7.3 Similarity by AA

FlexBooks® 2.0 > American HS Geometry > Similarity by AA

Last Modified: Dec 25, 2014

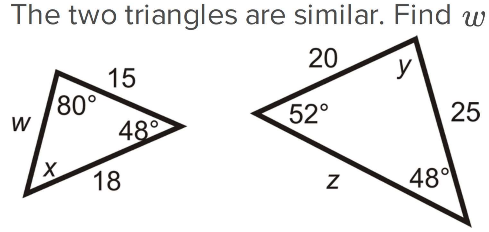
Learning Objectives

- Determine whether triangles are similar.
- Understand AA for similar triangles.
- Solve problems involving similar triangles.

Review Queue

a. Find the measures of x and y.

The two triangles are similar. Find w and z.



[Figure 1]

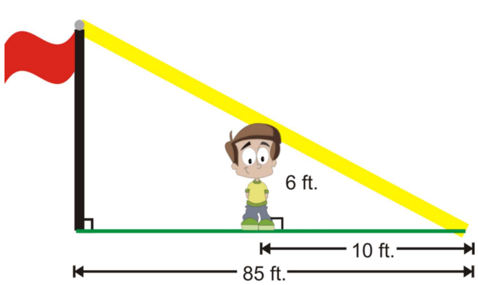
1. Use the true proportion $\frac{6}{8} = \frac{x}{28} = \frac{27}{y}$ to answer the following questions.

a. Find x and y.

b. Write another true proportion.

c. Is $\frac{28}{8} = \frac{6+x}{12}$ true? If you solve for x, is it the same as in part a?

Know What? George wants to measure the height of a flagpole. He is 6 feet tall and his shadow is 10 feet long. At the same time, the shadow of the flagpole was 85 feet long. How tall is the flagpole?



[Figure 2]

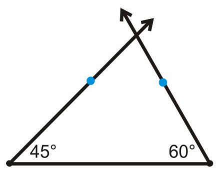
Angles in Similar Triangles

The Third Angle Theorem states if two angles are congruent to two angles in another triangle, the third angles are congruent too. Because a triangle has 180° , the third angle in any triangle is 180° minus the other two angle measures. Let's investigate what happens when two different triangles have the same angle measures. We will use Investigation 4-4 (Constructing a Triangle using ASA) to help us with this.

Investigation 7-1: Constructing Similar Triangles

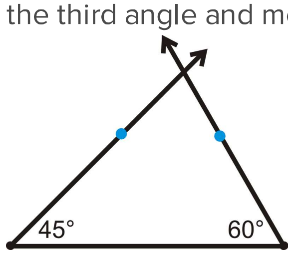
Tools Needed: pencil, paper, protractor, ruler

Draw a 45° angle. Extend the horizontal side and then draw a 60° angle on the other side of this side. Extend the other side of the 45° angle and the 60° angle so that they intersect to form a triangle. What is the measure of the third angle? Measure the length of each side.



[Figure 3]

Repeat Step 1 and make the horizontal side between the 45° and 60° angle at least 1 inch longer than in Step 1. This will make the entire triangle larger. Find the measure of the third angle and measure the length of each side.



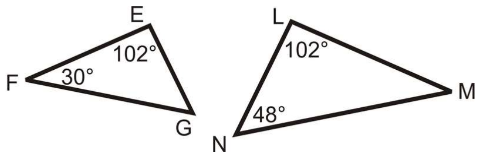
[Figure 4]

1. Find the ratio of the sides. Put the sides opposite the 45° angles over each other, the sides opposite the 60° angles over each other, and the sides opposite the third angles over each other. What happens?

AA Similarity Postulate: If two angles in one triangle are congruent to two angles in another triangle, the two triangles are similar.

The AA Similarity Postulate is a shortcut for showing that two ***triangles*** are similar. If you know that two angles in one triangle are congruent to two angles in another, which is now enough information to show that the two triangles are similar. Then, you can use the similarity to find the lengths of the sides.

Example 1: Determine if the following two triangles are similar. If so, write the similarity statement.

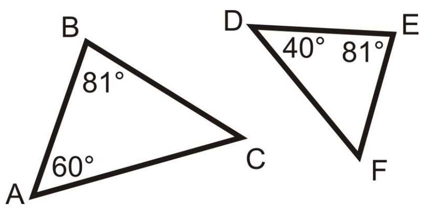


[Figure 5]

Solution: Find the measure of the third angle in each triangle. $m\angle G = 48^\circ$ and $m\angle M = 30^\circ$ by the Triangle Sum Theorem. Therefore, all three angles are congruent, so the two triangles are similar. $\triangle FEG \sim \triangle MLN$.

Example 2: Determine if the following two triangles are similar. If so, write the similarity statement.

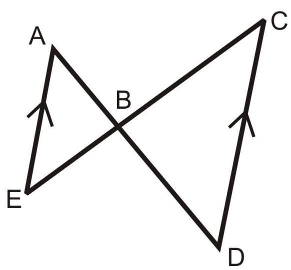
Solution: $m\angle C = 39^\circ$ and $m\angle F = 59^\circ$. The angles are not equal, $\triangle ABC$ and $\triangle DEF$ are not similar.



[Figure 6]

Example 3: Are the following triangles similar? If so, write the similarity statement.

Solution: Because $AE \,||\, CD, \angle A \cong \angle D$ and $\angle C \cong \angle E$ by the Alternate Interior Angles Theorem. Therefore, by the AA Similarity Postulate, $\triangle ABE \sim \triangle DBC$.



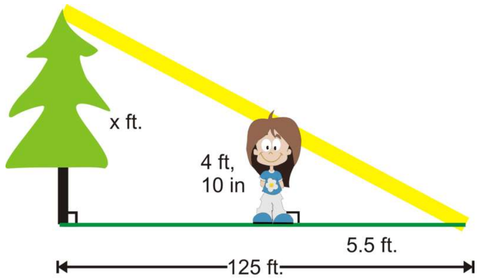
[Figure 7]

Indirect Measurement

An application of similar triangles is to measure lengths *indirectly*. The length to be measured would be some feature that was not easily accessible to a person, such as: the width of a river or canyon and the height of a tall object. To measure something indirectly, you would need to set up a pair of similar triangles.

Example 4: A tree outside Ellie's building casts a 125 foot shadow. At the same time of day, Ellie casts a 5.5 foot shadow. If Ellie is 4 feet 10 inches tall, how tall is the tree?

Solution: Draw a picture. From the picture to the right, we see that the tree and Ellie are parallel, therefore the two triangles are similar to each other. Write a proportion.



[Figure 8]

$$\frac{4ft, 10in}{xft} = \frac{5.5ft}{125ft}$$

Notice that our measurements are not all in the same units. Change both numerators to inches and then we can cross multiply.

$$\begin{aligned} \frac{58in}{xft} = \frac{66in}{125ft} \longrightarrow 58(125) &= 66(x) \\ 7250 &= 66x \\ x &\approx 109.85\ ft \end{aligned}$$

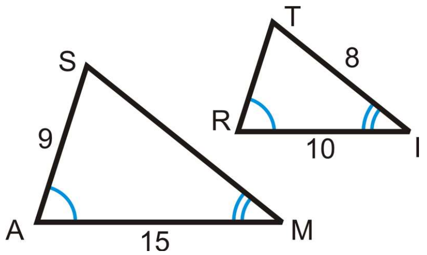
Know What? Revisited It is safe to assume that George and the flagpole stand vertically, making right angles with the ground. Also, the angle where the sun's rays hit the ground is the same for both. The two triangles are similar. Set up a proportion.

$$\begin{aligned} \frac{10}{85} = \frac{6}{x} \longrightarrow 10x &= 510 \\ x &= 51\ ft. \end{aligned}$$

The height of the flagpole is 51 feet.

Review Questions

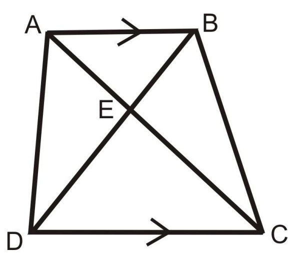
Use the diagram to complete each statement.



[Figure 9]

1. $\triangle SAM \sim \triangle$_____
2. $\frac{SA}{?} = \frac{SM}{?} = \frac{?}{RI}$
3. $SM =$ _____
4. $TR =$ _____
5. $\frac{9}{?} = \frac{?}{8}$

Answer questions 6-9 about trapezoid $ABCD$.

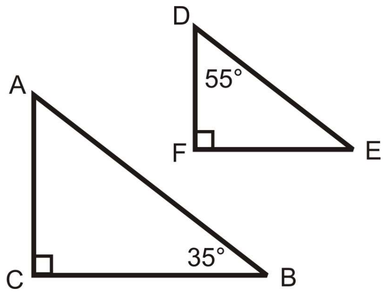


[Figure 10]

6. Name two similar triangles. How do you know they are similar?
7. Write a true proportion.
8. Name two other triangles that might *not* be similar.
9. If $AB = 10, AE = 7,$ and $DC = 22$, find AC. Be careful!
10. ***Writing*** How many angles need to be congruent to show that two triangles are similar? Why?
11. ***Writing*** How do congruent triangles and similar triangles differ? How are they the same?

Use the triangles below for questions 12-14.

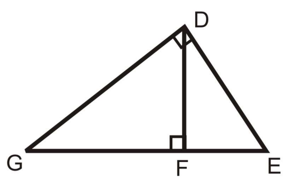
$AB = 20, DE = 15,$ and $BC = k$.



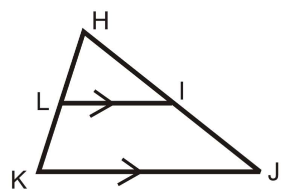
[Figure 11]

12. Are the two triangles similar? How do you know?
13. Write an expression for FE in terms of k.
14. If $FE = 12$, what is k?
15. Fill in the blanks: If an acute angle of a ________ triangle is congruent to an acute angle in another _________ triangle, then the two triangles are ________.

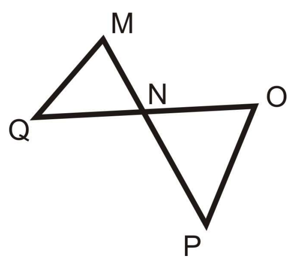
Are the following triangles similar? If so, write a similarity statement.



[Figure 12]

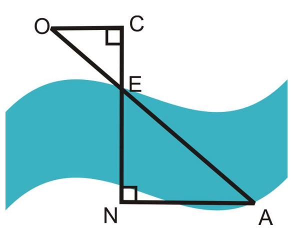


[Figure 13]



[Figure 14]

In order to estimate the width of a river, the following technique can be used. Use the diagram on the left.



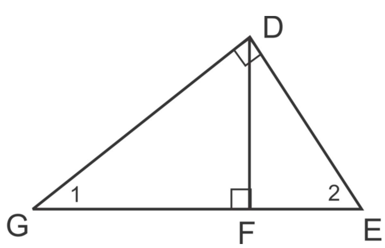
[Figure 15]

Place three markers, $O, C,$ and E on the upper bank of the river. E is on the edge of the river and $OC \perp CE$. Go across the river and place a marker, N so that it is collinear with C and E. Then, walk along the lower bank of the river and place marker A, so that $CN \perp NA$. $OC = 50 \text{ feet}, CE = 30 \text{ feet}, NA = 80 \text{ feet}$.

19. Is $OC \parallel NA$? How do you know?
20. Is $\triangle OCE \sim \triangle ANE$? How do you know?

21. What is the width of the river? Find EN.
22. Can we find EA? If so, find it. If not, explain.
23. Janet wants to measure the height of her apartment building. She places a pocket mirror on the ground 20 ft from the building and steps backwards until she can see the top of the build in the mirror. She is 18 in from the mirror and her eyes are 5 ft 3 in above the ground. The angle formed by her line of sight and the ground is congruent to the angle formed by the reflection of the building and the ground. You may wish to draw a diagram to illustrate this problem. How tall is the building?
24. Sebastian is curious to know how tall the announcer's box is on his school's football field. On a sunny day he measures the shadow of the box to be 45 ft and his own shadow is 9 ft. Sebastian is 5 ft 10 in tall. Find the height of the box.
25. Juanita wonders how tall the mast of a ship she spots in the harbor is. The deck of the ship is the same height as the pier on which she is standing. The shadow of the mast is on the pier and she measures it to be 18 ft long. Juanita is 5 ft 4 in tall and her shadow is 4 ft long. How tall is the ship's mast?
26. Use shadows or a mirror to measure the height of an object in your yard or on the school grounds. Draw a picture to illustrate your method.

Use the diagram below to answer questions 27-31.



[Figure 16]

27. Draw the three separate triangles in the diagram.
28. Explain why $\triangle GDE \cong \triangle DFE \cong \triangle GFD$.

Complete the following proportionality statements.

29. $\frac{GF}{DF} = \frac{?}{FE}$
30. $\frac{GF}{GD} = \frac{?}{GE}$
31. $\frac{GE}{DE} = \frac{DE}{?}$

Review Queue Answers

1. Answers:

   a. $x = 52^\circ, y = 80^\circ$

   b. $$\begin{aligned} \frac{w}{20} &= \frac{15}{25} \\ 25w &= 15(20) \\ 25w &= 300 \\ w &= 12 \end{aligned} \qquad \begin{aligned} \frac{15}{25} &= \frac{18}{z} \\ 25(18) &= 15z \\ 450 &= 15z \\ 30 &= z \end{aligned}$$

2. Answers:

   a. $$\begin{aligned} 168 &= 8x \\ x &= 21 \end{aligned} \qquad \begin{aligned} 6y &= 216 \\ y &= 36 \end{aligned}$$

   b. Answers will vary. One possibility: $\frac{28}{8} = \frac{21}{6}$

   c. $$\begin{aligned} 28(12) &= 8(6+x) \\ 336 &= 48 + 8x \\ 288 &= 8x \\ 36 &= x \quad \text{Because } x \neq 21, \text{ like in part a, this is not a true proportion.} \end{aligned}$$

7.4 Similarity by SSS and SAS

FlexBooks® 2.0 > American HS Geometry > Similarity by SSS and SAS

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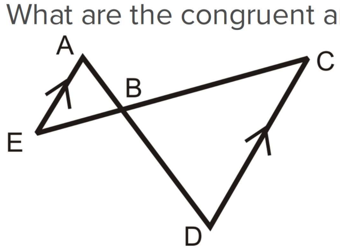
Learning Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

Review Queue

What are the congruent angles? List each pair.



[Figure 1]

a. Write the similarity statement.

b. If $AB = 8, BD = 20,$ and $BC = 25$, find BE .

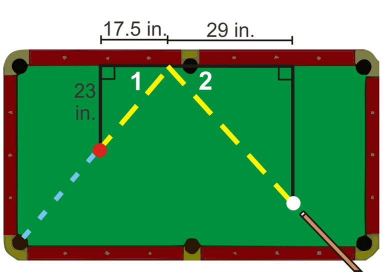
1. Solve the following proportions.

a. $\frac{6}{8} = \frac{21}{x}$

b. $\frac{x+2}{6} = \frac{2x-1}{15}$

c. $\frac{x-3}{9} = \frac{4}{x+2}$

Know What? Recall from Chapter 2, that the game of pool relies heavily on angles. In Section 2.5, we discovered that $m\angle 1 = m\angle 2$.



[Figure 2]

The dimensions of a pool table are 92 inches by 46 inches. You decide to hit the cue ball so it follows the yellow path to the right. The horizontal and vertical distances are in the picture. Are the two triangles similar? Why? How far did the cue ball travel to get to the red ball?

Link for an interactive game of pool: http://www.coolmath-games.com/0-poolgeometry/index.html

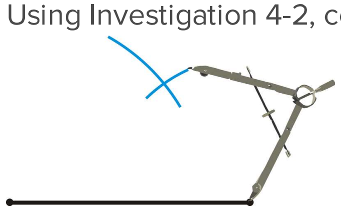
SSS for Similar Triangles

If you do not know any angle measures, can you say two triangles are similar? Let's investigate this to see. You will need to recall Investigation 4-2, Constructing a Triangle, given Three Sides.

Investigation 7-2: SSS Similarity

Tools Needed: ruler, compass, protractor, paper, pencil

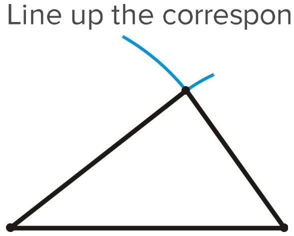
Using Investigation 4-2, construct a triangle with sides 6 cm, 8 cm, and 10 cm.



[Figure 3]

1. Construct a second triangle with sides 9 cm, 12 cm, and 15 cm.
2. Using your protractor, measure the angles in both triangles. What do you notice?

Line up the corresponding sides. Write down the ratios of these sides. What happens?



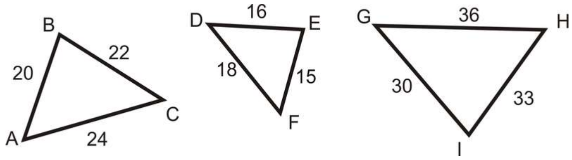
[Figure 4]

To see an animated construction of this, click: http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html

From #3, you should notice that the angles in the two triangles are equal. Second, when the corresponding sides are lined up, the sides are all in the same proportion, $\frac{6}{9} = \frac{8}{12} = \frac{10}{15}$. If you were to repeat this activity, for a 3-4-5 or 12-16-20 triangle, you will notice that they are all similar. That is because, each of these triangles are multiples of 3-4-5. If we generalize what we found in this investigation, we have the SSS Similarity Theorem.

SSS Similarity Theorem: If the corresponding sides of two triangles are proportional, then the two triangles are similar.

Example 1: Determine if any of the triangles below are similar.



[Figure 5]

Solution: Compare two triangles at a time. In the proportions, place the shortest sides over each other, the longest sides over each other, and the middle sides over each other. Then, determine if the proportions are equal.

$\triangle ABC$ and $\triangle DEF$: $\frac{20}{15}=\frac{22}{16}=\frac{24}{18}$

Reduce each fraction to see if they are equal. $\frac{20}{15}=\frac{4}{3},\frac{22}{16}=\frac{11}{8}$, and $\frac{24}{18}=\frac{4}{3}$. Because $\frac{4}{3}\neq\frac{11}{8},\triangle ABC$ and $\triangle DEF$ are **not** similar.

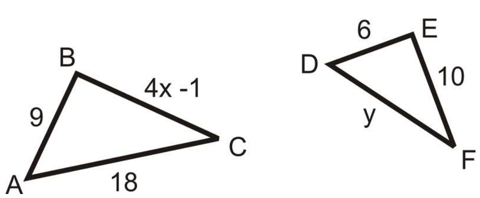
$\triangle DEF$ and $\triangle GHI$: $\frac{15}{30}=\frac{16}{33}=\frac{18}{36}$

$\frac{15}{30}=\frac{1}{2},\frac{16}{33}=\frac{16}{33},$ and $\frac{18}{36}=\frac{1}{2}$. Because $\frac{1}{2}\neq\frac{16}{33},\triangle DEF$ is not similar to $\triangle GHI$.

$\triangle ABC$ and $\triangle GHI$: $\frac{20}{30}=\frac{22}{33}=\frac{24}{36}$

$\frac{20}{30}=\frac{2}{3},\frac{22}{33}=\frac{2}{3},$ and $\frac{24}{36}=\frac{2}{3}$. Because all three ratios reduce to $\frac{2}{3},\triangle ABC\sim\triangle GIH$.

Example 2: *Algebra Connection* Find x and y, such that $\triangle ABC\sim\triangle DEF$.



[Figure 6]

Solution: According to the similarity statement, the corresponding sides are: $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$. Substituting in what we know, we have:

$$\frac{9}{6} = \frac{4x-1}{10} = \frac{18}{y}$$

$$\begin{aligned} \frac{9}{6} &= \frac{4x-1}{10} & \frac{9}{6} &= \frac{18}{y} \\ 9(10) &= 6(4x-1) & 9y &= 18(6) \\ 90 &= 24x-6 & 9y &= 108 \\ 96 &= 24x & y &= 12 \\ x &= 4 \end{aligned}$$

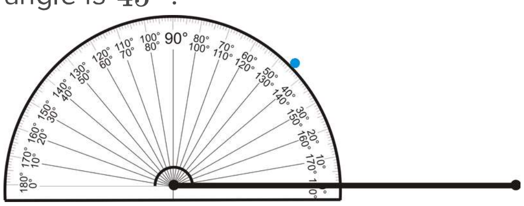
SAS for Similar Triangles

SAS is the last way to show two triangles are similar. If we know that two sides are proportional AND the included angles are congruent, then the two triangles are similar. For the following investigation, you will need to use Investigation 4-3, Constructing a Triangle with SAS.

Investigation 7-3: SAS Similarity

Tools Needed: paper, pencil, ruler, protractor, compass

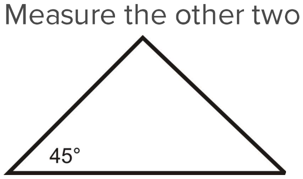
Using Investigation 4-3, construct a triangle with sides 6 cm and 4 cm and the *included* angle is 45° .



[Figure 7]

1. Repeat Step 1 and construct another triangle with sides 12 cm and 8 cm and the included angle is 45° .

   Measure the other two angles in both triangles. What do you notice?



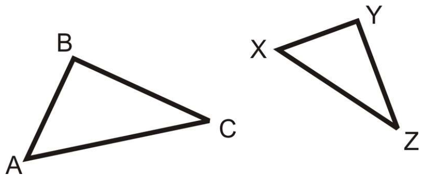
[Figure 8]

2. Measure the third side in each triangle. Make a ratio. Is this ratio the same as the ratios of the sides you were given?

SAS Similarity Theorem: If two sides in one triangle are proportional to two sides in another triangle and the included angle in the first triangle is congruent to the included angle in the second, then the two triangles are similar.

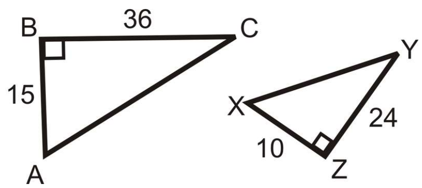
In other words,

If $\frac{AB}{XY} = \frac{AC}{XZ}$ and $\angle A \cong \angle X$, then $\triangle ABC \sim \triangle XYZ$.



[Figure 9]

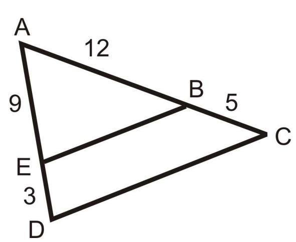
Example 3: Are the two triangles similar? How do you know?



[Figure 10]

Solution: $\angle B \cong \angle Z$ because they are both right angles. Second, $\frac{10}{15} = \frac{24}{36}$ because they both reduce to $\frac{2}{3}$. Therefore, $\frac{AB}{XZ} = \frac{BC}{ZY}$ and $\triangle ABC \sim \triangle XZY$.

Notice with this example that we can find the third sides of each triangle using the Pythagorean Theorem. If we were to find the third sides, $AC = 39$ and $XY = 26$. The ratio of these sides is $\frac{26}{39} = \frac{2}{3}$.



[Figure 11]

Example 4: Are there any similar triangles? How do you know?

Solution: $\angle A$ is shared by $\triangle EAB$ and $\triangle DAC$, so it is congruent to itself. If $\frac{AE}{AD} = \frac{AB}{AC}$ then, by SAS Similarity, the two triangles would be similar.

$$\frac{9}{9+3} = \frac{12}{12+5}$$
$$\frac{9}{12} = \frac{3}{4} \neq \frac{12}{17}$$

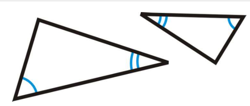
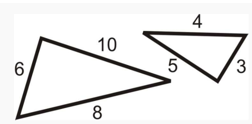
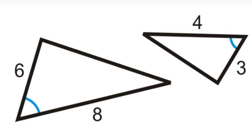
Because the proportion is not equal, the two triangles are not similar.

Example 5: From Example 4, what should BC equal for $\triangle EAB \sim \triangle DAC$?

Solution: The proportion we ended up with was $\frac{9}{12} = \frac{3}{4} \neq \frac{12}{17}$. AC needs to equal 16, so that $\frac{12}{16} = \frac{3}{4}$. Therefore, $AC = AB + BC$ and $16 = 12 + BC$. BC should equal 4 in order for $\triangle EAB \sim \triangle DAC$.

Similar Triangles Summary

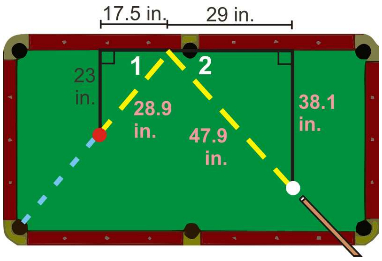
Let's summarize what we've found that guarantees two triangles are similar. Two triangles are **similar** if and only if:

Name	*Description*	*Picture*
AA	Two angles in one triangle are congruent to two angles in another triangle.	 [Figure 12]
SSS for Similar Triangles	All three sides in one triangle are proportional to three sides in another triangle.	 [Figure 13]
SAS for Similar Triangles	Two sides in one triangle are proportional with two sides in another triangle AND the included angles are congruent.	 [Figure 14]

Know What? Revisited First, we need to find the vertical length of the larger triangle. The two triangles are similar by AA, two right angles and $\angle 1 \cong \angle 2$. Set up a proportion.

$$\frac{17.5}{23} = \frac{29}{v}$$

Doing cross-multiplication, $v = 38.1$. Second, to find the distance that the cue ball travels, use the Pythagorean Theorem. $17.5^2 + 23^2 = d_1^2$ and $38.1^2 + 29^2 = d_2^2$, the lengths 28.9 and 47.9, and the total length is 76.8 inches.



[Figure 15]

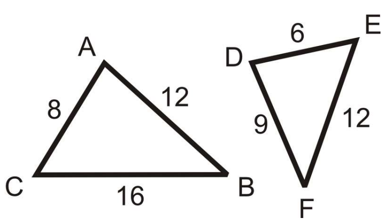
Review Questions

Use the following diagram for questions 1-3. *The diagram is to scale.*

[Figure 16]

1. Are the two triangles similar? Explain your answer.
2. Are the two triangles congruent? Explain your answer.
3. What is the scale factor for the two triangles?
4. ***Writing*** How come there is no ASA Similarity Theorem?

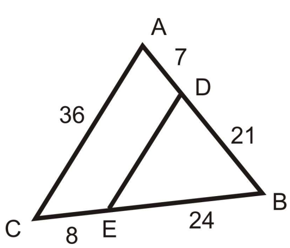
Fill in the blanks in the statements below. Use the diagram to the left.



[Figure 17]

5. $\triangle ABC \sim \triangle$_____
6. $\frac{AB}{?} = \frac{BC}{?} = \frac{AC}{?}$
7. If $\triangle ABC$ had an altitude, $AG = 10$, what would be the length of altitude DH?

Use the diagram to the right for questions 8-12.



[Figure 18]

8. $\triangle ABC \sim \triangle$_____

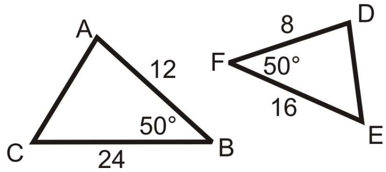
9. Why are the two triangles similar?

10. Find ED.

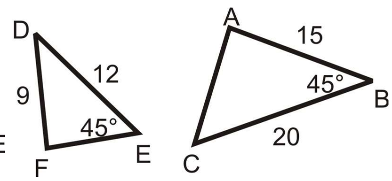
11. $\frac{BD}{?} = \frac{?}{BC} = \frac{DE}{?}$

12. Is $\frac{AD}{DB} = \frac{CE}{EB}$ a valid proportion? How do you know?

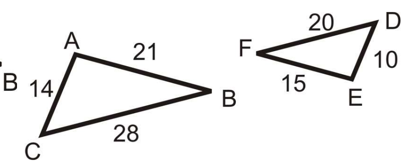
Determine if the following triangles are similar. If so, write the similarity theorem and statement.



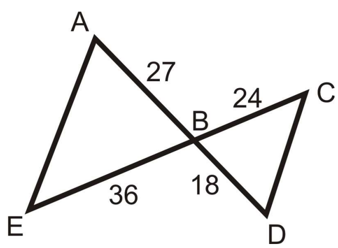
[Figure 19]



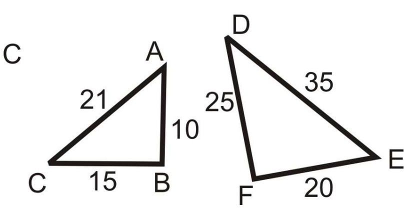
[Figure 20]



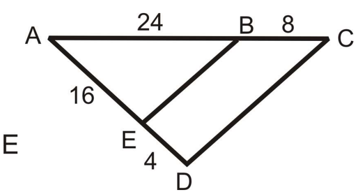
[Figure 21]



[Figure 22]

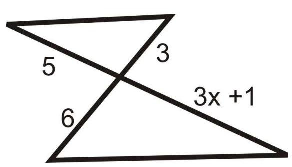


[Figure 23]

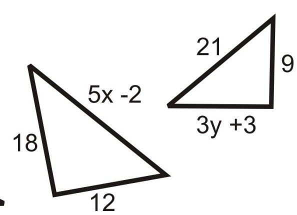


[Figure 24]

Algebra Connection Find the value of the missing variable(s) that makes the two triangles similar.



[Figure 25]



[Figure 26]

19. At a certain time of day, a building casts a 25 ft shadow. At the same time of day, a 6 ft tall stop sign casts a 15 ft shadow. How tall is the building?

20. A child who is 42 inches tall is standing next to the stop sign in #21. How long is her shadow?

21. An architect wants to build 3 similar right triangles such that the ratio of the middle triangle to the small triangle is the same as the ratio of the largest triangle to the middle triangle. The smallest one has side lengths 5, 12 and 13. The largest triangle has side lengths 45, 108 and 117. What are the lengths of the sides of the middle triangle?

22. Jaime wants to find the height of a radio tower in his neighborhood. He places a mirror on the ground 30 ft from the tower and walks backwards 3 ft until he can see the top of the tower in the mirror. Jaime is 5 ft 6 in tall. How tall is the radio tower?

For questions 25-27, use $\triangle ABC$ with $A(-3,0), B(-1.5,3)$ and $C(0,0)$ and $\triangle DEF$ with $D(0,2), E(1,4)$ and $F(2,2)$.

25. Find AB, BC, AC, DE, EF and DF.
26. Use these values to find the following proportions: $\frac{AB}{DE}, \frac{BC}{EF}$ and $\frac{AC}{DF}$.
27. Are these triangles similar? Justify your answer.

For questions 28-31, use $\triangle CAR$ with $C(-3,3), A(-3,-1)$ and $R(0,-1)$ and $\triangle LOT$ with $L(5,-2), O(5,6)$ and $T(-1,6)$.

28. Find the slopes of CA, AR, LO and OT.
29. What are the measures of $\angle A$ and $\angle O$? Explain.
30. Find LO, OT, CA and AR. Use these values to write the ratios $LO : CA$ and $OT : AR$.
31. Are the triangles similar? Justify your answer.

Review Queue Answer

1. Answers:
   a. $\angle A \cong \angle D, \angle E \cong \angle C$
   b. $\triangle ABE \sim \triangle DBC$
   c. $BE = 10$
2. Answers:
   a. $\frac{6}{8} = \frac{21}{x}, x = 28$
   b. $$\begin{aligned} 15(x+2) &= 6(2x-1) \\ 15x+30 &= 12x-6 \\ 3x &= -36 \\ x &= -12 \end{aligned}$$
   c. $$\begin{aligned} (x-3)(x+2) &= 36 \\ x^2-x-6 &= 36 \\ x^2-x-42 &= 0 \\ (x-7)(x+6) &= 0 \\ x &= 7,-6 \end{aligned}$$