3.3 Proving Lines Parallel

FlexBooks® 2.0 > American HS Geometry > Proving Lines Parallel

Last Modified: Dec 25, 2014

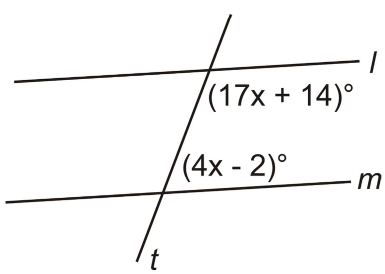
Learning Objectives

- Use the *converses* of the Corresponding Angles Postulate, Alternate Interior Angles Theorem, Alternate Exterior Angles Theorem, and the Same Side Interior Angles Theorem to show that lines are parallel.
- Construct parallel lines using the above converses.
- Use the Parallel Lines Property.

Review Queue

Answer the following questions.

1. Write the converse of the following statements:
   a. *If it is summer, then I am out of school.*
   b. *I will go to the mall when I am done with my homework.*
   c. *If two parallel lines are cut by a transversal, then the corresponding angles are congruent.*
2. Are any of the three converses from #1 true? Why or why not? Give a counterexample.
3. Determine the value of x if $l || m$.



[Figure 1]

Know What? Here is a picture of the support beams for the Coronado Bridge in San Diego. This particular bridge, called a girder bridge, is usually used in straight, horizontal situations. The Coronado Bridge is diagonal, so the beams are subject to twisting forces (called torque). This can be fixed by building a curved bridge deck. To aid the curved bridge deck, the support beams should not be parallel. If they are, the bridge would be too fragile and susceptible to damage.

[Figure 2]

This bridge was designed so that $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$. Are the support beams parallel?

Corresponding Angles Converse

Recall that the converse of a statement switches the conclusion and the hypothesis. So, if a , then b becomes if b , then a . We will find the converse of all the theorems from the last section and will determine if they are true.

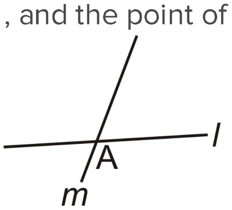
The Corresponding Angles Postulate says: *If two lines are parallel, then the corresponding angles are congruent.* The converse is:

Converse of Corresponding Angles Postulate: If corresponding angles are congruent when two lines are cut by a transversal, then the lines are parallel.

Is this true? For example, if the corresponding angles both measured 60° , would the lines be parallel? YES. All eight angles created by l, m and the transversal are either 60° or 120° , making the slopes of l and m the same which makes them parallel. This can also be seen by using a construction.

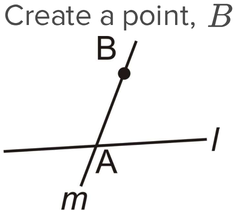
Investigation 3-5: Creating Parallel Lines using Corresponding Angles

Draw two intersecting lines. Make sure they are not perpendicular. Label them l and m , and the point of intersection, A , as shown.



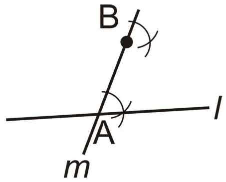
[Figure 3]

Create a point, B , on line m , above A .



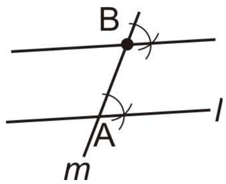
[Figure 4]

Copy the acute angle at A (the angle to the right of m) at point B. See Investigation 2-2 in Chapter 2 for the directions on how to copy an angle.



[Figure 5]

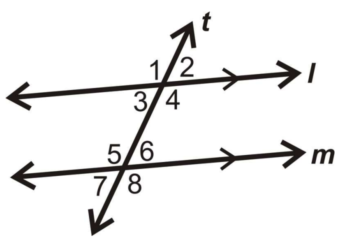
Draw the line from the arc intersections to point B.



[Figure 6]

From this construction, we can see that the lines are parallel.

Example 1: If $m\angle 8 = 110^\circ$ and $m\angle 4 = 110^\circ$, then what do we know about lines l and m?



[Figure 7]

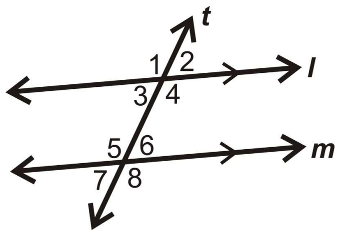
Solution: $\angle 8$ and $\angle 4$ are corresponding angles. Since $m\angle 8 = m\angle 4$, we can conclude that $l \,||\, m$.

Alternate Interior Angles Converse

We also know, from the last lesson, that when parallel lines are cut by a transversal, the alternate interior angles are congruent. The converse of this theorem is also true:

Converse of Alternate Interior Angles Theorem: If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Example 3: Prove the Converse of the Alternate Interior Angles Theorem.



[Figure 8]

Given: l and m and transversal t

$\angle 3 \cong \angle 6$

Prove: $l \,||\, m$

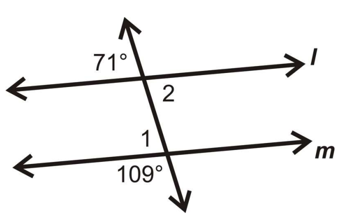
Solution:

Statement	***Reason***
1. l and m and transversal $t \angle 3 \cong \angle 6$	Given
2. $\angle 3 \cong \angle 2$	Vertical Angles Theorem
3. $\angle 2 \cong \angle 6$	Transitive PoC
4. $l \,||\, m$	Converse of the Corresponding Angles Postulate

Prove Move: Shorten the names of these theorems. Discuss with your teacher an appropriate abbreviations. For example, the Converse of the Corresponding Angles Theorem could be “Converse CA Thm” or “ConvCA.”

Notice that the Corresponding Angles Postulate was not used in this proof. The Transitive Property is the reason for Step 3 because we do not know if l is parallel to m until we are done with the proof. You could conclude that if we are trying to prove two lines are parallel, the converse theorems will be used. And, if we are proving two angles are congruent, we must be given that the two lines are parallel.

Example 4: Is $l \,||\, m$?

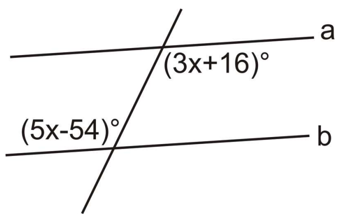


[Figure 9]

Solution: First, find $m\angle 1$. We know its linear pair is 109° . By the Linear Pair Postulate, these two angles add up to 180° , so $m\angle 1 = 180^\circ - 109^\circ = 71^\circ$. This means that $l \,||\, m$, by the Converse of the Corresponding Angles Postulate.

Example 5: ***Algebra Connection*** What does x have to be to make $a \mid\mid b$?

Solution: Because these are alternate interior angles, they must be equal for $a \mid\mid b$. Set the expressions equal to each other and solve.



[Figure 10]

$$\begin{aligned} 3x + 16^\circ &= 5x - 54^\circ \\ 70^\circ &= 2x \\ 35^\circ &= x \qquad \text{To make } a \mid\mid b,\ x = 35^\circ. \end{aligned}$$

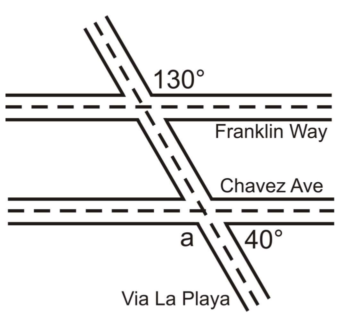
Converse of Alternate Exterior Angles & Consecutive Interior Angles

You have probably guessed that the converse of the Alternate Exterior Angles Theorem and the Consecutive Interior Angles Theorem areal so true.

Converse of the Alternate Exterior Angles Theorem: If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Example 6: ***Real-World Situation*** The map below shows three roads in Julio's town.

Julio used a surveying tool to measure two angles at the intersections in this picture he drew (NOT to scale). ***Julio wants to know if Franklin Way is parallel to Chavez Avenue.***



[Figure 11]

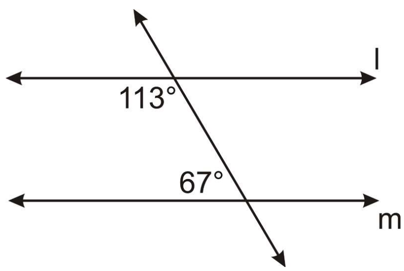
Solution: The labeled 130° angle and $\angle a$ are alternate exterior angles. If $m\angle a = 130^\circ$, then the lines are parallel. To find $m\angle a$, use the other labeled angle which is 40° , and its linear pair. Therefore, $\angle a + 40^\circ = 180^\circ$ and $\angle a = 140^\circ$. $140^\circ \neq 130^\circ$, so Franklin Way and Chavez Avenue are <u>not</u> parallel streets.

The final converse theorem is of the Same Side Interior Angles Theorem. Remember that these angles are not congruent when lines are parallel, they are **supplementary**.

Converse of the Same Side Interior Angles Theorem: If two lines are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Example 7: Is $l \,||\, m$? How do you know?

Solution: These are Same Side Interior Angles. So, if they add up to 180° , then $l \,||\, m$. $113^\circ + 67^\circ = 180^\circ$, therefore $l \,||\, m$.



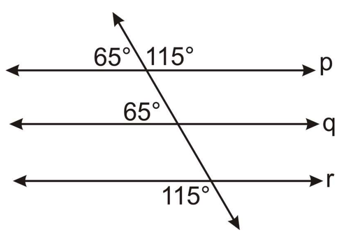
[Figure 12]

Parallel Lines Property

The Parallel Lines Property is a transitive property that can be applied to parallel lines. Remember the Transitive Property of Equality is: If $a = b$ and $b = c$, then $a = c$. The Parallel Lines Property changes = to $||$.

Parallel Lines Property: If lines $l \,||\, m$ and $m \,||\, n$, then $l \,||\, n$.

Example 8: Are lines q and r parallel?



[Figure 13]

Solution: First find if $p \,||\, q$, followed by $p \,||\, r$. If so, then $q \,||\, r$.

$p \,||\, q$ by the Converse of the Corresponding Angles Postulate, the corresponding angles are 65° . $p \,||\, r$ by the Converse of the Alternate Exterior Angles Theorem, the alternate exterior angles are 115° . Therefore, by the Parallel Lines Property, $q \,||\, r$.

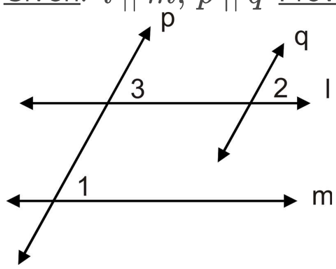
Know What? Revisited: The CoronadoBridge has $\angle 1$ and $\angle 2$, which are corresponding angles. These angles must be equal for the beams to be parallel. $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$ and $92^\circ \neq 88^\circ$, so the beams are <u>not</u> parallel, therefore a sturdy and safe girder bridge.

Review Questions

1. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing congruent alternate interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy the angle in a different location.
2. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing supplementary consecutive interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy a different angle.

For Questions 3-5, fill in the blanks in the proofs below.

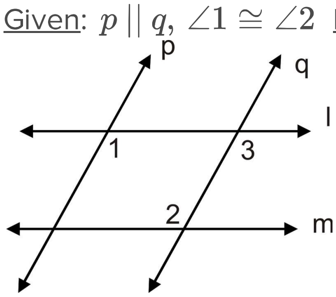
Given: $l \,||\, m,\ p \,||\, q$ Prove: $\angle 1 \cong \angle 2$



[Figure 14]

Statement	*Reason*
1. $l \,\|\|\, m$	1.
2.	2. Corresponding Angles Postulate
3. $p \,\|\|\, q$	3.
4.	4.
5. $\angle 1 \cong \angle 2$	5.

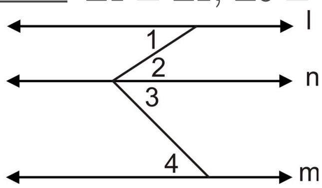
Given: $p \,||\, q,\ \angle 1 \cong \angle 2$ Prove: $l \,||\, m$



[Figure 15]

Statement	*Reason*
1. $p \mid\mid q$	1.
2.	2. Corresponding Angles Postulate
3. $\angle 1 \cong \angle 2$	3.
4.	4. Transitive PoC
5.	5. Converse of Alternate Interior Angles Theorem

Given: $\angle 1 \cong \angle 2, \ \angle 3 \cong \angle 4$ Prove: $l \mid\mid m$

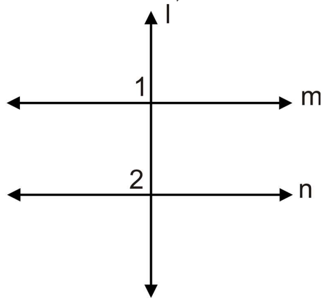


[Figure 16]

Statement	*Reason*
1. $\angle 1 \cong \angle 2$	1.
2. $l \mid\mid n$	2.
3. $\angle 3 \cong \angle 4$	3.
4.	4. Converse of Alternate Interior Angles Theorem
5. $l \mid\mid m$	5.

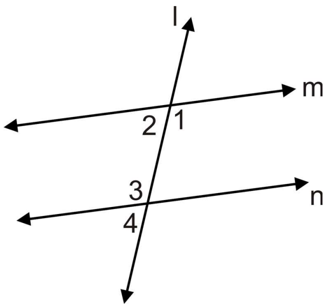
For Questions 6-9, create your own two column proof.

Given: $m \perp l, n \perp l$ Prove: $m || n$



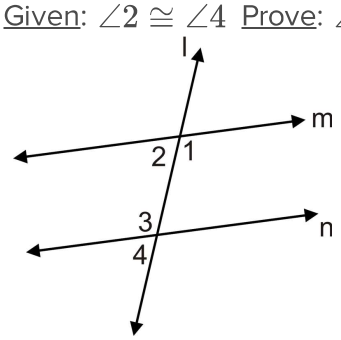
[Figure 17]

Given: $\angle 1 \cong \angle 3$ Prove: $\angle 1$ and $\angle 4$ are supplementary



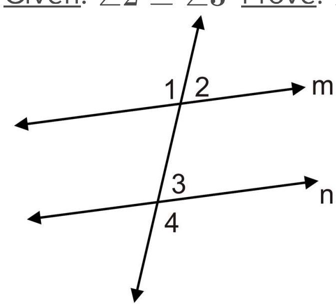
[Figure 18]

Given: $\angle 2 \cong \angle 4$ Prove: $\angle 1 \cong \angle 3$



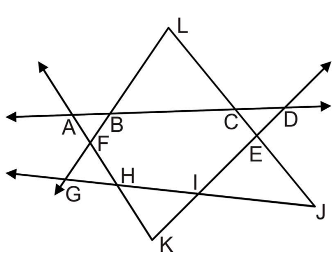
[Figure 19]

Given: $\angle 2 \cong \angle 3$ Prove: $\angle 1 \cong \angle 4$



[Figure 20]

In 10-15, use the given information to determine which lines are parallel. If there are none, write *none*. Consider each question individually.

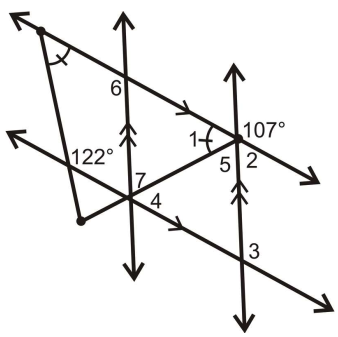


[Figure 21]

10. $\angle LCD \cong \angle CJI$
11. $\angle BCE\ and\ \angle BAF$ are supplementary

12. $\angle FGH \cong \angle EIJ$
13. $\angle BFH \cong \angle CEI$
14. $\angle LBA \cong \angle IHK$
15. $\angle ABG \cong \angle BGH$

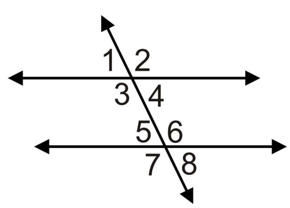
In 16-22, find the measure of the lettered angles below.



[Figure 22]

16. $m\angle 1$
17. $m\angle 2$
18. $m\angle 3$
19. $m\angle 4$
20. $m\angle 5$
21. $m\angle 6$
22. $m\angle 7$

For 23-27, what does x have to measure to make the lines parallel?

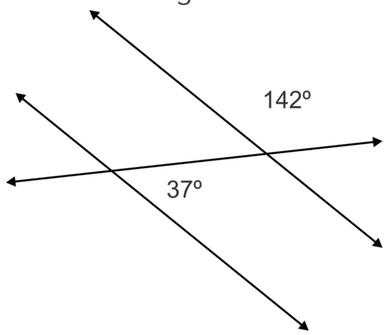


[Figure 23]

23. $m\angle 3 = (3x+25)^\circ$ and $m\angle 5 = (4x-55)^\circ$
24. $m\angle 2 = (8x)^\circ$ and $m\angle 7 = (11x-36)^\circ$
25. $m\angle 1 = (6x-5)^\circ$ and $m\angle 5 = (5x+7)^\circ$
26. $m\angle 4 = (3x-7)^\circ$ and $m\angle 7 = (5x-21)^\circ$
27. $m\angle 1 = (9x)^\circ$ and $m\angle 6 = (37x)^\circ$

28. ***Construction*** Draw a straight line. Construct a line perpendicular to this line through a point on the line. Now, construct a perpendicular line to this new line. What can you conclude about the original line and this final line?
29. How could you prove your conjecture from problem 28?

What is wrong in the following diagram, given that $j || k$?



[Figure 24]

Review Queue Answers

1. Answers:
   a. *If I am out of school, then it is summer.*
   b. *If I go to the mall, then I am done with my homework.*
   c. *If corresponding angles created by two lines cut by a transversal are congruent, then the two lines are parallel.*
2. Answers:
   a. Not true, I could be out of school on any school holiday or weekend during the school year.
   b. Not true, I don't have to be done with my homework to go to the mall.
   c. Yes, because if two corresponding angles are congruent, then the slopes of these two lines have to be the same, making the lines parallel.
3. The two angles are supplementary.

$$\begin{aligned}(17x+14)^\circ + (4x-2)^\circ &= 180^\circ \\ 21x + 12^\circ &= 180^\circ \\ 21x &= 168^\circ \\ x &= 8^\circ\end{aligned}$$