# 3.5 Parallel and Perpendicular Lines in the Coordinate Plane

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## Learning Objectives

- Compute slope.
- Determine the equation of parallel and perpendicular lines to a given line.
- Graph parallel and perpendicular lines in slope-intercept and standard form.

## Review Queue

Find the slope between the following points.

1. (-3, 5) and (2, -5)

2. (7, -1) and (-2, 2)

3. Is $x = 3$ horizontal or vertical? How do you know?

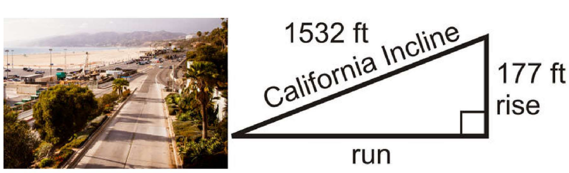
Graph the following lines on an $x - y$ plane.

4. $y = -2x + 3$

5. $y = \frac{1}{4}x - 2$

**Know What?** The picture to the right is the California Incline, a short piece of road that connects Highway 1 with the city of Santa Monica. The length of the road is 1532 feet and has an elevation of 177 feet. *You may assume that the base of this incline is sea level, or zero feet.* **Can you find the slope of the California Incline?**

HINT: You will need to use the Pythagorean Theorem, which has not been introduced in this class, but you may have seen it in a previous math class.

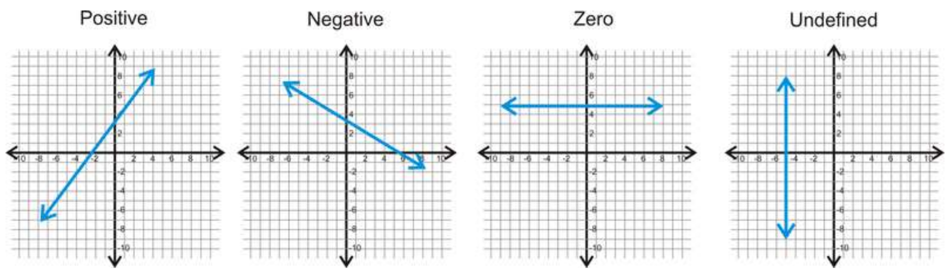



[Figure 1]

## Slope in the Coordinate Plane

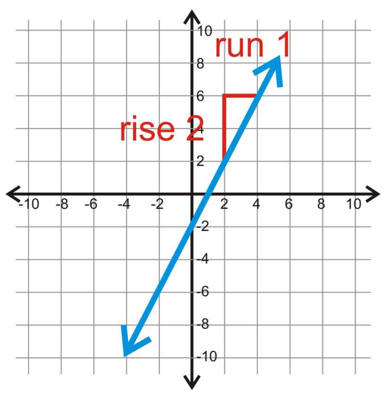
Recall from Algebra I, The slope of the line between two points $(x_1, y_1)$ and $(x_2, y_2)$ is $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

Different Types of Slope:




[Figure 2]

**Example 1:** What is the slope of the line through (2, 2) and (4, 6)?




[Figure 3]

**Solution:** Use the slope formula to determine the slope. Use (2, 2) as $(x_1, y_1)$ and (4, 6) as $(x_2, y_2)$.

$$m = \frac{6-2}{4-2} = \frac{4}{2} = 2$$

Therefore, the slope of this line is 2.

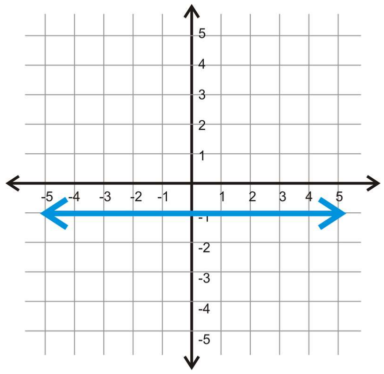
This slope is positive. Recall that slope can also be the “rise over run.” In this case we “rise”, or go up 2, and “run” in the positive direction 1.

**Example 2:** Find the slope between (-8, 3) and (2, -2).

**Solution:** $m = \frac{-2-3}{2-(-8)} = \frac{-5}{10} = -\frac{1}{2}$

This is a negative slope. Instead of "rising," the negative slope means that you would "fall," when finding points on the line.

**Example 3:** Find the slope between (-5, -1) and (3, -1).



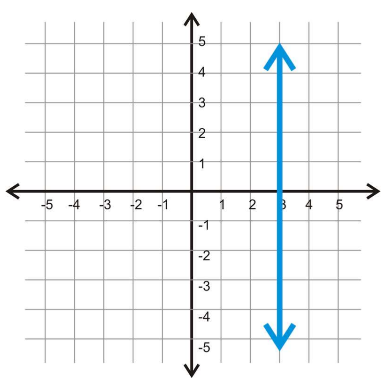

[Figure 4]

**Solution:**

$$m = \frac{-1-(-1)}{3-(-5)} = \frac{0}{8} = 0$$

Therefore, the slope of this line is 0, which means that it is a horizontal line. Horizontal lines always pass through the $y-$ axis. Notice that the $y-$ coordinate for both points is -1. In fact, the $y-$ coordinate for *any* point on this line is -1. This means that the horizontal line must cross $y = -1$ .

**Example 4:** What is the slope of the line through (3, 2) and (3, 6)?




[Figure 5]

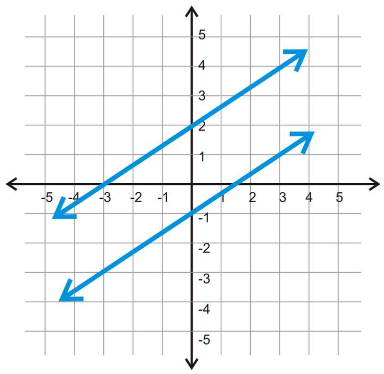
**Solution:**

$$m = \frac{6-2}{3-3} = \frac{4}{0} = undefined$$

Therefore, the slope of this line is undefined, which means that it is a *vertical* line. Vertical lines always pass through the $x-$ axis. Notice that the $x-$ coordinate for both points is 3. In fact, the $x-$ coordinate for *any* point on this line is 3. This means that the vertical line must cross $x = 3$ .

## Slopes of Parallel Lines

Recall from earlier in the chapter that the definition of parallel is two lines that never intersect. In the coordinate plane, that would look like this:




[Figure 6]

If we take a closer look at these two lines, we see that the slopes of both are $\frac{2}{3}$ .

This can be generalized to any pair of parallel lines in the coordinate plane.

***Parallel lines have the same slope.***

**Example 5:** Find the equation of the line that is parallel to $y = -\frac{1}{3}x + 4$ and passes through (9, -5).

Recall that the equation of a line in this form is called the slope-intercept form and is written as $y = mx + b$ where $m$ is the slope and $b$ is the $y-$ intercept. Here, $x$ and $y$ represent any coordinate pair, $(x, y)$ on the line.

**Solution:** We know that parallel lines have the same slope, so the line we are trying to find also has $m = -\frac{1}{3}$ . Now, we need to find the $y-$ intercept. 4 is the $y-$ intercept of the given line, *not our new line*. We need to plug in 9 for $x$ and -5 for $y$ (this is our given coordinate pair that needs to be on the line) to solve for the *new* $y-$ intercept $(b)$ .

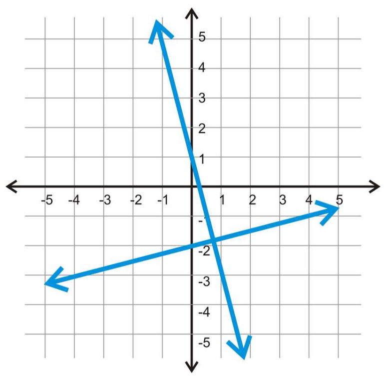
$$-5 = -\frac{1}{3}(9) + b$$
$$-5 = -3 + b \qquad \text{Therefore, the equation of line is } y = -\frac{1}{3}x - 2.$$
$$-2 = b$$

Reminder: the final equation contains the variables $x$ and $y$ to indicate that the line contains and infinite number of points or coordinate pairs that satisfy the equation.

***Parallel lines always have the <u>same slope</u> and <u>different</u> $y-$ <u>intercepts</u>.***

## Slopes of Perpendicular Lines

Recall from Chapter 1 that the definition of perpendicular is two lines that intersect at a $90^\circ$, or right, angle. In the coordinate plane, that would look like this:




[Figure 7]

If we take a closer look at these two lines, we see that the slope of one is -4 and the other is $\frac{1}{4}$.

This can be generalized to any pair of perpendicular lines in the coordinate plane.

***The slopes of perpendicular lines are opposite signs and reciprocals of each other.***

**Example 6:** Find the slope of the perpendicular lines to the lines below.

a) $y = 2x + 3$

b) $y = -\frac{2}{3}x - 5$

c) $y = x + 2$

**Solution:** We are only concerned with the slope for each of these.

a) $m = 2$ , so $m_{\perp}$ is the reciprocal and negative, $m_{\perp} = -\frac{1}{2}$ .

b) $m = -\frac{2}{3}$ , take the reciprocal and make the slope positive, $m_{\perp} = \frac{3}{2}$ .

c) Because there is no number in front of $x$ , the slope is 1. The reciprocal of 1 is 1, so the only thing to do is make it negative, $m_{\perp} = -1$ .

**Example 7:** Find the equation of the line that is perpendicular to $y = -\frac{1}{3}x + 4$ and passes through (9, -5).

**Solution:** First, the slope is the reciprocal and opposite sign of $-\frac{1}{3}$ . So, $m = 3$ . Now, we need to find the $y-$ intercept. 4 is the $y-$ intercept of the given line, *not our new line*. We need to plug in 9 for $x$ and -5 for $y$ to solve for the *new* $y-$ intercept $(b)$ .

$$\begin{aligned} -5 &= 3(9) + b \\ -5 &= 27 + b \qquad \text{Therefore, the equation of line is } y = 3x - 32. \\ -32 &= b \end{aligned}$$

## Graphing Parallel and Perpendicular Lines

**Example 8:** Find the equations of the lines below and determine if they are parallel, perpendicular or neither.

[Figure 8]

**Solution:** To find the equation of each line, start with the $y-$ intercept. The top line has a $y-$ intercept of 1. From there, determine the slope triangle, or the "rise over run." From the $y-$ intercept, if you go up 1 and over 2, you hit the line again. Therefore, the slope of this line is $\frac{1}{2}$ . The equation is $y = \frac{1}{2}x + 1$ . For the second line, the $y-$ intercept is -3. Again, start here to determine the slope and if you "rise" 1 and "run" 2, you run into the line again, making the slope $\frac{1}{2}$ . The equation of this line is $y = \frac{1}{2}x - 3$ . The lines are parallel because they have the same slope.

**Example 9:** Graph $3x - 4y = 8$ and $4x + 3y = 15$ . Determine if they are parallel, perpendicular, or neither.

**Solution:** First, we have to change each equation into slope-intercept form. In other words, we need to solve each equation for $y$ .

$$\begin{aligned} 3x - 4y &= 8 \qquad & 4x + 3y &= 15 \\ -4y &= -3x + 8 & 3y &= -4x + 15 \\ y &= \frac{3}{4}x - 2 & y &= -\frac{4}{3}x + 5 \end{aligned}$$

Now that the lines are in slope-intercept form (also called $y-$ intercept form), we can tell they are perpendicular because the slopes are opposites signs and reciprocals.

To graph the two lines, plot the $y-$ intercept on the $y-$ axis. From there, use the slope to rise and then run. For the first line, you would plot -2 and then rise 3 and run 4, making the next point on the line (1, 4). For the second line, plot 5 and then fall (because the slop is negative) 4 and run 3, making the next point on the line (1, 3).

[Figure 9]

**Know What? Revisited** In order to find the slope, we need to first find the horizontal distance in the triangle to the right. This triangle represents the incline and the elevation. To find the horizontal distance, or the run, we need to use the Pythagorean Theorem, $a^2 + b^2 = c^2$ , where $c$ is the hypotenuse.

[Figure 10]

$$\begin{aligned} 177^2 + run^2 &= 1532^2 \\ 31,329 + run^2 &= 2,347,024 \\ run^2 &= 2,315,695 \\ run &\approx 1521.75 \end{aligned}$$

The slope is then $\frac{177}{1521.75}$ , which is roughly $\frac{3}{25}$ .

## Review Questions

Find the slope between the two given points.

1. (4, -1) and (-2, -3)
2. (-9, 5) and (-6, 2)
3. (7, 2) and (-7, -2)
4. (-6, 0) and (-1, -10)
5. (1, -2) and (3, 6)
6. (-4, 5) and (-4, -3)

Determine if each pair of lines are parallel, perpendicular, or neither. Then, graph each pair on the same set of axes.

7. $y = -2x + 3$ and $y = \frac{1}{2}x + 3$
8. $y = 4x - 2$ and $y = 4x + 5$
9. $y = -x + 5$ and $y = x + 1$
10. $y = -3x + 1$ and $y = 3x - 1$
11. $2x - 3y = 6$ and $3x + 2y = 6$
12. $5x + 2y = -4$ and $5x + 2y = 8$
13. $x - 3y = -3$ and $x + 3y = 9$
14. $x + y = 6$ and $4x + 4y = -16$

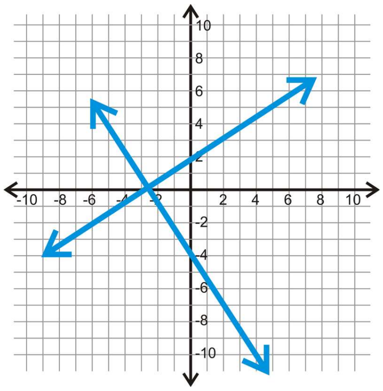
Determine the equation of the line that is ***parallel*** to the given line, through the given point.

15. $y = -5x + 1;\ (-2,\ 3)$
16. $y = \frac{2}{3}x - 2;\ (9, 1)$
17. $x - 4y = 12;\ (-16,\ -2)$
18. $3x + 2y = 10;\ (8,\ -11)$
19. $2x - y = 15;\ (3,\ 7)$
20. $y = x - 5;\ (9,\ -1)$

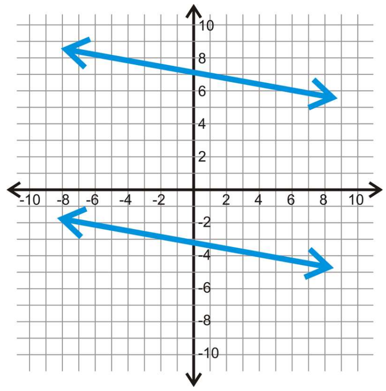
Determine the equation of the line that is ***perpendicular*** to the given line, through the given point.

21. $y = x - 1;\ (-6,\ 2)$
22. $y = 3x + 4;\ (9,\ -7)$
23. $5x - 2y = 6;\ (5,\ 5)$
24. $y = 4;\ (-1,\ 3)$
25. $x = -3;\ (1,\ 8)$
26. $x - 3y = 11;\ (0,\ 13)$

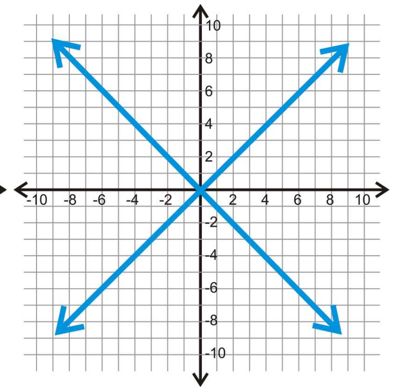
Find the equation of the two lines in each graph below. Then, determine if the two lines are parallel, perpendicular or neither.



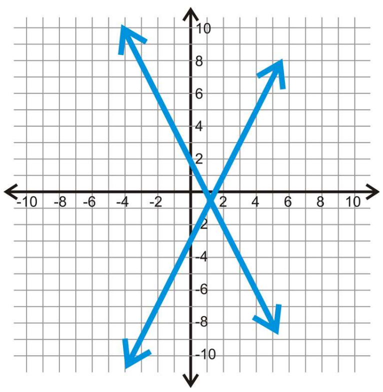

[Figure 11]




[Figure 12]




[Figure 13]

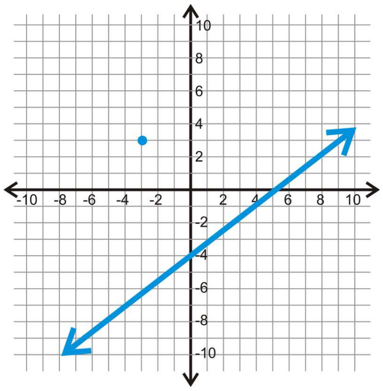



[Figure 14]

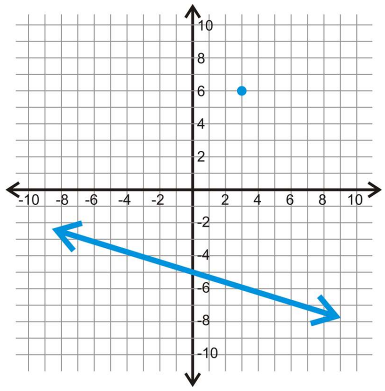
For the line and point below, find:

a) A parallel line, through the given point.

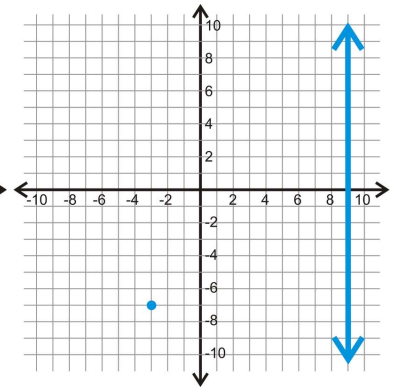
b) A perpendicular line, through the given point.



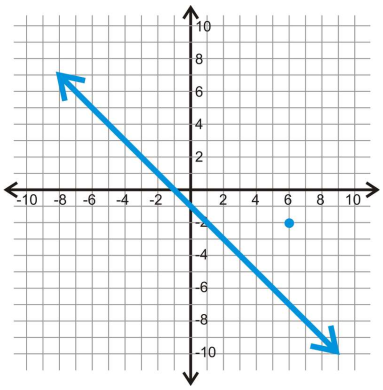

[Figure 15]




[Figure 16]



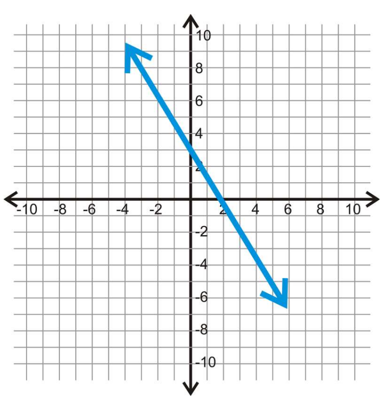

[Figure 17]



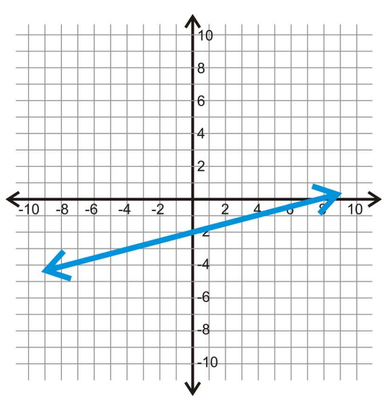

[Figure 18]

## Review Queue Answers

1. $m = \frac{-5-5}{2+3} = \frac{-10}{2} = -5$
2. $m = \frac{2+1}{-2-7} = \frac{3}{-9} = -\frac{1}{3}$
3. Vertical because it has to pass through $x = 3$ on the $x-$ axis and doesn't pass through $y$ at all.




[Figure 19]




[Figure 20]