

# 6.4 Rectangles, Rhombuses and Squares

FlexBooks® 2.0 > American HS Geometry > Rectangles, Rhombuses and Squares

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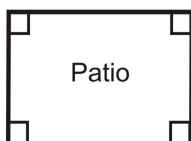
## Learning Objectives

- Define and analyze a rectangle, rhombus, and square.
- Determine if a parallelogram is a rectangle, rhombus, or square in the coordinate plane.
- Analyze the properties of the diagonals of a rectangle, rhombus, and square.

## Review Queue

1. Define rectangle in your own words. Is a rectangle a parallelogram?
2. Define square in your own words. Is a square a parallelogram? Is it a rectangle?
3. List five examples where you might see a square, rectangle, or rhombus in real life.

**Know What?** You are designing a patio for you backyard. You decide to mark it off using your tape measure. Two sides are **21 feet** long and two sides are **28 feet** long. Explain how you would only use the tape measure to make your patio a rectangle.



[Figure 1]

## Defining Special Parallelograms

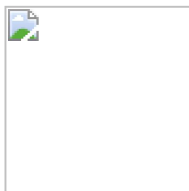
Rectangles, Rhombuses (the plural is also Rhombi) and Squares are all more specific versions of parallelograms, also called special parallelograms. Taking the theorems we learned in the previous two sections, we have three more new theorems.

**Rectangle Theorem:** A quadrilateral is a rectangle if and only if it has four right (congruent) angles.



[Figure 2]

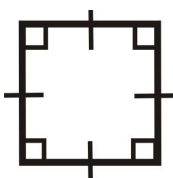
**Rhombus Theorem:** A quadrilateral is a rhombus if and only if it has four congruent sides.



[Figure 3]

**Square Theorem:** A quadrilateral is a square if and only if it has four right angles and four congruent sides.

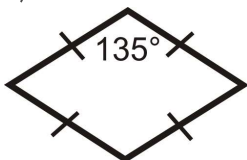
From the Square Theorem, we can also conclude that a ***square is a rectangle and a rhombus.***



[Figure 4]

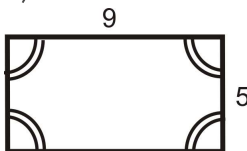
**Example 1:** What type of parallelogram are the ones below?

a)



[Figure 5]

b)



[Figure 6]

**Solution:**

a) All sides are congruent and one angle is  $135^\circ$ , meaning that the angles are not congruent. By the Rhombus Theorem, this is a rhombus.

b) This quadrilateral has four congruent angles and all the sides are not congruent. By the Rectangle Theorem, this is a rectangle.

**Example 2:** Is a rhombus SOMETIMES, ALWAYS, or NEVER a square? Explain your reasoning.

**Solution:** A rhombus has four congruent sides, while a square has four congruent sides and angles. Therefore, a rhombus is only a square when it also has congruent angles. So, a rhombus is SOMETIMES a square.

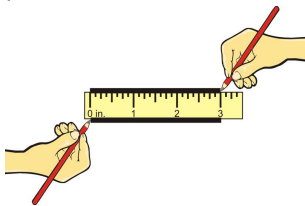
## Diagonals in Special Parallelograms

Recall from previous lessons that the **diagonals in a parallelogram bisect each other**. Therefore, the diagonals of a rectangle, square and rhombus also bisect each other. The diagonals of these parallelograms also have additional properties.

### Investigation 6-3: Drawing a Rectangle

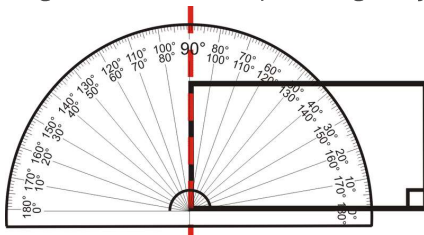
Tools Needed: pencil, paper, protractor, ruler

Like with Investigation 6-2, draw two lines on either side of your ruler, to ensure they are parallel. Make these lines 3 inches long.



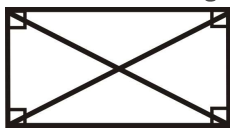
[Figure 7]

Remove the ruler and mark two  $90^\circ$  angles, 2.5 inches apart on the bottom line drawn in Step 1. Then, draw the angles to intersect the top line. This will ensure that all four angles are  $90^\circ$ . Depending on your ruler, the sides should be 2.5 inches and 1 inch.



[Figure 8]

Draw in the diagonals and measure them. What do you discover?



[Figure 9]

**Theorem 6-14:** A parallelogram is a rectangle if and only if the diagonals are congruent.

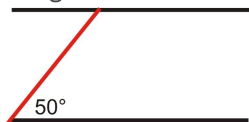
Notice, we did not say *any quadrilateral*. There are quadrilaterals that have congruent diagonals and are not parallelograms.

### Investigation 6-4: Drawing a Rhombus

Tools Needed: pencil, paper, protractor, ruler

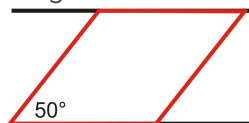
1. Like with Investigation 6-2 and 6-3, draw two lines on either side of your ruler, to ensure they are parallel. Make these lines 3 inches long.

Remove the ruler and mark a  $50^\circ$  angle, at the left end of the bottom line drawn in Step 1. Draw the other side of the angle and make sure it intersects the top line. Measure the length of this side.



[Figure 10]

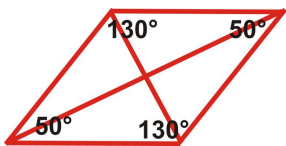
The measure of the diagonal (red) side should be about 1.3 inches (if your ruler is 1 inch wide). Mark this length on the bottom line and the top line from the point of intersection with the  $50^\circ$  angle. Draw in the fourth side. It will connect the two endpoints of these lengths.



[Figure 11]

2. By the way we drew this parallelogram; it is a rhombus because all four sides are 1.3 inches long. Draw in the diagonals.

Measure the angles created by the diagonals: the angles at their point of intersection and the angles created by the sides and each diagonal. You should find the measure of 12 angles total. What do you discover?



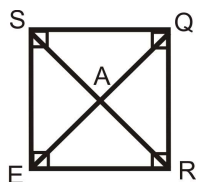
[Figure 12]

**Theorem 6-15:** A parallelogram is a rhombus if and only if the diagonals are perpendicular.

**Theorem 6-16:** A parallelogram is a rhombus if and only if the diagonals bisect each angle.

There are no theorems about the diagonals of a square. We know that a square is a rhombus and a rectangle. So, the diagonals of a square have the properties of a rhombus and a rectangle.

**Example 3:** List everything you know about the square *SQRE*.



[Figure 13]

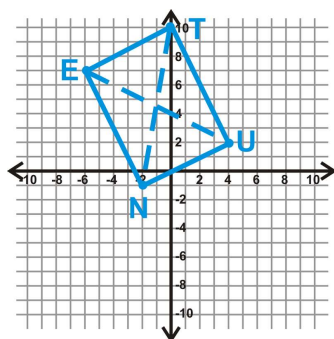
**Solution:** A square has all the properties of a parallelogram, rectangle and rhombus.

Properties of Parallelograms	Properties of Rhombuses	Properties of Rectangles
• $\overline{SQ} \parallel \overline{ER}$	• $SQ \cong ER \cong SE \cong QR$	• $\angle SER \cong \angle SQR \cong \angle QSE \cong \angle QRE$
• $\overline{SE} \parallel \overline{QR}$	• $SR \perp QE$	• $SR \cong QE$
• $SQ \cong ER$	• $\angle SEQ \cong \angle QER \cong \angle SQE \cong \angle EQR$	• $\overline{SA} \cong \overline{AR} \cong \overline{QA} \cong \overline{AE}$
• $SE \cong QR$	• $\angle QSR \cong \angle RSE \cong \angle QRS \cong \angle SRE$	
• $\overline{SA} \cong \overline{AR}$		
• $\overline{QA} \cong \overline{AE}$		
• $\angle SER \cong \angle SQR$		
• $\angle QSE \cong \angle QRE$		

## Parallelograms in the Coordinate Plane

**Example 4:** Determine what type of parallelogram  $TUNE$  is:

$T(0, 10)$ ,  $U(4, 2)$ ,  $N(-2, -1)$ , and  $E(-6, 7)$ .



[Figure 14]

**Solution:** First, let's double-check and make sure the diagonals bisect each other.

$$\text{Midpoint of } EU = \left( \frac{-6 + 4}{2}, \frac{7 + 2}{2} \right) = (-1, 4.5)$$

$$\text{Midpoint of } TN = \left( \frac{0-2}{2}, \frac{10-1}{2} \right) = (-1, 4.5)$$

Now, let's see if the diagonals are equal. If they are, then  $TUNE$  is a rectangle.

$$\begin{aligned} EU &= \sqrt{(-6-4)^2 + (7-2)^2} & TN &= \sqrt{(0+2)^2 + (10+1)^2} \\ &= \sqrt{(-10)^2 + 5^2} & &= \sqrt{2^2 + 11^2} \\ &= \sqrt{100 + 25} & &= \sqrt{4 + 121} \\ &= \sqrt{125} & &= \sqrt{125} \end{aligned}$$

If the diagonals are perpendicular, then  $TUNE$  is a square.

$$\text{Slope of } EU = \frac{7-2}{-6-4} = -\frac{5}{10} = -\frac{1}{2} \quad \text{Slope of } TN = \frac{10+1}{0+2} = \frac{11}{2}$$

The slope of  $EU$  is not the opposite reciprocal of the slope of  $TN$ , so we can conclude that  $TUNE$  is not a square, it is a rectangle.

Here are the steps to determine if a quadrilateral is a parallelogram, rectangle, rhombus, or square.

1. See if the diagonals bisect each other by using the midpoint formula.

**Yes: Parallelogram**, continue to #2. **No: A quadrilateral**, done.

2. Determine if the diagonals are equal by using the distance formula.

**Yes: Rectangle**, skip to #4. **No:** Could be a rhombus, continue to #3.

3. Determine if the sides are congruent by using the distance formula.

**Yes: Rhombus**, done. **No: Parallelogram**, done.

4. See if the diagonals are perpendicular by finding their slopes.

**Yes: Square**, done. **No: Rectangle**, done.

NOTE: This is just one list of steps to take to determine what type of parallelogram a quadrilateral is. There are several other steps that you could take based on the theorems we have learned.

**Know What? Revisited** In order for the patio to be a rectangle, first the opposite sides must be congruent. So, two sides are 21ft and two are 28 ft. To ensure that the parallelogram is a rectangle *without* measuring the angles, the diagonals must be equal. You can find the length of the diagonals by using the Pythagorean Theorem.

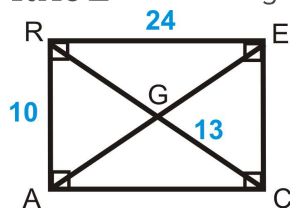
[Figure 15]

$$d^2 = 21^2 + 28^2 = 441 + 784 = 1225$$

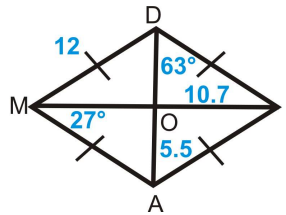
$$d = \sqrt{1225} = 35 \text{ ft}$$

## Review Questions

$RACE$  is a rectangle. Find:  $DIAM$  is a rhombus. Find:



[Figure 16]



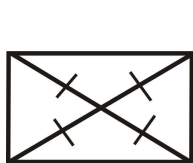
[Figure 17]

- |                  |                  |
|------------------|------------------|
| a. $RG$          | a. $MA$          |
| b. $AE$          | b. $MI$          |
| c. $AC$          | c. $DA$          |
| d. $EC$          | d. $m\angle DIA$ |
| e. $m\angle RAC$ | e. $m\angle MOA$ |

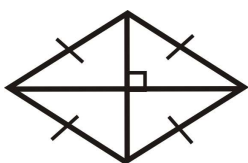
1. Draw a square and label it  $CUBE$ . Mark the point of intersection of the diagonals  $Y$ . Find:

- $m\angle UCE$
- $m\angle EYB$
- $m\angle UBY$
- $m\angle UEB$

For questions 4-12, determine if the quadrilateral is a parallelogram, rectangle, rhombus, square or none. Explain your reasoning.



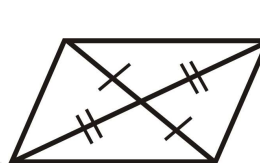
[Figure 18]



[Figure 19]



[Figure 20]



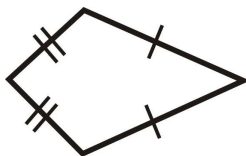
[Figure 21]



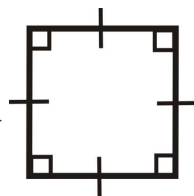
[Figure 22]



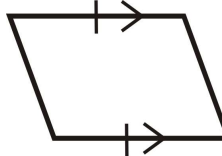
[Figure 23]



[Figure 24]



[Figure 25]



[Figure 26]

For questions 13-18 determine if the following are ALWAYS, SOMETIME, or NEVER true. Explain your reasoning.

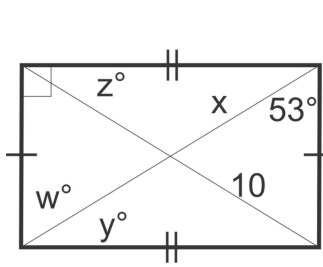
13. A rectangle is a rhombus.
14. A square is a parallelogram.
15. A parallelogram is regular.
16. A square is a rectangle.
17. A rhombus is equiangular.
18. A quadrilateral is a pentagon.

For questions 19-22, determine what type of quadrilateral  $ABCD$  is.

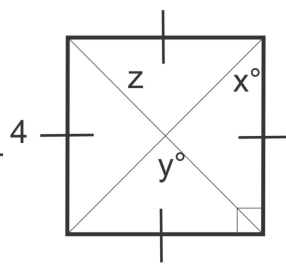
19.  $A(-2, 4)$ ,  $B(-1, 2)$ ,  $C(-3, 1)$ ,  $D(-4, 3)$
20.  $A(-2, 3)$ ,  $B(3, 4)$ ,  $C(2, -1)$ ,  $D(-3, -2)$
21.  $A(1, -1)$ ,  $B(7, 1)$ ,  $C(8, -2)$ ,  $D(2, -4)$
22.  $A(10, 4)$ ,  $B(8, -2)$ ,  $C(2, 2)$ ,  $D(4, 8)$

23. **Writing** From 19-22, you used the coordinate plane to determine what type of quadrilateral  $ABCD$  is. Other than the method described in this section, describe another way to determine if  $ABCD$  is a square.

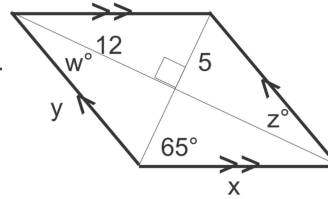
For problems 24-26, find the value of each variable in the figures.



[Figure 27]

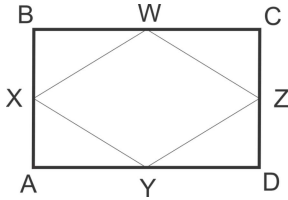


[Figure 28]



[Figure 29]

Given:  $ABCD$  is a rectangle  $W$ ,  $X$ ,  $Y$  and  $Z$  are midpoints of  $BC$ ,  $AB$ ,  $AD$  and  $CD$  respectively Prove: quadrilateral  $WXYZ$  is a rhombus



[Figure 30]

Statements	Reasons
7. $ABCD$ is a rectangle	1. Given
8. $\overline{BW} \cong \overline{WC}$ , _____, _____, _____	2. Definition of a midpoint
9. $BD = AC$	3.
10. $XY$ is a midsegment in $\triangle ABD$ _____, _____, _____	4. Definition of a midsegment in a triangle
11. $XY = \frac{1}{2}BD = WZ$ and _____	5. Midsegment in a triangle is half the length of the parallel side.
12. $\frac{1}{2}BD = \frac{1}{2}AC$	6.
13. $XY = WZ = YZ = XW$	7.
14. $WXYZ$ is a rhombus	8.

28. Explain why a quadrilateral formed by the segments connecting the midpoints of the sides in a rhombus is always a rectangle.
29. Explain why a quadrilateral formed by the segments connecting the midpoints of the sides in a square is always a square.
30. Construct a rhombus with diagonals of lengths 2 inches and 1.5 inches.
31. Construct a rectangle with diagonal length inches.

## Review Queue Answers

1. A rectangle has all equal angles and opposite sides are congruent. It is a parallelogram.

2. A square has equal angles and sides. It is a parallelogram and a rectangle (and a rhombus).
3. Possibilities: picture frame, door, baseball diamond, windows, walls, floor tiles, book cover, pages/paper, table/desk top, black/white board, the diamond suit (in a deck of cards).