6.2 Properties of Parallelograms

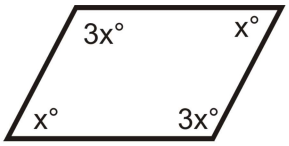
Last Modified: Dec 25, 2014

Learning Objectives

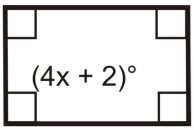
- Define a parallelogram.
- Understand the properties of a parallelogram
- Apply theorems about a parallelogram's sides, angles and diagonals.

Review Queue

1. Draw a quadrilateral with one set of parallel sides.
2. Draw a quadrilateral with two sets of parallel sides.
3. Find the measure of the missing angles in the quadrilaterals below.

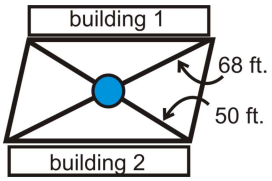


[Figure 1]



[Figure 2]

Know What? A college has a parallelogram-shaped courtyard between two buildings. The school wants to build two walkways on the diagonals of the parallelogram with a fountain where they intersect. The walkways are going to be 50 feet and 68 feet long. Where would the fountain be?

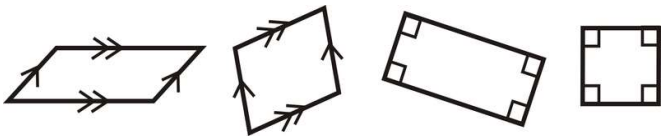


[Figure 3]

What is a Parallelogram?

Parallelogram: A quadrilateral with two pairs of parallel sides.

Here are some examples:



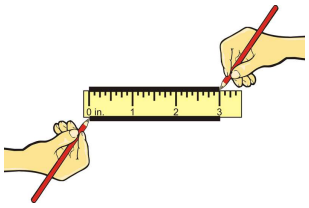
[Figure 4]

Notice that each pair of sides is marked parallel. As is the case with the rectangle and square, recall that two lines are parallel when they are perpendicular to the same line. Once we know that a quadrilateral is a parallelogram, we can discover some additional properties.

Investigation 6-2: Properties of Parallelograms

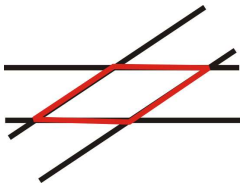
Tools Needed: Paper, pencil, ruler, protractor

Draw a set of parallel lines by placing your ruler on the paper and drawing a line on either side of it. Make your lines 3 inches long.



[Figure 5]

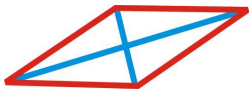
Rotate the ruler and repeat this so that you have a parallelogram. Your second set of parallel lines can be any length. If you have colored pencils, outline the parallelogram in another color.



[Figure 6]

1. Measure the four interior angles of the parallelogram as well as the length of each side. Can you conclude anything about parallelograms, other than opposite sides are parallel?

   Draw the diagonals. Measure each and then measure the lengths from the point of intersection to each vertex.



[Figure 7]

To continue to explore the properties of a parallelogram, see the website:

http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php

In the above investigation, we drew a parallelogram. From this investigation we can conclude:

- The sides that are parallel are also congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Opposite Sides Theorem: If a quadrilateral is a parallelogram, then the opposite sides are congruent.

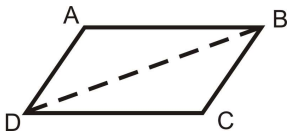
Opposite Angles Theorem: If a quadrilateral is a parallelogram, then the opposite angles are congruent.

Consecutive Angles Theorem: If a quadrilateral is a parallelogram, then the consecutive angles are supplementary.

Parallelogram Diagonals Theorem: If a quadrilateral is a parallelogram, then the diagonals bisect each other.

To prove the first three theorems, one of the diagonals must be added to the figure and then the two triangles can be proved congruent.

Proof of Opposite Sides Theorem



[Figure 8]

Given: $ABCD$ is a parallelogram with diagonal BD

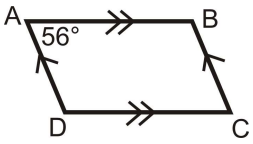
Prove: $AB \cong DC, AD \cong BC$

Statement	*Reason*
1. $ABCD$ is a parallelogram with diagonal BD	Given
2. $AB \parallel DC, AD \parallel BC$	Definition of a parallelogram
3. $\angle ABD \cong BDC, \angle ADB \cong DBC$	Alternate Interior Angles Theorem
4. $DB \cong DB$	Reflexive PoC
5. $\triangle ABD \cong \triangle CDB$	ASA
6. $AB \cong DC, AD \cong BC$	CPCTC

The proof of the Opposite Angles Theorem is almost identical. For the last step, the angles are congruent by CPCTC. You will prove the other three theorems in the review questions.

Example 1: $ABCD$ is a parallelogram. If $m\angle A = 56^\circ$, find the measure of the other three angles.

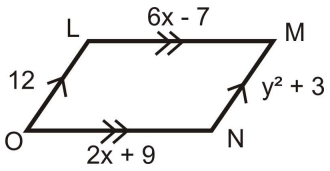
Solution: Draw a picture. When labeling the vertices, the letters are listed, in order, clockwise.



[Figure 9]

If $m\angle A = 56^\circ$, then $m\angle C = 56^\circ$ because they are opposite angles. $\angle B$ and $\angle D$ are consecutive angles with $\angle A$, so they are both supplementary to $\angle A$.
$m\angle A + m\angle B = 180^\circ, 56^\circ + m\angle B = 180^\circ, m\angle B = 124^\circ$. $m\angle D = 124^\circ$.

Example 2: ***Algebra Connection*** Find the values of x and y.



[Figure 10]

Solution: Opposite sides are congruent, so we can set each pair equal to each other and solve both equations.

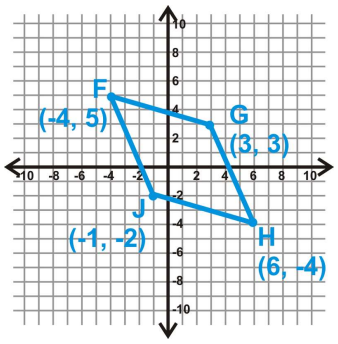
$$\begin{aligned} 6x - 7 &= 2x + 9 \\ 4x &= 16 \\ x &= 4 \end{aligned} \qquad \begin{aligned} y^2 + 3 &= 12 \\ y^2 &= 9 \\ y &= 3 \; or \; -3 \end{aligned}$$

Even though $y = 3$ or -3, lengths cannot be negative, so $y = 3$.

Diagonals in a Parallelogram

From the Parallelogram Diagonals Theorem, we know that the diagonals of a parallelogram bisect each other.

Example 3: Show that the diagonals of $FGHJ$ bisect each other.

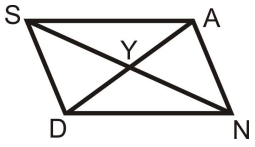


[Figure 11]

Solution: The easiest way to show this is to find the midpoint of each diagonal. If it is the same point, you know they intersect at each other's midpoint and, by definition, cuts a line in half.

$$\text{Midpoint of } FH: \left(\frac{-4+6}{2}, \frac{5-4}{2}\right) = (1, 0.5)$$
$$\text{Midpoint of } GJ: \left(\frac{3-1}{2}, \frac{3-2}{2}\right) = (1, 0.5)$$

Example 4: ***Algebra Connection*** $SAND$ is a parallelogram and $SY = 4x - 11$ and $YN = x + 10$. Solve for x.

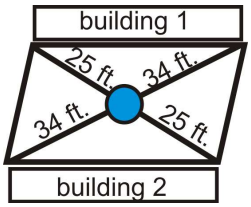


[Figure 12]

Solution: AD and SN bisect each other, so $SY = YN$.

$$\begin{aligned} 4x - 11 &= x + 10 \\ 3x &= 21 \\ x &= 7 \end{aligned}$$

Know What? Revisited By the Parallelogram Diagonals Theorem, the fountain is going to be 34 feet from either endpoint on the 68 foot diagonal and 25 feet from either endpoint on the 50 foot diagonal.

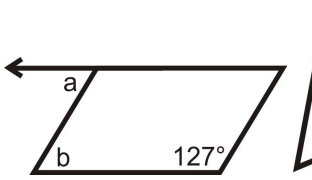


[Figure 13]

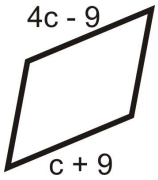
Review Questions

1. If $m\angle B = 72^\circ$ in parallelogram $ABCD$, find the other three angles.
2. If $m\angle S = 143^\circ$ in parallelogram $PQRS$, find the other three angles.
3. If $AB \perp BC$ in parallelogram $ABCD$, find the measure of all four angles.
4. If $m\angle F = x^\circ$ in parallelogram $EFGH$, find expressions for the other three angles in terms of x.

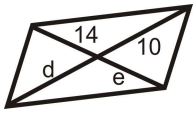
For questions 5-13, find the measures of the variable(s). All the figures below are parallelograms.



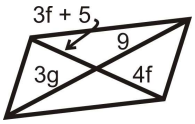
[Figure 14]



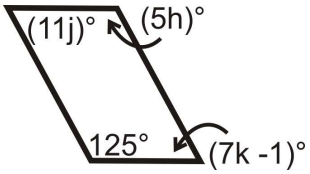
[Figure 15]



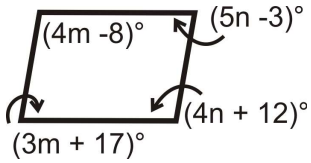
[Figure 16]



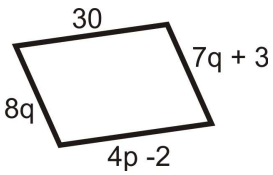
[Figure 17]



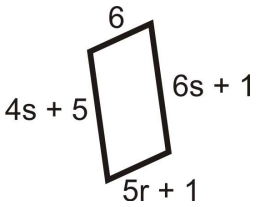
[Figure 18]



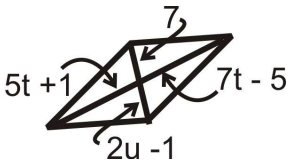
[Figure 19]



[Figure 20]

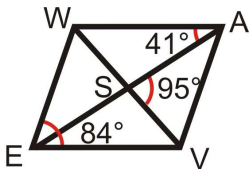


[Figure 21]



[Figure 22]

Use the parallelogram $WAVE$ to find:

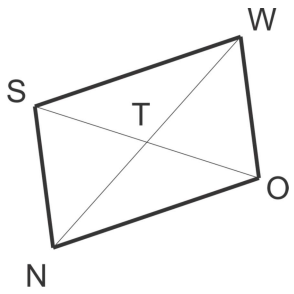


[Figure 23]

14. $m\angle AWE$
15. $m\angle ESV$
16. $m\angle WEA$

17. $m\angle AVW$

In the parallelogram $SNOW, ST = 6, NW = 4, m\angle OSW = 36^\circ, m\angle SNW = 58^\circ$ and $m\angle NTS = 80^\circ$. (*diagram is not drawn to scale*)



[Figure 24]

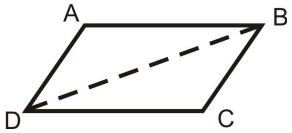
18. SO
19. NT
20. $m\angle NWS$
21. $m\angle SOW$

Plot the points $E(-1,3), F(3,4), G(5,-1), H(1,-2)$ and use parallelogram $EFGH$ for problems 22-25.

22. Find the coordinates of the point at which the diagonals intersect. How did you do this?
23. Find the slopes of all four sides. What do you notice?
24. Use the distance formula to find the lengths of all four sides. What do you notice?
25. Make a conjecture about how you might determine whether a quadrilateral in the coordinate is a parallelogram.

Write a two-column proof.

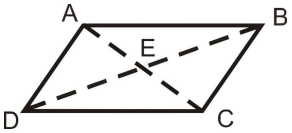
Opposite Angles Theorem



[Figure 25]

Given: $ABCD$ is a parallelogram with diagonal BD Prove: $\angle A \cong \angle C$

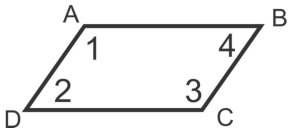
Parallelogram Diagonals Theorem



[Figure 26]

Given: $ABCD$ is a parallelogram with diagonals BD and AC Prove: $AE \cong EC$, $DE \cong EB$

Fill in the blanks for the proof of the ***Consecutive Angles Theorem***

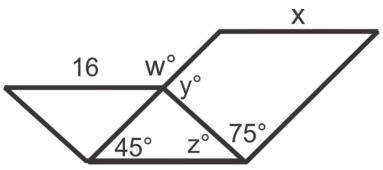


[Figure 27]

Given: $ABCD$ is a parallelogram Prove: $m\angle 1 + m\angle 2 = 180^\circ$

Statements	***Reasons***
1.	Given
2. $m\angle 1 = m\angle 3$ and _____	
3. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$	
4. $m\angle 1 + m\angle 2 + m\angle 1 + m\angle 2 = 360^\circ$	
5. $2(m\angle 1 + m\angle 2) = 360^\circ$	
6.	Division POE

Use the diagram below to find the indicated lengths or angle measures for problems 29-32. The two quadrilaterals that share a side are parallelograms.



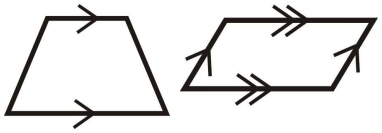
[Figure 28]

29. w

30. x

31. y
32. z

Review Queue Answers



[Figure 29] [Figure 30]

1. Answers:

a.
$$\begin{aligned} 3x + x + 3x + x &= 360^\circ \\ 8x &= 360^\circ \\ x &= 45^\circ \end{aligned}$$

b.
$$\begin{aligned} 4x + 2 &= 90^\circ \\ 4x &= 88^\circ \\ x &= 22^\circ \end{aligned}$$