2.5 Proofs about Angle Pairs and Segments

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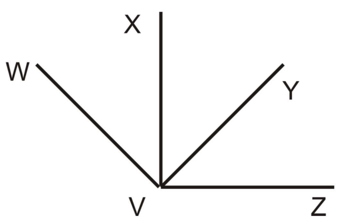
Learning Objectives

- Use theorems about special pairs of angles.
- Use theorems about right angles and midpoints.

Review Queue

Write a 2-column proof



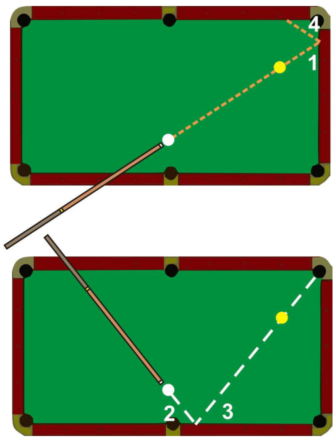
[Figure 1]

1. Given: $\overline{VX}$ is the angle bisector of $\angle WVY$.

$\overline{VY}$ is the angle bisector of $\angle XVZ$.

Prove: $\angle WVX \cong \angle YVZ$

Know What? The game of pool relies heavily on angles. The angle at which you hit the cue ball with your cue determines if a) you hit the yellow ball and b) if you can hit it into a pocket.



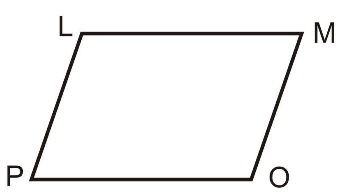
[Figure 2]

The top picture on the right illustrates if you were to hit the cue ball straight on and then hit the yellow ball. The orange line shows the path that the cue ball and then the yellow ball would take. You notice that $m\angle 1 = 56^\circ$. With a little focus, you notice that it makes more sense to approach the ball from the other side of the table and bank it off of the opposite side (see lower picture with the white path). You measure and need to hit the cue ball so that it hits the side of the table at a 50° angle (this would be $m\angle 2$). $\angle 3$ and $\angle 4$ are called the angles of reflection. Find the measures of these angles and how they relate to $\angle 1$ and $\angle 2$.

If you would like to play with the angles of pool, click the link for an interactive game. http://www.coolmath-games.com/0-poolgeometry/index.html

Naming Angles

As we learned in Chapter 1, angles can be addressed by numbers and three letters, where the letter in the middle is the vertex. We can shorten this label to one letter if there is only one angle with that vertex.



[Figure 3]

All of the angles in this parallelogram can be labeled by one letter, the vertex, instead of three.

$\angle MLP$ can be $\angle L$ $\quad$ $\angle LMO$ can be $\angle M$

$\angle MOP$ can be $\angle O$ $\quad$ $\angle OPL$ can be $\angle P$

This shortcut will now be used when applicable.

Right Angle Theorem: If two angles are right angles, then the angles are congruent.

Proof of the Right Angle Theorem

Given: $\angle A$ and $\angle B$ are right angles

Prove: $\angle A \cong \angle B$

Statement	*Reason*
1. $\angle A$ and $\angle B$ are right angles	Given
2. $m\angle A = 90^\circ$ and $m\angle B = 90^\circ$	Definition of right angles
3. $m\angle A = m\angle B$	Transitive PoE
4. $\angle A \cong \angle B$	$\cong$ angles have = measures

This theorem may seem redundant, but anytime right angles are mentioned, you need to use this theorem to say the angles are congruent.

Same Angle Supplements Theorem: If two angles are supplementary to the same angle then the angles are congruent.

So, if $m\angle A + m\angle B = 180^\circ$ and $m\angle C + m\angle B = 180^\circ$, then $m\angle A = m\angle C$. Using numbers to illustrate, we could say that if $\angle A$ is supplementary to an angle measuring 56° , then $m\angle A = 124^\circ$. $\angle C$ is also supplementary to 56° , so it too is 124° . Therefore, $m\angle A = m\angle C$. This example, however, does not constitute a proof.

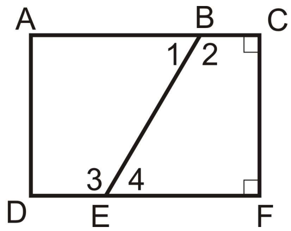
Proof of the Same Angles Supplements Theorem

Given: $\angle A$ and $\angle B$ are supplementary angles. $\angle B$ and $\angle C$ are supplementary angles.

Prove: $\angle A \cong \angle C$

Statement	*Reason*
1. $\angle A$ and $\angle B$ are supplementary $\angle B$ and $\angle C$ are supplementary	Given
2. $m\angle A + m\angle B = 180^\circ$ $m\angle B + m\angle C = 180^\circ$	Definition of supplementary angles
3. $m\angle A + m\angle B = m\angle B + m\angle C$	Substitution PoE
4. $m\angle A = m\angle C$	Subtraction PoE
5. $\angle A \cong \angle C$	$\cong$ angles have = measures

Example 1: Given that $\angle 1 \cong \angle 4$ and $\angle C$ and $\angle F$ are right angles, show which angles are congruent.



[Figure 4]

Solution: By the Right Angle Theorem, $\angle C \cong \angle F$. Also, $\angle 2 \cong \angle 3$ by the Same Angles Supplements Theorem. $\angle 1$ and $\angle 2$ are a linear pair, so they add up to 180° . $\angle 3$ and $\angle 4$ are also a linear pair and add up to 180° . Because $\angle 1 \cong \angle 4$, we can substitute $\angle 1$ in for $\angle 4$ and then $\angle 2$ and $\angle 3$ are supplementary to the same angle, making them congruent.

This is an example of a **paragraph proof**. Instead of organizing the proof in two columns, you explain everything in sentences.

Same Angle Complements Theorem: If two angles are complementary to the same angle then the angles are congruent.

So, if $m\angle A + m\angle B = 90^\circ$ and $m\angle C + m\angle B = 90^\circ$, then $m\angle A = m\angle C$. Using numbers, we could say that if $\angle A$ is supplementary to an angle measuring 56° , then $m\angle A = 34^\circ$. $\angle C$ is also supplementary to 56° , so it too is 34° . Therefore, $m\angle A = m\angle C$.

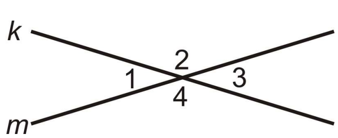
The proof of the Same Angles Complements Theorem is in the Review Questions. Use the proof of the Same Angles Supplements Theorem to help you.

Vertical Angles Theorem

Recall the Vertical Angles Theorem from Chapter 1. We will do a formal proof here.

Given: Lines k and m intersect.

Prove: $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



[Figure 5]

Statement	*Reason*
1. Lines k and m intersect	Given
2. $\angle 1$ and $\angle 2$ are a linear pair $\angle 2$ and $\angle 3$ are a linear pair $\angle 3$ and $\angle 4$ are a linear pair	Definition of a Linear Pair
3. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary $\angle 3$ and $\angle 4$ are supplementary	Linear Pair Postulate
4. $m\angle 1+m\angle 2=180^\circ$ $m\angle 2+m\angle 3=180^\circ$ $m\angle 3+m\angle 4=180^\circ$	Definition of Supplementary Angles
5. $m\angle 1+m\angle 2=m\angle 2+m\angle 3$ $m\angle 2+m\angle 3=m\angle 3+m\angle 4$	Substitution PoE
6. $m\angle 1=m\angle 3, m\angle 2=m\angle 4$	Subtraction PoE
7. $\angle 1\cong\angle 3, \angle 2\cong\angle 4$	$\cong$ angles have = measures

In this proof we combined everything. You could have done two separate proofs, one for $\angle 1\cong\angle 3$ and one for $\angle 2\cong\angle 4$.

Example 2: In the picture $\angle 2\cong\angle 3$ and $k\perp p$.

Each pair below is congruent. State why.

a) $\angle 1$ and $\angle 5$

b) $\angle 1$ and $\angle 4$

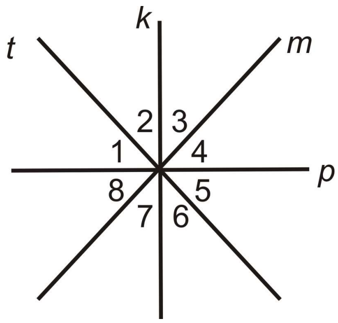
c) $\angle 2$ and $\angle 6$

d) $\angle 3$ and $\angle 7$

e) $\angle 6$ and $\angle 7$

f) $\angle 3$ and $\angle 6$

g) $\angle 4$ and $\angle 5$



[Figure 6]

Solution:

a), c) and d) Vertical Angles Theorem

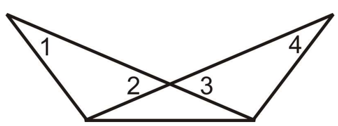
b) and g) Same Angles Complements Theorem

e) and f) Vertical Angles Theorem followed by the Transitive Property

Example 3: Write a two-column proof.

Given: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$

Prove: $\angle 1 \cong \angle 4$



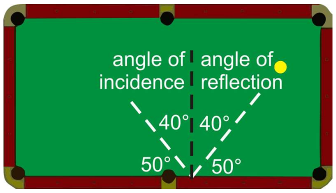
[Figure 7]

Solution:

Statement	*Reason*
1. $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$	Given
2. $\angle 2 \cong \angle 3$	Vertical Angles Theorem
3. $\angle 1 \cong \angle 4$	Transitive PoC

Know What? Revisited If $m\angle 2 = 50^\circ$, then

$m\angle 3 = 50^\circ$. Draw a perpendicular line at the point of reflection and the laws of reflection state that the angle of incidence is equal to the angle of reflection. So, this is a case of the Same Angles Complements Theorem. $\angle 2 \cong \angle 3$ because the angle of incidence and the angle of reflection are equal. We can also use this to find $m\angle 4$, which is 56° .

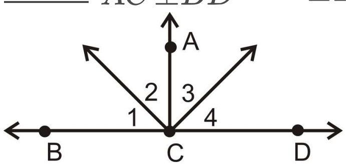


[Figure 8]

Review Questions

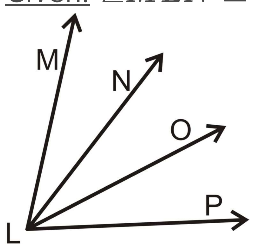
Write a two-column proof for questions 1-10.

Given: $AC \perp BD$ and $\angle 1 \cong \angle 4$ Prove: $\angle 2 \cong \angle 3$



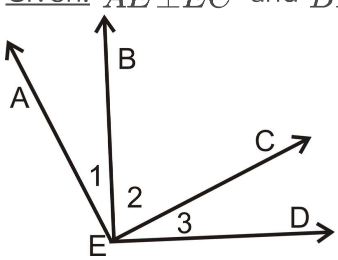
[Figure 9]

Given: $\angle MLN \cong \angle\ OLP$ Prove: $\angle MLO \cong \angle NLP$



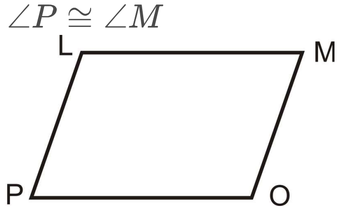
[Figure 10]

Given: $AE \perp EC$ and $BE \perp ED$ Prove: $\angle 1 \cong \angle 3$



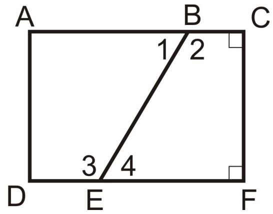
[Figure 11]

Given: $\angle L$ is supplementary to $\angle M$, $\angle P$ is supplementary to $\angle O$, $\angle L \cong \angle O$ Prove: $\angle P \cong \angle M$



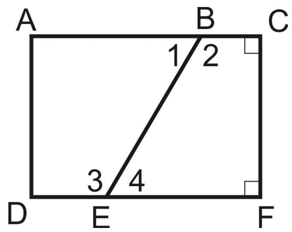
[Figure 12]

Given: $\angle 1 \cong \angle \ 4$ Prove: $\angle 2 \cong \angle 3$



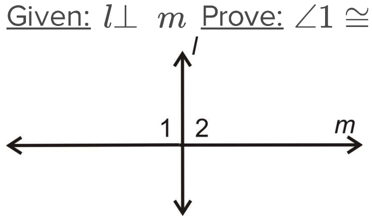
[Figure 13]

Given: $\angle C$ and $\angle F$ are right angles Prove: $m\angle C + m\angle F = 180^\circ$



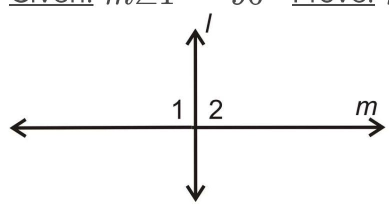
[Figure 14]

Given: $l \perp \ m$ Prove: $\angle 1 \cong \angle 2$



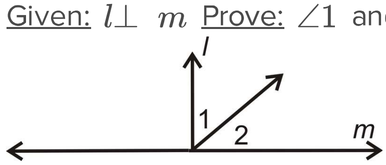
[Figure 15]

Given: $m\angle 1 = \ 90^\circ$ Prove: $m\angle 2 = 90^\circ$



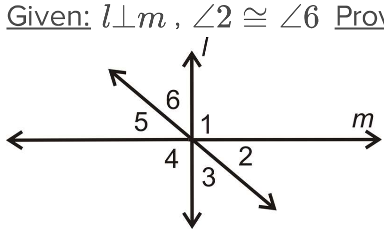
[Figure 16]

Given: $l \perp \ m$ Prove: $\angle 1$ and $\angle 2$ are complements



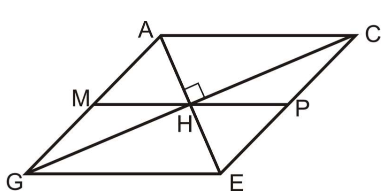
[Figure 17]

Given: $l \perp m$, $\angle 2 \cong \angle 6$ Prove: $\angle 6 \cong \angle 5$



[Figure 18]

Use the picture for questions 11-20.



[Figure 19]

Given: H is the midpoint of AE, MP and GC

M is the midpoint of GA

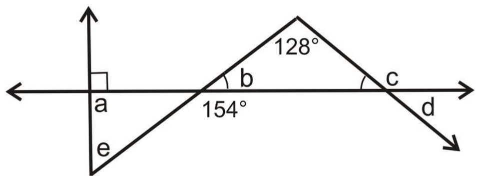
P is the midpoint of CE

$AE \perp GC$

11. List two pairs of vertical angles.
12. List all the pairs of congruent segments.
13. List two linear pairs that do not have H as the vertex.
14. List a right angle.
15. List two pairs of adjacent angles that are NOT linear pairs.
16. What is the perpendicular bisector of AE ?
17. List two bisectors of MP .
18. List a pair of complementary angles.
19. If GC is an angle bisector of $\angle AGE$, what two angles are congruent?
20. Fill in the blanks for the proof below. Given: Picture above and $\angle ACH \cong \angle ECH$ Prove: CH is the angle bisector of $\angle ACE$

Statement	*Reason*
1. $\angle ACH \cong \angle ECH$ CH is on the interior of $\angle ACE$	
2. $m\angle ACH = m\angle ECH$	
3.	Angle Addition Postulate
4.	Substitution
5. $m\angle ACE = 2m\angle ACH$	
6.	Division PoE
7.	

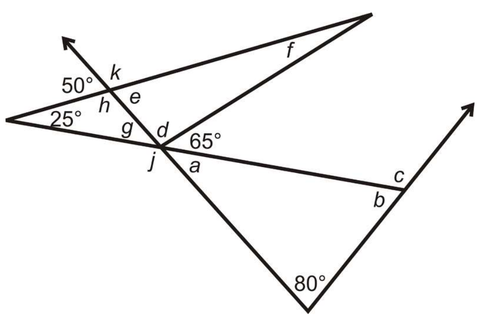
For questions 21-25, find the measure of the lettered angles in the picture below.



[Figure 20]

21. a
22. b
23. c
24. d
25. e (hint: e is complementary to b)

For questions 26-35, find the measure of the lettered angles in the picture below. *Hint: Recall the sum of the three angles in a triangle is* 180° .



[Figure 21]

26. a
27. b
28. c
29. d
30. e
31. f
32. g
33. h
34. j
35. k

Review Queue Answers

1.

Statement	Reason
1. VX is an $\angle$ bisector of $\angle WVY$ VY is an $\angle$ bisector of $\angle XVZ$	Given
2. $\angle WVX \cong \angle XVY$ $\angle XVY \cong \angle YVZ$	Definition of an angle bisector
3. $\angle WVX \cong \angle YVZ$	Transitive Property