12.4 Rotations

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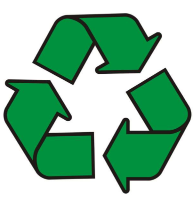
Learning Objectives

- Find the image of a figure in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.

Review Queue

1. Reflect $\triangle XYZ$ with vertices $X(9,2), Y(2,4)$ and $Z(7,8)$ over the $y-$ axis. What are the vertices of $\triangle X'Y'Z'$?
2. Reflect $\triangle X'Y'Z'$ over the $x-$ axis. What are the vertices of $\triangle X''Y''Z''$?
3. How do the coordinates of $\triangle X''Y''Z''$ relate to $\triangle XYZ$?

Know What? The international symbol for recycling appears below. It is three arrows rotated around a point. Let's assume that the arrow on the top is the preimage and the other two are its images. Find the center of rotation and the angle of rotation for each image.



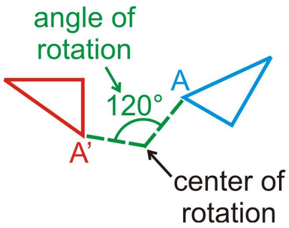
[Figure 1]

Defining Rotations

Rotation: A transformation by which a figure is turned around a fixed point to create an image.

Center of Rotation: The fixed point that a figure is rotated around.

Lines can be drawn from the preimage to the center of rotation, and from the center of rotation to the image. The angle formed by these lines is the angle of ***rotation.***



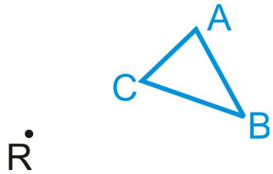
[Figure 2]

In this section, our center of rotation will always be the ***origin.*** Rotations can also be clockwise or counterclockwise. We will only do ***counterclockwise*** rotations, to go along with the way the quadrants are numbered.

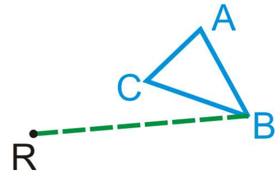
Investigation 12-1: Drawing a Rotation of 100°

Tools Needed: pencil, paper, protractor, ruler

Draw $\triangle ABC$ and a point R outside the circle. Draw the line segment RB.

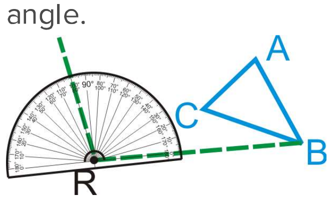


[Figure 3]



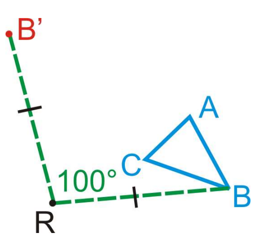
[Figure 4]

Take your protractor, place the center on R and the initial side on RB. Mark a 100° angle.

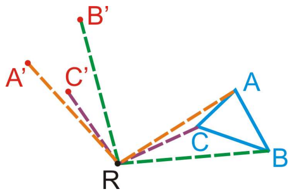


[Figure 5]

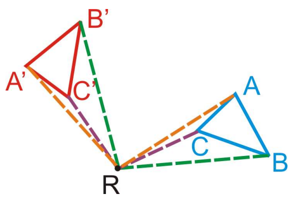
1. Find B' such that $RB = RB'$.
2. Repeat steps 2-4 with points A and C.
3. Connect $A', B',$ and C' to form $\triangle A'B'C'$.



[Figure 6]



[Figure 7]



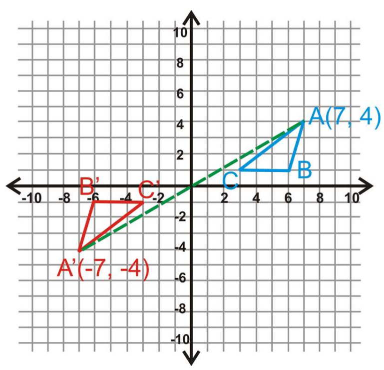
[Figure 8]

This is the process you would follow to rotate any figure 100° counterclockwise. If it was a different angle measure, then in Step 3, you would mark a different angle. You will need to repeat steps 2-4 for every vertex of the shape.

180° Rotation

To rotate a figure 180° in the coordinate plane, we use the origin as the center of the rotation. Recall, that a 180° angle is the same as a straight line. So, a rotation of a point over the origin of 180° will be on the same line and the same distance away from the origin.

Example 1: Rotate $\triangle ABC$, with vertices $A(7,4), B(6,1),$ and $C(3,1)$ 180°. Find the coordinates of $\triangle A'B'C'$.



[Figure 9]

Solution: You can either use Investigation 12-1 or the hint given above to find $\triangle A'B'C'$. It is very helpful to graph the triangle. Using the hint, if A is $(7,4)$, that means it is 7 units to the right of the origin and 4 units up. A' would then be 7 units to the ***left*** of the origin and 4 units ***down.*** The vertices are:

$$A(7,4) \rightarrow A'(-7,-4)$$
$$B(6,1) \rightarrow B'(-6,-1)$$
$$C(3,1) \rightarrow C'(-3,-1)$$

The image has vertices that are the negative of the preimage. This will happen every time a figure is rotated 180° .

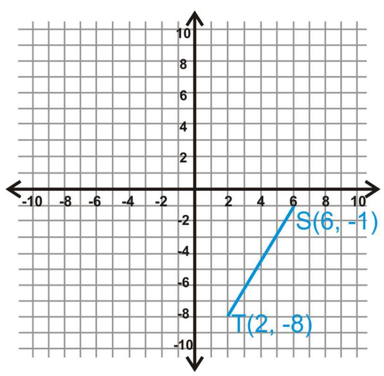
Rotation of 180° **:** If (x,y) is rotated 180° around the origin, then the image will be $(-x,-y)$.

From this example, we can also see that ***a rotation is an isometry.*** This means that $\triangle ABC \cong \triangle A'B'C'$. You can use the distance formula to verify that our assertion holds true.

90° Rotation

Similar to the 180° rotation, a 90° rotation (counterclockwise) is an isometry. Each image will be the same distance away from the origin as its preimage, but rotated 90° .

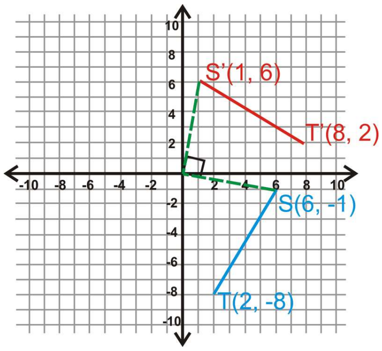
Example 2: Rotate ST 90° .



[Figure 10]

Solution: When we rotate something 90° , you can use Investigation 12-1. Draw lines from the origin to S and T . The line from each point to the origin is going to be ***perpendicular*** to the line from the origin to its image. Therefore, if S is 6 units to the ***right*** of the origin and 1 unit ***down,*** S' will be 6 units ***up*** and 1 to the ***right.***

Using this pattern, T' is (8, 2).



[Figure 11]

If you were to write the slope of each point to the origin, S would be $\frac{-1}{6} \rightarrow \frac{y}{x}$, and S' must be $\frac{6}{1} \rightarrow \frac{y'}{x'}$. Again, they are perpendicular slopes, following along with the 90° rotation. Therefore, the x and the y values switch and the new $x-$ value is the opposite sign of the original $y-$ value.

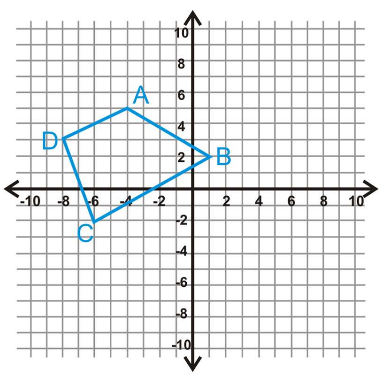
Rotation of 90° **:** If (x, y) is rotated 90° around the origin, then the image will be $(-y, x)$.

Rotation of 270°

A rotation of 270° counterclockwise would be the same as a clockwise rotation of 90° . We also know that a 90° rotation and a 270° rotation are 180° apart. We know that for every 180° rotation, the x and y values are negated. So, if the values of a 90° rotation are $(-y, x)$, then a 270° rotation would be the opposite sign of each, or $(y, -x)$.

Rotation of 270° **:** If (x, y) is rotated 270° around the origin, then the image will be $(y, -x)$.

Example 3: Find the coordinates of $ABCD$ after a 270° rotation.



[Figure 12]

Solution: Using the rule, we have:

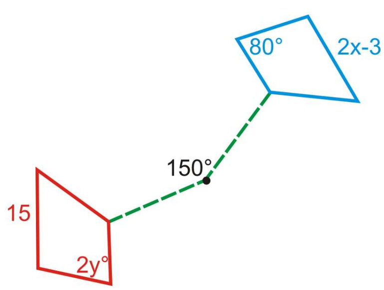
$$\begin{aligned}(x,y) &\rightarrow (y,-x)\\ A(-4,5) &\rightarrow A'(5,4)\\ B(1,2) &\rightarrow B'(2,-1)\\ C(-6,-2) &\rightarrow C'(-2,6)\\ D(-8,3) &\rightarrow D'(3,8)\end{aligned}$$

While we can rotate any image any amount of degrees, only $90^\circ, 180^\circ$ and 270° have special rules. To rotate a figure by an angle measure other than these three, you must use Investigation 12-1.

Example 4: ***Algebra Connection*** The rotation of a quadrilateral is shown below. What is the measure of x and y?

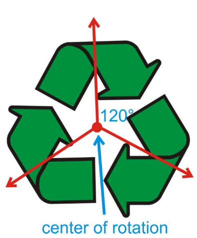
Solution: Because a rotation is an isometry, we can set up two equations to solve for x and y.

$$\begin{aligned}2y &= 80^\circ & 2x-3 &= 15\\ y &= 40^\circ & 2x &= 18\\ & & x &= 9\end{aligned}$$



[Figure 13]

Know What? Revisited The center of rotation is shown in the picture below. If we draw rays to the same point in each arrow, we see that the two images are a 120° rotation in either direction.



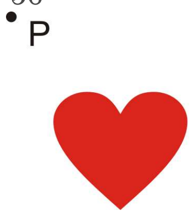
[Figure 14]

Review Questions

In the questions below, every rotation is *counterclockwise,* unless otherwise stated.

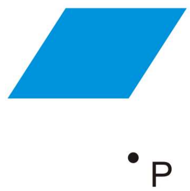
Using Investigation 12-1, rotate each figure around point P the given angle measure.

50°



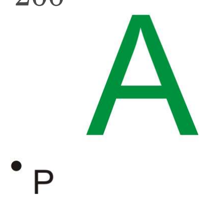
[Figure 15]

120°



[Figure 16]

200°

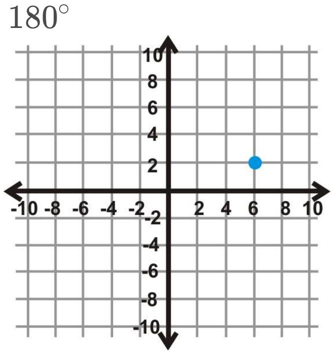


[Figure 17]

1. If you rotated the letter p 180° counterclockwise, what letter would you have?
2. If you rotated the letter p 180° *clockwise,* what letter would you have? Why do you think that is?
3. A 90° clockwise rotation is the same as what counterclockwise rotation?
4. A 270° clockwise rotation is the same as what counterclockwise rotation?
5. Rotating a figure 360° is the same as what other rotation?

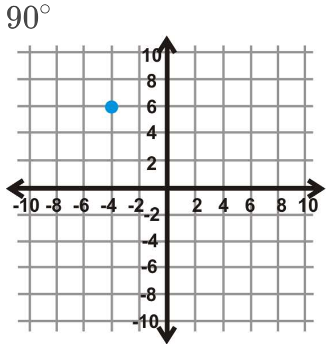
Rotate each figure in the coordinate plane the given angle measure. The center of rotation is the origin.

180°



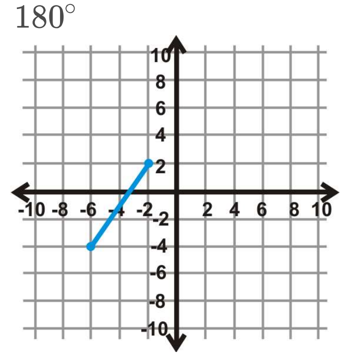
[Figure 18]

90°



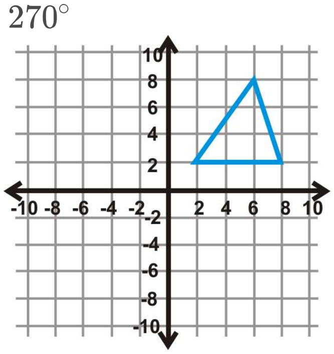
[Figure 19]

180°



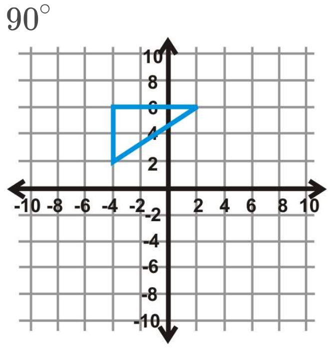
[Figure 20]

270°



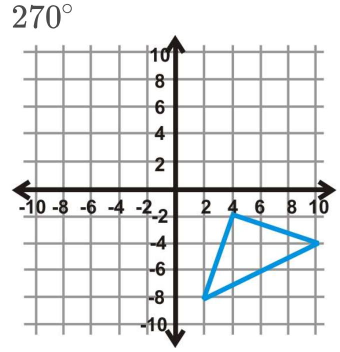
[Figure 21]

90°

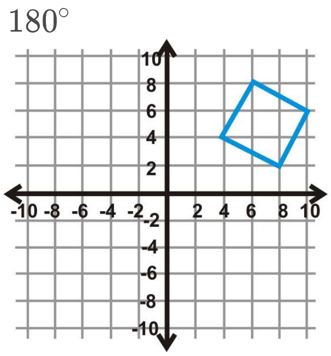


[Figure 22]

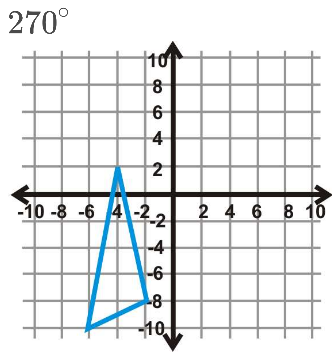
270°



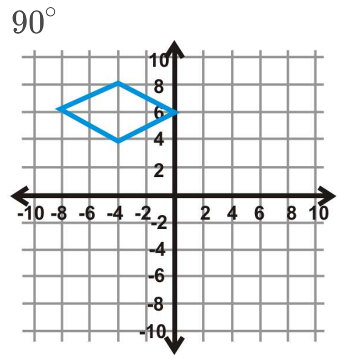
[Figure 23]



[Figure 24]

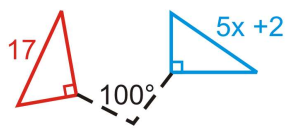


[Figure 25]

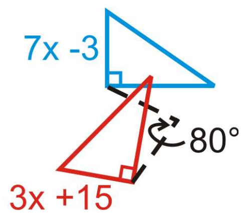


[Figure 26]

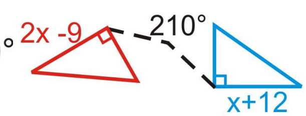
Algebra Connection Find the measure of x in the rotations below. The blue figure is the preimage.



[Figure 27]

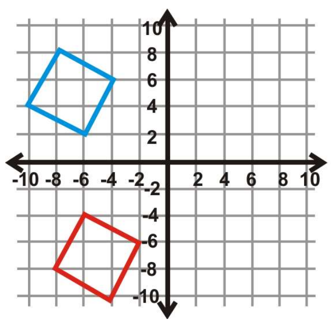


[Figure 28]

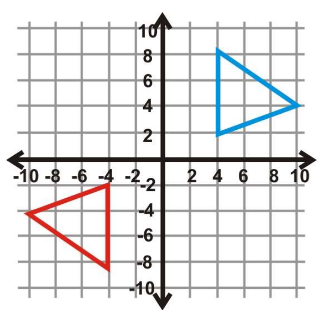


[Figure 29]

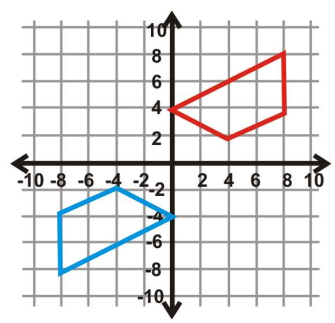
Find the angle of rotation for the graphs below. The center of rotation is the origin and the blue figure is the preimage.



[Figure 30]



[Figure 31]



[Figure 32]

Two Reflections The vertices of $\triangle GHI$ are $G(-2,2), H(8,2)$ and $I(6,8)$. Use this information to answer questions 24-27.

24. Plot $\triangle GHI$ on the coordinate plane.

25. Reflect $\triangle GHI$ over the $x-$ axis. Find the coordinates of $\triangle G'H'I'$.

26. Reflect $\triangle G'H'I'$ over the $y-$ axis. Find the coordinates of $\triangle G''H''I''$.

27. What ***one*** transformation would be the same as this double reflection?

Multistep Construction Problem

28. Draw two lines that intersect, m and n, and $\triangle ABC$. Reflect $\triangle ABC$ over line m to make $\triangle A'B'C'$. Reflect $\triangle A'B'C'$ over line n to get $\triangle A''B''C''$. Make sure $\triangle ABC$ does not intersect either line.
29. Draw segments from the intersection point of lines m and n to A and A''. Measure the angle between these segments. This is the angle of rotation between $\triangle ABC$ and $\triangle A''B''C''$.
30. Measure the angle between lines m and n. Make sure it is the angle which contains $\triangle A'B'C'$ in the interior of the angle.
31. What is the relationship between the angle of rotation and the angle between the two lines of reflection?

Review Queue Answers

1. $X'(-9,2), Y'(-2,4), Z'(-7,8)$
2. $X''(-9,-2), Y''(-2,-4), Z''(-7,-8)$
3. $\triangle X''Y''Z''$ is the double negative of $\triangle XYZ; (x,y) \rightarrow (-x,-y)$