7.2 Similar Polygons

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Learning Objectives

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a similarity statement.
- Calculate and apply scale factors.

Review Queue

1. Solve the proportions.

   a. $\frac{6}{x}=\frac{10}{15}$

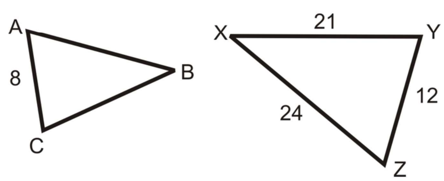
   b. $\frac{4}{7}=\frac{2x+1}{42}$

   c. $\frac{5}{8}=\frac{x-2}{2x}$

2. In the picture, $\frac{AB}{XZ}=\frac{BC}{XY}=\frac{AC}{YZ}$.

   a. Find AB .

   b. Find BC .



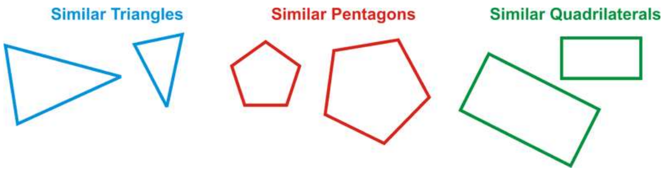
[Figure 1]

Know What? A baseball diamond is a square with 90 foot sides. A softball diamond is a square with 60 foot sides. Are the two diamonds similar? If so, what is the scale factor? Explain your answer.

Similar Polygons

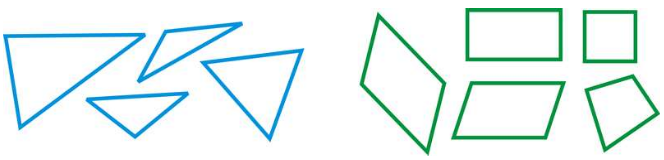
Similar Polygons: Two polygons with the same shape, but not the same size.

Think about similar polygons as an enlargement or shrinking of the same shape. So, more specifically, similar polygons have to have the same number of sides, the corresponding angles are congruent, and the corresponding sides are proportional. The symbol $\sim$ is used to represent similar. Here are some examples:



[Figure 2]

These polygons are not similar:



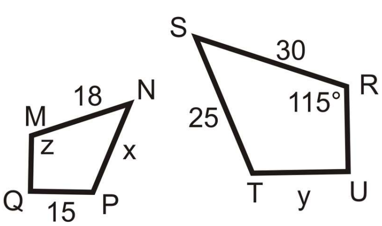
[Figure 3]

Example 1: Suppose $\triangle ABC \sim \triangle JKL$. Based on the similarity statement, which angles are congruent and which sides are proportional?

Solution: Just like a congruence statement, the congruent angles line up within the statement. So, $\angle A \cong \angle J, \angle B \cong \angle K,$ and $\angle C \cong \angle L$. The same is true of the proportional sides. We write the sides in a proportion, $\frac{AB}{JK} = \frac{BC}{KL} = \frac{AC}{JL}$.

Because of the corollaries we learned in the last section, the proportions in Example 1 could be written several different ways. For example, $\frac{AB}{BC} = \frac{JK}{KL}$. Make sure to line up the corresponding proportional sides.

Example 2: $MNPQ \sim RSTU$. What are the values of x, y and z?



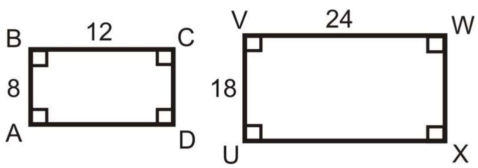
[Figure 4]

Solution: In the similarity statement, $\angle M \cong \angle R$, so $z = 115^\circ$. For x and y, set up a proportion.

$$\frac{18}{30}=\frac{x}{25} \qquad \frac{18}{30}=\frac{15}{y}$$
$$450=30x \qquad 450=18y$$
$$x=15 \qquad y=25$$

Specific types of triangles, quadrilaterals, and polygons will always be similar. For example, because all the angles and sides are congruent, ***all equilateral triangles are similar.*** For the same reason, ***all squares are similar.*** We can take this one step further and say that all regular polygons (with the same number of sides) are similar.

Example 3: $ABCD$ is a rectangle with length 12 and width 8. $UVWX$ is a rectangle with length 24 and width 18. Are these two rectangles similar?



[Figure 5]

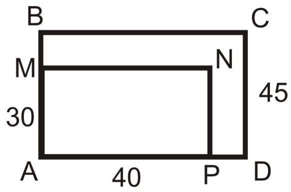
Solution: Draw a picture. First, all the corresponding angles need to be congruent. In rectangles, all the angles are congruent, so this condition is satisfied. Now, let's see if the sides are proportional. $\frac{8}{12}=\frac{2}{3},\frac{18}{24}=\frac{3}{4}$. $\frac{2}{3}\neq\frac{3}{4}$. This tells us that the sides are not in the same proportion, so the rectangles are not similar. We could have also set up the proportion as $\frac{12}{24}=\frac{1}{2}$ and $\frac{8}{18}=\frac{4}{9}$. $\frac{1}{2}\neq\frac{4}{9}$, so you would end up with the same conclusion.

Scale Factors

If two polygons are similar, we know the lengths of corresponding sides are proportional. If k is the length of a side in one polygon, and m is the length of the corresponding side in the other polygon, then the ratio $\frac{k}{m}$ is the ***scale factor*** relating the first polygon to the second.

Scale Factor: In similar polygons, the ratio of one side of a polygon to the corresponding side of the other.

Example 5: $ABCD \sim AMNP$. Find the scale factor and the length of BC .



[Figure 6]

Solution: Line up the corresponding proportional sides. $AB : AM$, so the scale factor is $\frac{30}{45} = \frac{2}{3}$ or $\frac{3}{2}$. Because BC is in the bigger rectangle, we will multiply 40 by $\frac{3}{2}$ because it is greater than 1. $BC = \frac{3}{2}(40) = 60$.

Example 6: Find the perimeters of $ABCD$ and $AMNP$. Then find the ratio of the perimeters.

Solution: Perimeter of $ABCD = 60 + 45 + 60 + 45 = 210$

Perimeter of $AMNP = 40 + 30 + 40 + 30 = 140$

The ratio of the perimeters is 140:210, which reduces to 2:3.

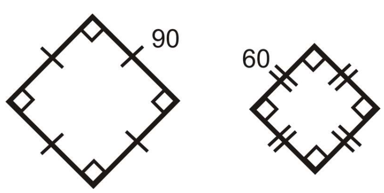
Theorem 7-2: The ratio of the perimeters of two similar polygons is the same as the ratio of the sides.

In addition the perimeter being in the same ratio as the sides, all parts of a polygon are in the same ratio as the sides. This includes diagonals, medians, midsegments, altitudes, and others.

Example 7: $\triangle ABC \sim \triangle MNP$. The perimeter of $\triangle ABC$ is 150 and $AB = 32$ and $MN = 48$. Find the perimeter of $\triangle MNP$.

Solution: From the similarity statement, AB and MN are corresponding sides. So, the scale factor is $\frac{32}{48} = \frac{2}{3}$ or $\frac{3}{2}$. The perimeter of $\triangle MNP$ is $\frac{2}{3}(150) = 100$.

Know What? Revisited All of the sides in the baseball diamond are 90 feet long and 60 feet long in the softball diamond. This means all the sides are in a $\frac{90}{60} = \frac{3}{2}$ ratio. All the angles in a square are congruent, all the angles in both diamonds are congruent. The two squares are similar and the scale factor is $\frac{3}{2}$.



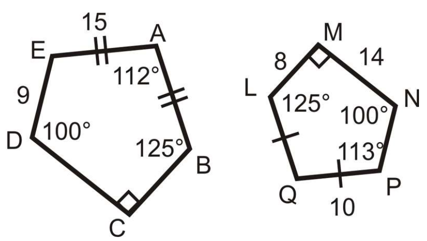
[Figure 7]

Review Questions

Determine if the following statements are true or false.

1. All equilateral triangles are similar.
2. All isosceles triangles are similar.
3. All rectangles are similar.
4. All rhombuses are similar.
5. All squares are similar.
6. All congruent polygons are similar.
7. All similar polygons are congruent.
8. All regular pentagons are similar.
9. $\triangle BIG \sim \triangle HAT$. List the congruent angles and proportions for the sides.
10. If $BI = 9$ and $HA = 15$, find the scale factor.
11. If $BG = 21$, find HT .
12. If $AT = 45$, find IG .
13. Find the perimeter of $\triangle BIG$ and $\triangle HAT$. What is the ratio of the perimeters?

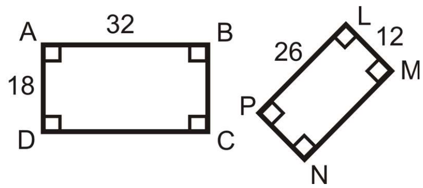
Use the picture to the right to answer questions 14-18.



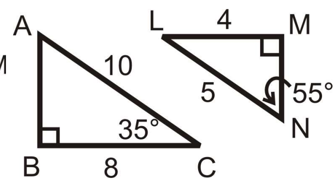
[Figure 8]

14. Find $m\angle E$ and $m\angle Q$.
15. $ABCDE \sim QLMNP$, find the scale factor.
16. Find BC .
17. Find CD .
18. Find NP .

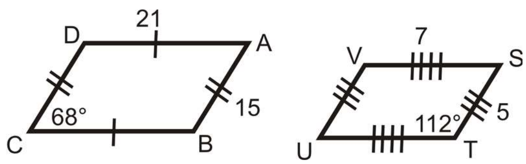
Determine if the following triangles and quadrilaterals are similar. If they are, write the similarity statement.



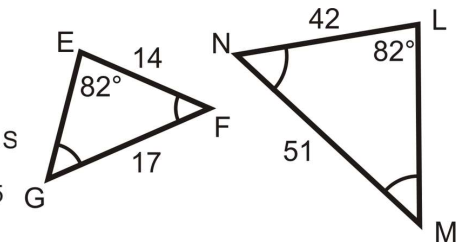
[Figure 9]



[Figure 10]

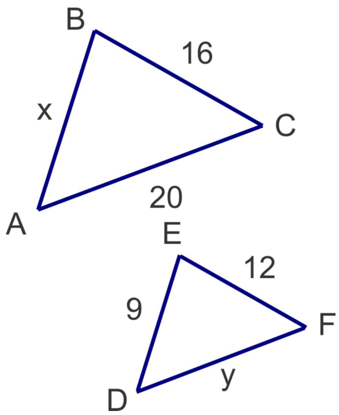


[Figure 11]



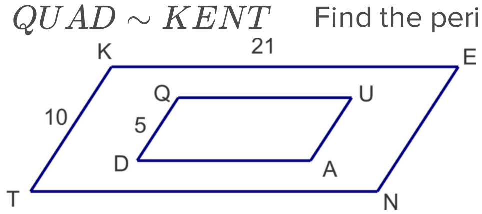
[Figure 12]

$\triangle ABC \sim \triangle DEF$ Solve for x and y.



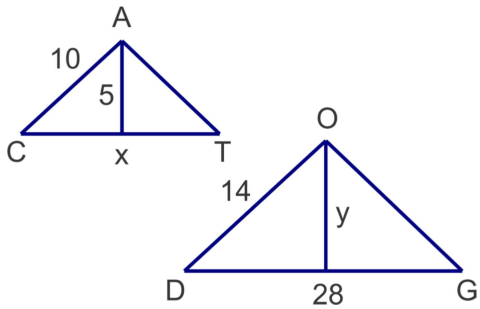
[Figure 13]

$QUAD \sim KENT$ Find the perimeter of $QUAD$.



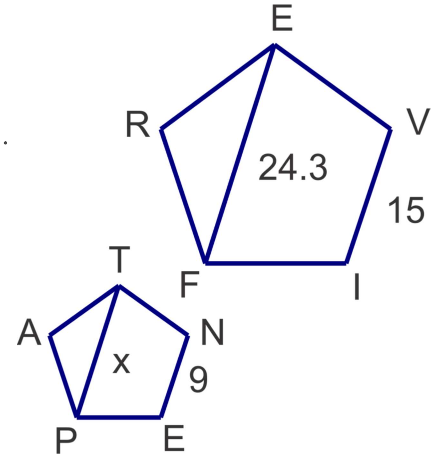
[Figure 14]

$\triangle CAT \sim \triangle DOG$ Solve for x and y.



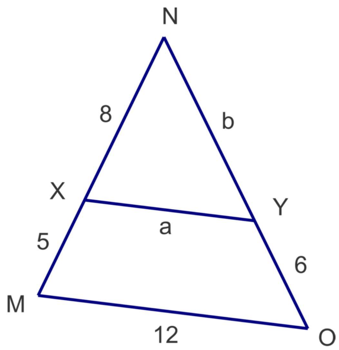
[Figure 15]

$PENTA \sim FIVER$ Solve for x.



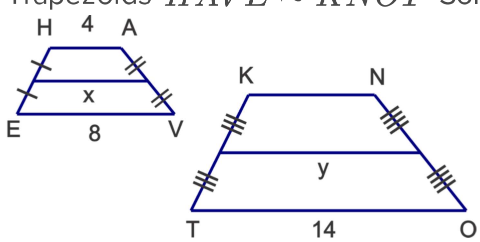
[Figure 16]

$\triangle MNO \sim \triangle XNY$ Solve for a and b.



[Figure 17]

Trapezoids $HAVE \sim KNOT$ Solve for x and y.



[Figure 18]

19. Two similar octagons have a scale factor of $\frac{9}{11}$. If the perimeter of the smaller octagon is 99 meters, what is the perimeter of the larger octagon?
20. Two right triangles are similar. The legs of one of the triangles are 5 and 12. The second right triangle has a hypotenuse of length 39. What is the scale factor between the two triangles?
21. What is the area of the smaller triangle in problem 30? What is the area of the larger triangle in problem 30? What is the ratio of the areas? How does it compare to the ratio of the lengths (or scale factor)? Recall that the area of a triangle is $A = \frac{1}{2}bh$.

Review Queue Answers

1. Answers:
   a. $x = 9$
   b. $x = 11.5$
   c. $x = 8$
2. Answers:
   a. $AB = 16$
   b. $BC = 14$