

Given: $\angle 1 \cong \angle 3$ Prove: $\angle 1$ and $\angle 4$ are supplementary



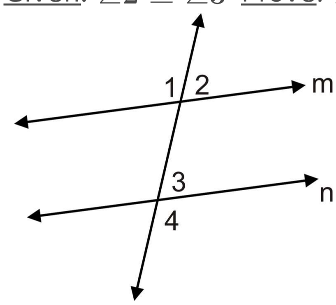
[Figure 18]

Given: $\angle 2 \cong \angle 4$ Prove: $\angle 1 \cong \angle 3$



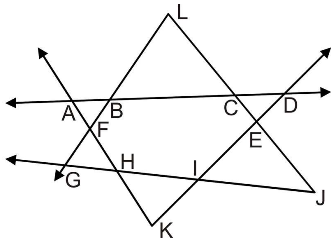
[Figure 19]

Given: $\angle 2 \cong \angle 3$ Prove: $\angle 1 \cong \angle 4$



[Figure 20]

In 10-15, use the given information to determine which lines are parallel. If there are none, write *none*. Consider each question individually.

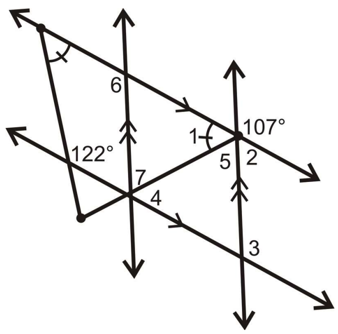


[Figure 21]

10. $\angle LCD \cong \angle CJI$
11. $\angle BCE \, and \, \angle BAF$ are supplementary

12. $\angle FGH \cong \angle EIJ$
13. $\angle BFH \cong \angle CEI$
14. $\angle LBA \cong \angle IHK$
15. $\angle ABG \cong \angle BGH$

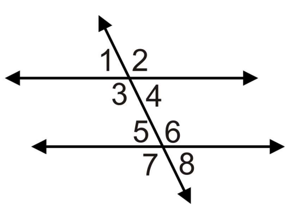
In 16-22, find the measure of the lettered angles below.



[Figure 22]

16. $m\angle 1$
17. $m\angle 2$
18. $m\angle 3$
19. $m\angle 4$
20. $m\angle 5$
21. $m\angle 6$
22. $m\angle 7$

For 23-27, what does x have to measure to make the lines parallel?

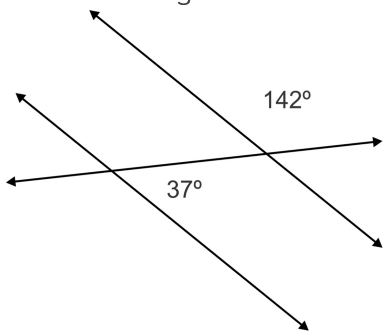


[Figure 23]

23. $m\angle 3 = (3x+25)^\circ$ and $m\angle 5 = (4x-55)^\circ$
24. $m\angle 2 = (8x)^\circ$ and $m\angle 7 = (11x-36)^\circ$
25. $m\angle 1 = (6x-5)^\circ$ and $m\angle 5 = (5x+7)^\circ$
26. $m\angle 4 = (3x-7)^\circ$ and $m\angle 7 = (5x-21)^\circ$
27. $m\angle 1 = (9x)^\circ$ and $m\angle 6 = (37x)^\circ$

28. ***Construction*** Draw a straight line. Construct a line perpendicular to this line through a point on the line. Now, construct a perpendicular line to this new line. What can you conclude about the original line and this final line?
29. How could you prove your conjecture from problem 28?

What is wrong in the following diagram, given that $j || k$?



[Figure 24]

Review Queue Answers

1. Answers:

a. *If I am out of school, then it is summer.*

b. *If I go to the mall, then I am done with my homework.*

c. *If corresponding angles created by two lines cut by a transversal are congruent, then the two lines are parallel.*

2. Answers:

a. Not true, I could be out of school on any school holiday or weekend during the school year.

b. Not true, I don't have to be done with my homework to go to the mall.

c. Yes, because if two corresponding angles are congruent, then the slopes of these two lines have to be the same, making the lines parallel.

3. The two angles are supplementary.

$$\begin{aligned}(17x+14)^\circ + (4x-2)^\circ &= 180^\circ \\ 21x+12^\circ &= 180^\circ \\ 21x &= 168^\circ \\ x &= 8^\circ\end{aligned}$$

3.4 Properties of Perpendicular Lines

FlexBooks® 2.0 > American HS Geometry > Properties of Perpendicular Lines

Last Modified: Dec 25, 2014

Learning Objectives

- Understand the properties of perpendicular lines.
- Explore problems with parallel lines and a perpendicular transversal.
- Solve problems involving complementary adjacent angles.

Review Queue

Determine if the following statements are true or false. If they are true, write the converse. If they are false, find a counter example.

1. Perpendicular lines form four right angles.

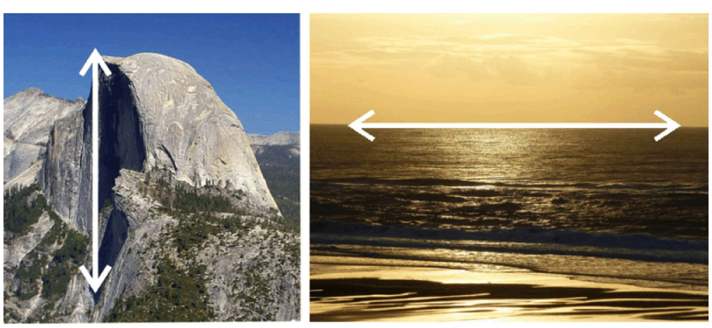
2. A right angle is greater than or equal to 90° .

Find the slope between the two given points.

3. (-3, 4) and (-3, 1)

4. (6, 7) and (-5, 7)

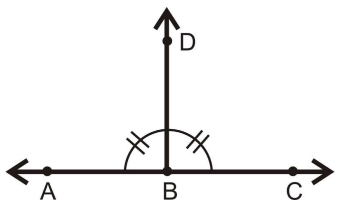
Know What? There are several examples of slope in nature. Below are pictures of Half Dome in Yosemite National Park and the horizon over the Pacific Ocean. These are examples of horizontal and vertical lines in real life. Can you determine the slope of these lines?



[Figure 1]

Congruent Linear Pairs

Recall that a linear pair is a pair of adjacent angles whose outer sides form a straight line. The Linear Pair Postulate says that the angles in a linear pair are supplementary. What happens when the angles in a linear pair are congruent?

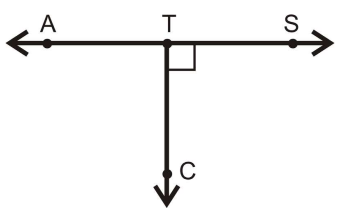


[Figure 2]

$$\begin{aligned} m\angle ABD + m\angle DBC &= 180^\circ && \text{Linear Pair Postulate} \\ m\angle ABD &= m\angle DBC && \text{The two angles are congruent} \\ m\angle ABD + m\angle ABD &= 180^\circ && \text{Substitution PoE} \\ 2m\angle ABD &= 180^\circ && \text{Combine like terms} \\ m\angle ABD &= 90^\circ && \text{Division PoE} \end{aligned}$$

So, anytime a linear pair is congruent, the angles are both 90° .

Example 1: Find $m\angle CTA$.



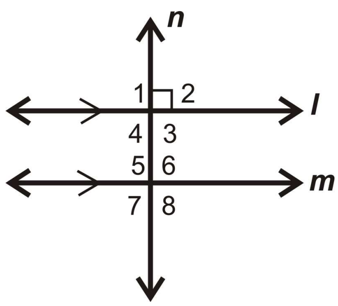
[Figure 3]

Solution: First, these two angles form a linear pair. Second, from the marking, we know that $\angle STC$ is a right angle. Therefore, $m\angle STC = 90^\circ$. So, $m\angle CTA$ is also 90° .

Perpendicular Transversals

Recall that when two lines intersect, four angles are created. If the two lines are perpendicular, then all four angles are right angles, even though only one needs to be marked with the square. Therefore, all four angles are 90° .

When a parallel line is added, then there are eight angles formed. If $l || m$ and $n \perp l$, is $n \perp m$? Let's prove it here.



[Figure 4]

Given: $l \mid\mid m, l \perp n$

Prove: $n \perp m$

Statement	*Reason*
1. $l \mid\mid m, l \perp n$	Given
2. $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are right angles	Definition of perpendicular lines
3. $m\angle 1 = 90^\circ$	Definition of a right angle
4. $m\angle 1 = m\angle 5$	Corresponding Angles Postulate
5. $m\angle 5 = 90^\circ$	Transitive PoE
6. $m\angle 6 = m\angle 7 = 90^\circ$	Congruent Linear Pairs
7. $m\angle 8 = 90^\circ$	Vertical Angles Theorem
8. $\angle 5, \angle 6, \angle 7$, and $\angle 8$ are right angles	Definition of right angle
9. $n \perp m$	Definition of perpendicular lines

Theorem 3-1: If two lines are parallel and a third line is perpendicular to one of the parallel lines, it is also perpendicular to the other parallel line.

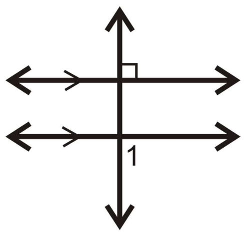
Or, if $l \mid\mid m$ and $l \perp n$, then $n \perp m$.

Theorem 3-2: If two lines are perpendicular to the same line, they are parallel to each other.

Or, if $l \perp n$ and $n \perp m$, then $l \mid\mid m$. You will prove this theorem in the review questions.

From these two theorems, we can now assume that any angle formed by two parallel lines and a perpendicular transversal will always be 90° .

Example 2: Determine the measure of $\angle 1$.



[Figure 5]

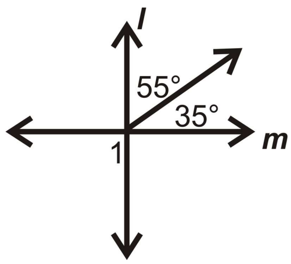
Solution: From Theorem 3-1, we know that the lower parallel line is also perpendicular to the transversal. Therefore, $m\angle 1 = 90^\circ$.

Adjacent Complementary Angles

Recall that complementary angles add up to 90° . If complementary angles are adjacent, their nonadjacent sides are perpendicular rays. What you have learned about perpendicular lines can be applied to this situation.

Example 3: Find $m\angle 1$.

Solution: The two adjacent angles add up to 90° , so $l \perp m$. Therefore, $m\angle 1 = 90^\circ$.

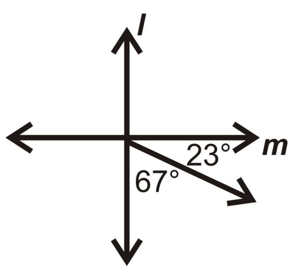


[Figure 6]

Example 4: Is $l \perp m$? Explain why or why not.

Solution: If the two adjacent angles add up to 90° , then l and m are perpendicular.

$23^\circ + 67^\circ = 90^\circ$. Therefore, $l \perp m$.



[Figure 7]

Know What? Revisited

Half Dome is vertical and the slope of any vertical line is undefined. Thousands of people flock to Half Dome to attempt to scale the rock. This front side is very difficult to climb because it is vertical. The only way to scale the front side is to use the provided cables at the base of the rock. http://www.nps.gov/yose/index.htm

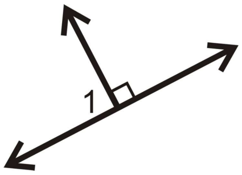
[Figure 8]

[Figure 9]

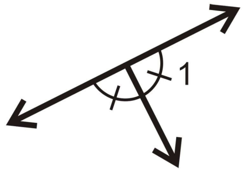
Any horizon over an ocean is horizontal, which has a slope of zero, or no slope. There is no steepness, so no incline or decline. The complete opposite of Half Dome. Actually, if Half Dome was placed on top of an ocean or flat ground, the two would be perpendicular!

Review Questions

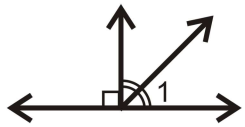
Find the measure of $\angle 1$ for each problem below.



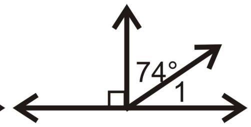
[Figure 10]



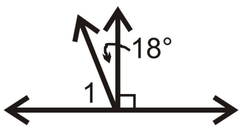
[Figure 11]



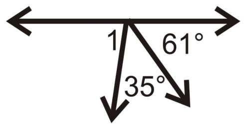
[Figure 12]



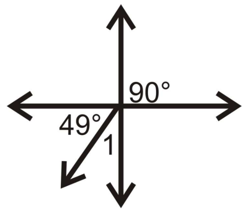
[Figure 13]



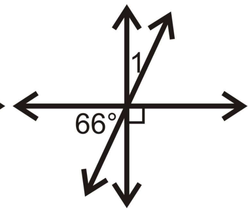
[Figure 14]



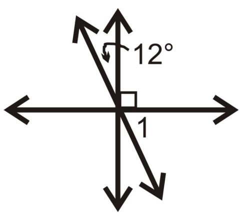
[Figure 15]



[Figure 16]

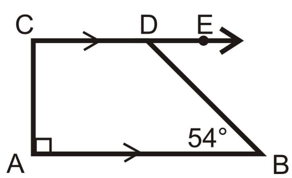


[Figure 17]



[Figure 18]

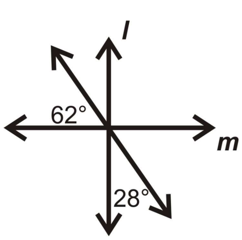
For questions 10-13, use the picture below.



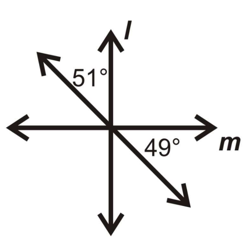
[Figure 19]

10. Find $m\angle ACD$.
11. Find $m\angle CDB$.
12. Find $m\angle EDB$.
13. Find $m\angle CDE$.

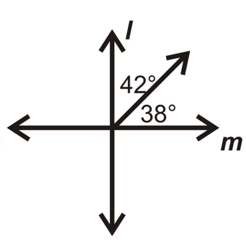
In questions 14-17, determine if $l \perp m$.



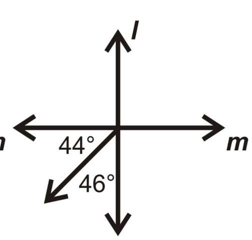
[Figure 20]



[Figure 21]

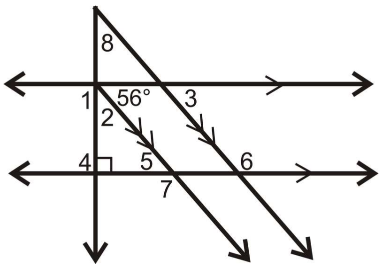


[Figure 22]



[Figure 23]

For questions 18-25, use the picture below.

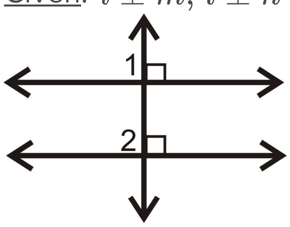


[Figure 24]

18. Find $m\angle 1$.
19. Find $m\angle 2$.
20. Find $m\angle 3$.
21. Find $m\angle 4$.
22. Find $m\angle 5$.
23. Find $m\angle 6$.
24. Find $m\angle 7$.
25. Find $m\angle 8$.

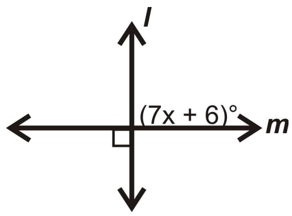
Complete the proof.

Given: $l \perp m, l \perp n$ Prove: $m \,||\, n$

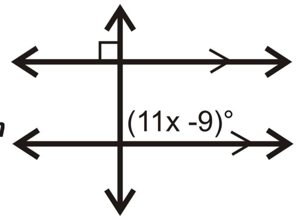


[Figure 25]

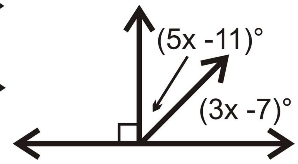
Algebra Connection Find the value of x .



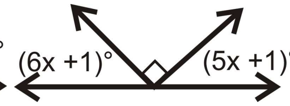
[Figure 26]



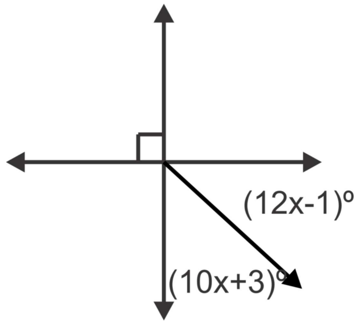
[Figure 27]



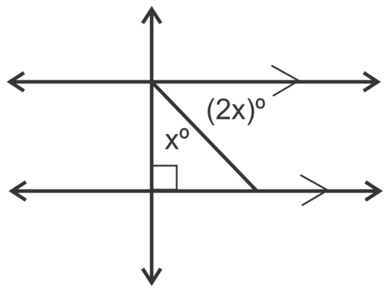
[Figure 28]



[Figure 29]



[Figure 30]



[Figure 31]

Review Queue Answers

1. True; If four right angles are formed by two intersecting lines, then the lines are perpendicular.
2. False; 95° is not a right angle.
3. Undefined slope; this is a vertical line.
4. Zero slope; this would be a horizontal line.