12.3 Reflections

Last Modified: Dec 25, 2014

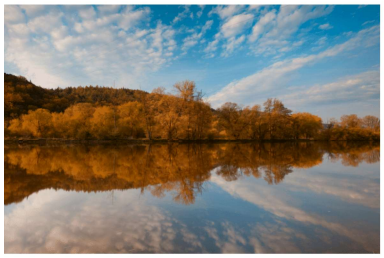
Learning Objectives

- Reflect a figure over a given line.
- Determine the rules of reflections in the coordinate plane.

Review Queue

1. Define reflection in your own words.
2. Plot $A(-3,2)$. Translate A such that $(x,y) \rightarrow (x+6,y)$.
3. What line is halfway between A and A'?
4. Translate A such that $(x,y) \rightarrow (x,y-4)$. Call this point A''.
5. What line is halfway between A and A''?

Know What? A lake can act like a mirror in nature. Describe the line of reflection in the photo below. If this image were on the coordinate plane, what could the equation of the line of reflection be? (There could be more than one correct answer, depending on where you place the origin.)



[Figure 1]

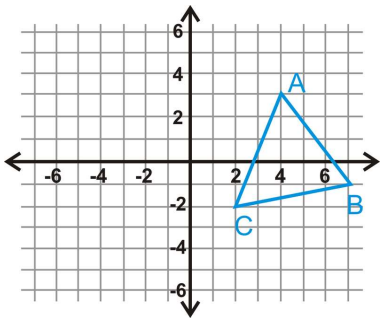
Reflections over an Axis

The next transformation is a reflection. Another way to describe a reflection is a "flip."

Reflection: A transformation that turns a figure into its mirror image by flipping it over a line.

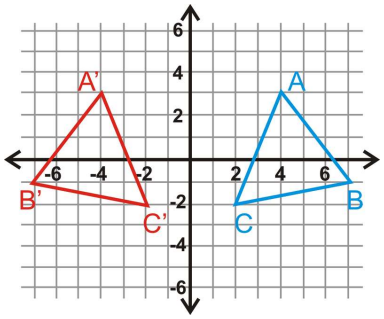
Line of Reflection: The line that a figure is reflected over.

Example 1: Reflect $\triangle ABC$ over the $y-$ axis. Find the coordinates of the image.



[Figure 2]

Solution: To reflect $\triangle ABC$ over the $y-$ axis the $y-$ coordinates will remain the same. The $x-$ coordinates will be the same distance away from the $y-$ axis, but on the other side of the $y-$ axis.

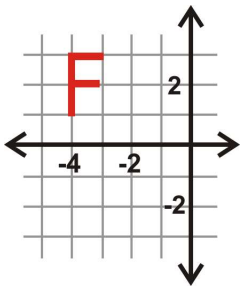


[Figure 3]

$$A(4,3) \rightarrow A'(-4,3)$$
$$B(7,-1) \rightarrow B'(-7,-1)$$
$$C(2,-2) \rightarrow C'(-2,-2)$$

From this example, we can generalize a rule for reflecting a figure over the $y-$ axis.

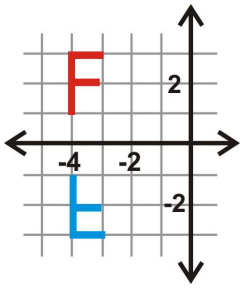
Reflection over the $y-$ axis: If (x,y) is reflected over the $y-$ axis, then the image is $(-x,y)$.



[Figure 4]

Example 2: Reflect the letter "F" over the $x-$ axis.

Solution: To reflect the letter F over the $x-$ axis, now the $x-$ coordinates will remain the same and the $y-$ coordinates will be the same distance away from the $x-$ axis on the other side.



[Figure 5]

The generalized rule for reflecting a figure over the $x-$ axis:

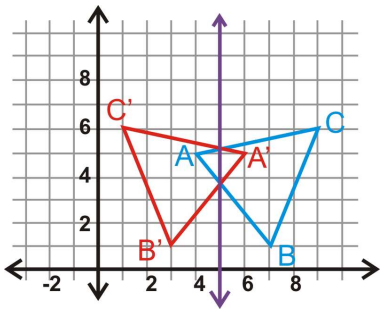
Reflection over the $x-$ axis: If (x, y) is reflected over the $x-$ axis, then the image is $(x, -y)$.

Reflections over Horizontal and Vertical Lines

Other than the x and y axes, we can reflect a figure over any vertical or horizontal line.

Example 3: Reflect the triangle $\triangle ABC$ with vertices $A(4,5), B(7,1)$ and $C(9,6)$ over the line $x = 5$.

Solution: Notice that this vertical line is through our preimage. Therefore, the image's vertices are the same distance away from $x = 5$ as the preimage. As with reflecting over the $y-$ axis (or $x = 0$), the $y-$ coordinates will stay the same.



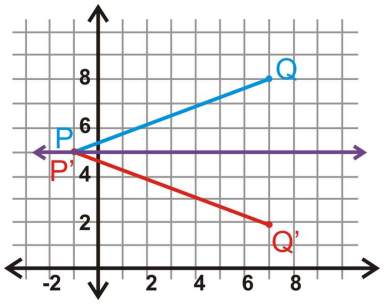
[Figure 6]

$$A(4,5) \rightarrow A'(6,5)$$
$$B(7,1) \rightarrow B'(3,1)$$
$$C(9,6) \rightarrow C'(1,6)$$

Example 4: Reflect the line segment PQ with endpoints $P(-1,5)$ and $Q(7,8)$ over the line $y = 5$.

Solution: Here, the line of reflection is on P, which means P' has the same coordinates. Q' has the same $x-$ coordinate as Q and is the same distance away from $y = 5$, but on the other side.

$$P(-1,5) \rightarrow P'(-1,5)$$
$$Q(7,8) \rightarrow Q'(7,2)$$



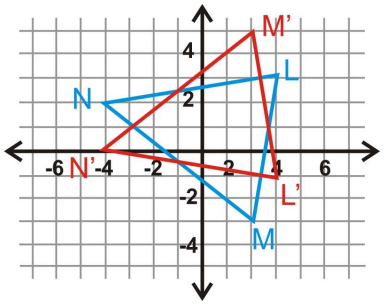
[Figure 7]

Reflection over $x = a$ **:** If (x,y) is reflected over the vertical line $x = a$, then the image is $(2a - x, y)$.

Reflection over $y = b$ **:** If (x,y) is reflected over the horizontal line $y = b$, then the image is $(x, 2b - y)$.

From these examples we also learned that if a point is on the line of reflection then the image is the same as the original point.

Example 5: A triangle $\triangle LMN$ and its reflection, $\triangle L'M'N'$ are to the left. What is the line of reflection?



[Figure 8]

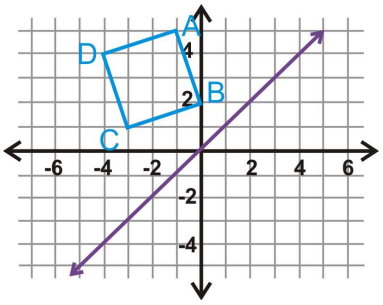
Solution: Looking at the graph, we see that the preimage and image intersect when $y = 1$. Therefore, this is the line of reflection.

If the image does not intersect the preimage, find the midpoint between a preimage and its image. This point is on the line of reflection. You will need to determine if the line is vertical or horizontal.

Reflections over $y = x$ and $y = -x$

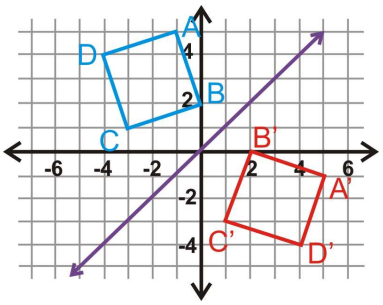
Technically, any line can be a line of reflection. We are going to study two more cases of reflections, reflecting over $y = x$ and over $y = -x$.

Example 6: Reflect square $ABCD$ over the line $y = x$.



[Figure 9]

Solution: The purple line is $y = x$. To reflect an image over a line that is not vertical or horizontal, you can fold the graph on the line of reflection.



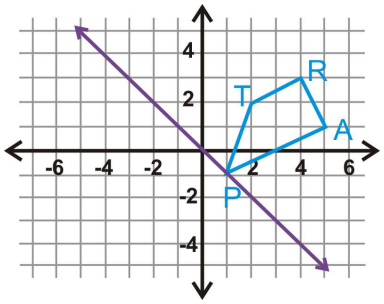
[Figure 10]

$$A(-1,5) \rightarrow A'(5,-1)$$
$$B(0,2) \rightarrow B'(2,0)$$
$$C(-3,1) \rightarrow C'(1,-3)$$
$$D(-4,4) \rightarrow D'(4,-4)$$

From this example, we see that the x and y values are switched when a figure is reflected over the line $y = x$.

Reflection over $y = x$: If (x, y) is reflected over the line $y = x$, then the image is (y, x).

Example 7: Reflect the trapezoid *TRAP* over the line $y = -x$.

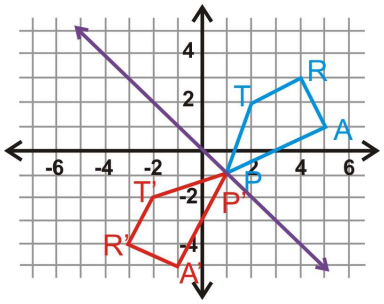


[Figure 11]

Solution: The purple line is $y = -x$. You can reflect the trapezoid over this line just like we did in Example 6.

$$T(2,2) \rightarrow T'(-2,-2)$$
$$R(4,3) \rightarrow R'(-3,-4)$$
$$A(5,1) \rightarrow A'(-1,-5)$$
$$P(1,-1) \rightarrow P'(1,-1)$$

From this example, we see that the x and y values are switched and the signs are changed when a figure is reflected over the line $y = x$.



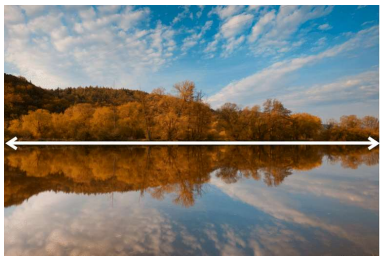
[Figure 12]

Reflection over $y = -x$: If (x, y) is reflected over the line $y = -x$, then the image is $(-y, -x)$.

At first glance, it does not look like P and P' follow the rule above. However, when you switch 1 and -1 you would have (-1, 1). Then, take the opposite sign of both, (1, -1). Therefore, when a point is on the line of reflection, it will be its own reflection.

From all of these examples, we notice that ***a reflection is an isometry***.

Know What? Revisited The white line in the picture is the line of reflection. This line coincides with the water's edge. If we were to place this picture on the coordinate plane, the line of reflection would be any horizontal line. One example could be the $x-$ axis.



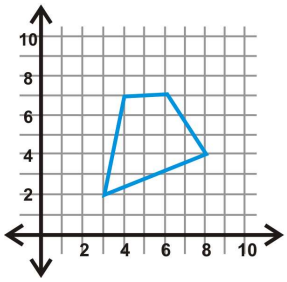
[Figure 13]

Review Questions

1. Which letter is a reflection over a vertical line of the letter "b"?
2. Which letter is a reflection over a horizontal line of the letter "b"?

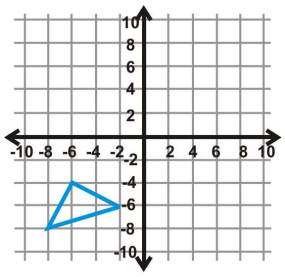
Reflect each shape over the given line.

$y-$ axis



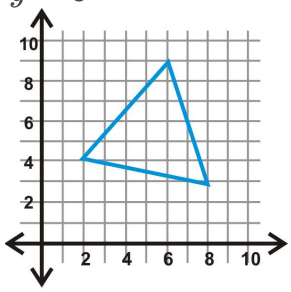
[Figure 14]

$x-$ axis



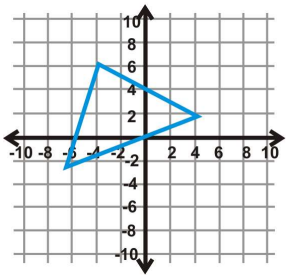
[Figure 15]

$y = 3$



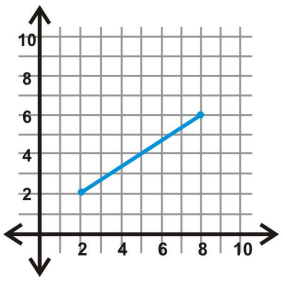
[Figure 16]

$x = -1$



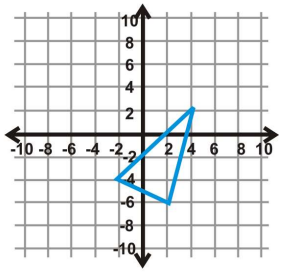
[Figure 17]

$x-$ axis



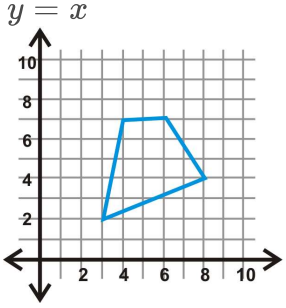
[Figure 18]

$y-$ axis



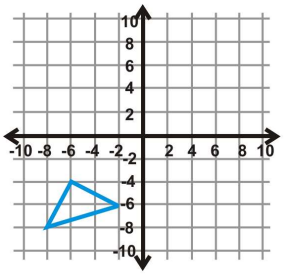
[Figure 19]

$y = x$



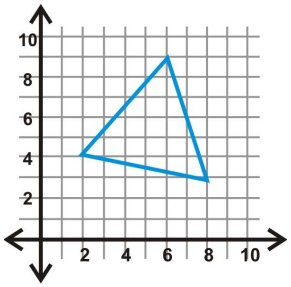
[Figure 20]

$y = -x$



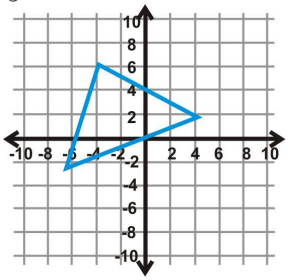
[Figure 21]

$x = 2$



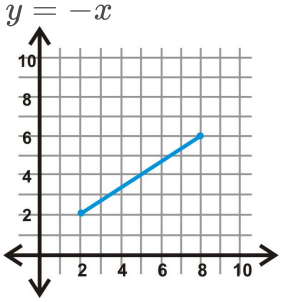
[Figure 22]

$y = -4$



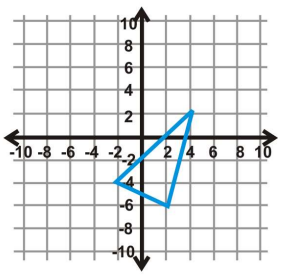
[Figure 23]

$y = -x$



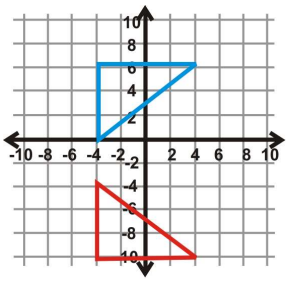
[Figure 24]

$y = x$

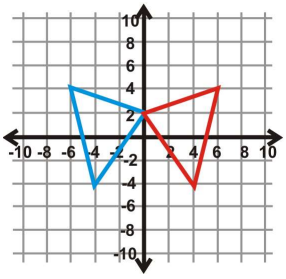


[Figure 25]

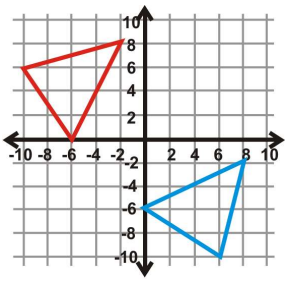
Find the line of reflection of the blue triangle (preimage) and the red triangle (image).



[Figure 26]



[Figure 27]



[Figure 28]

Two Reflections The vertices of $\triangle ABC$ are $A(-5,1), B(-3,6)$, and $C(2,3)$. Use this information to answer questions 18-21.

18. Plot $\triangle ABC$ on the coordinate plane.
19. Reflect $\triangle ABC$ over $y = 1$. Find the coordinates of $\triangle A'B'C'$.
20. Reflect $\triangle A'B'C'$ over $y = -3$. Find the coordinates of $\triangle A''B''C''$.
21. What ***one*** transformation would be the same as this double reflection?

Two Reflections The vertices of $\triangle DEF$ are $D(6,-2), E(8,-4)$, and $F(3,-7)$. Use this information to answer questions 22-25.

22. Plot $\triangle DEF$ on the coordinate plane.
23. Reflect $\triangle DEF$ over $x = 2$. Find the coordinates of $\triangle D'E'F'$.
24. Reflect $\triangle D'E'F'$ over $x = -4$. Find the coordinates of $\triangle D''E''F''$.
25. What ***one*** transformation would be the same as this double reflection?

Two Reflections The vertices of $\triangle GHI$ are $G(1,1), H(5,1)$, and $I(5,4)$. Use this information to answer questions 26-29.

26. Plot $\triangle GHI$ on the coordinate plane.
27. Reflect $\triangle GHI$ over the $x-$ axis. Find the coordinates of $\triangle G'H'I'$.
28. Reflect $\triangle G'H'I'$ over the $y-$ axis. Find the coordinates of $\triangle G''H''I''$.
29. What ***one*** transformation would be the same as this double reflection?
30. Following the steps to reflect a triangle using a compass and straightedge.

    a. Make a triangle on a piece of paper. Label the vertices A, B and C .

    b. Make a line next to your triangle (this will be your line of reflection).

c. Construct perpendiculars from each vertex of your triangle through the line of reflection.

d. Use your compass to mark off points on the other side of the line that are the same distance from the line as the original A, B and C. Label the points A', B' and C'.

e. Connect the new points to make the image $\triangle A'B'C'$.

31. Describe the relationship between the line of reflection and the segments connecting the preimage and image points.

32. Repeat the steps from problem 28 with a line of reflection that passes ***through*** the triangle.

Review Queue Answers

1. *Examples are:* To flip an image over a line; A mirror image.
2. $A'(3, 2)$
3. the $y-$ axis
4. $A''(-3, -2)$
5. the $x-$ axis