11.2 Surface Area of Prisms and Cylinders

FlexBooks® 2.0 > American HS Geometry > Surface Area of Prisms and Cylinders

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Learning Objectives

- Find the surface area of a prism.

- Find the surface area of a cylinder.

Review Queue

1. Find the area of a rectangle with sides:

    a. 6 and 9

    b. 11 and 4

    c. $5\sqrt{2}$ and $8\sqrt{6}$

2. If the area of a square is $36\ units^2$, what are the lengths of the sides?

3. If the area of a square is $45\ units^2$, what are the lengths of the sides?

    Find the area of the shape. All sides are perpendicular. (Split the shape up into rectangles.)
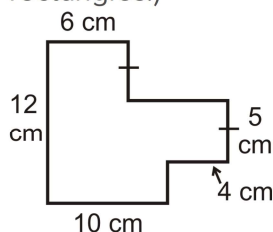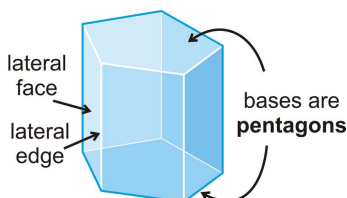

[Figure 1]

Know What? Your parents decide they want to put a pool in the backyard. They agree on a pool where the shallow end will be 4 ft. and the deep end will be 8 ft. The pool will be 10 ft. by 25 ft. How much siding do they need to buy to cover the sides and bottom of the pool? If the siding is $25.00 a square yard, how much will it cost to enclose the pool?
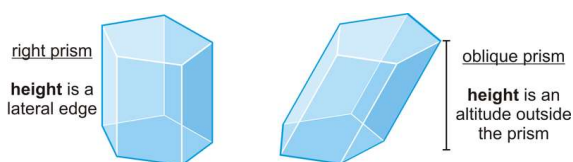
[Figure 2]

Parts of a Prism

lateral
face
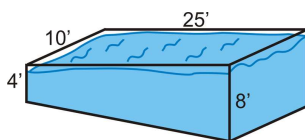
lateral
edge

bases are
pentagons

[Figure 3]

In the last section, we defined a prism as a 3-dimensional figure with 2 congruent bases, in parallel planes with rectangular lateral faces. The edges between the *lateral faces* are called *lateral edges*. All prisms are named by their bases, so the prism to the right is a pentagonal prism. This particular prism is called a *right prism* because the lateral faces are perpendicular to the bases. *Oblique prisms* lean to one side or the other and the height is outside the prism.


right prism

height is a
lateral edge

oblique prism

height is an
altitude outside
the prism

[Figure 4]

Surface Area of a Prism

Surface Area: The sum of the areas of the faces.

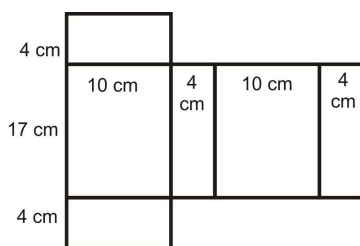**Lateral Area:** The sum of the areas of the *lateral* faces.

You can use a net and the Area Addition Postulate to find the surface area of a right prism.

Example 1: Find the surface area of the prism below.

17 cm

4 cm

10 cm
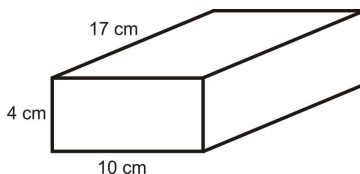
[Figure 5]

Solution: Open up the prism and draw the net. Determine the measurements for each rectangle in the net.
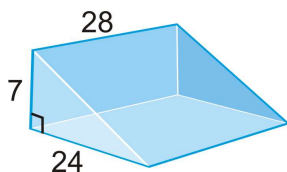
[Figure 6]

Using the net, we have:

$$
\begin{aligned}
SA_{prism} &= 2(4)(10) + 2(10)(17) + 2(17)(4) \\
&= 80 + 340 + 136 \\
&= 556\ cm^2
\end{aligned}
$$

Because this is still area, the units are squared.

Surface Area of a Right Prism: The surface area of a right prism is the sum of the area of the bases and the area of each rectangular lateral face.

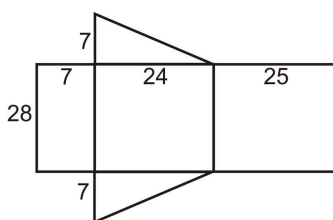**Example 2:** Find the surface area of the prism below.

[Figure 7]

Solution: This is a right triangular prism. To find the surface area, we need to find the length of the hypotenuse of the base because it is the width of one of the lateral faces. Using the Pythagorean Theorem, the hypotenuse is

$$
\begin{aligned}
7^2 + 24^2 &= c^2 \\
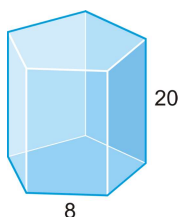49 + 576 &= c^2 \\
625 &= c^2 \\
c &= 25
\end{aligned}
$$

Looking at the net, the surface area is:

$$
\begin{aligned}
SA &= 28(7) + 28(24) + 28(25) + 2\left(\frac{1}{2} \cdot 7 \cdot 24\right) \\
SA &= 196 + 672 + 700 + 168 = 1736
\end{aligned}
$$

https://flexbooks.ck12.org/flx/b/18708492/11170312

[Figure 8]

Example 3: Find the surface area of the regular pentagonal prism.

[Figure 9]

Solution: For this prism, each lateral face has an area of $160 \ units^2$. Then, we need to find the area of the regular pentagonal bases. Recall that the area of a regular polygon is $\frac{1}{2}asn$. $s = 8$ and $n = 5$, so we need to find a, the apothem.
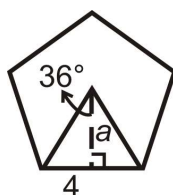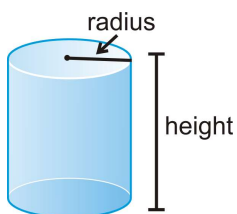

[Figure 10]

$$\tan 36° = \frac{4}{a}$$

$$a = \frac{4}{\tan 36°} \approx 5.51$$

$$SA = 5(160) + 2\left(\frac{1}{2} \cdot 5.51 \cdot 8 \cdot 5\right) = 1020.4$$
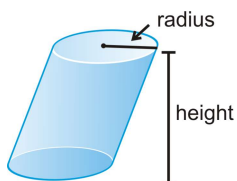
Cylinders

Cylinder: A solid with congruent circular bases that are in parallel planes. The space between the circles is enclosed.

Just like a circle, the cylinder has a radius for each of the circular bases. Also, like a prism, a cylinder can be oblique, like the one to the right.
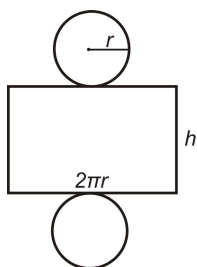
[Figure 11]

[Figure 12]

Surface Area of a Right Cylinder

Let's find the net of a right cylinder. One way for you to do this is to take a label off of a soup can or can of vegetables. When you take this label off, we see that it is a rectangle where the height is the height of the cylinder and the base is the circumference of the base. This rectangle and the two circular bases make up the net of a cylinder.

[Figure 13]

[Figure 14]

From the net, we can see that the surface area of a right cylinder is

$$\underbrace{2\pi r^2}_{\substack{\text{area of} \\ \text{both} \\ \text{circles}}} + \underbrace{2\pi r\, h}_{\substack{\text{length} \\ \text{of} \\ \text{rectangle}}}$$

Surface Area of a Right Cylinder: If r is the radius of the base and h is the height of the cylinder, then the surface area is $SA = 2\pi r^2 + 2\pi rh$.

To see an animation of the surface area, click
http://www.rkm.com.au/ANIMATIONS/animation-Cylinder-Surface-Area-Derivation.html, by Russell Knightley.

Example 4: Find the surface area of the cylinder.
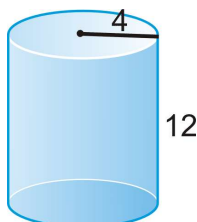
[Figure 15]

Solution: $r = 4$ and $h = 12$. Plug these into the formula.

$$SA = 2\pi(4)^2 + 2\pi(4)(12)$$
$$= 32\pi + 96\pi$$
$$= 128\pi$$

Example 5: The circumference of the base of a cylinder is 16π and the height is 21. Find the surface area of the cylinder.
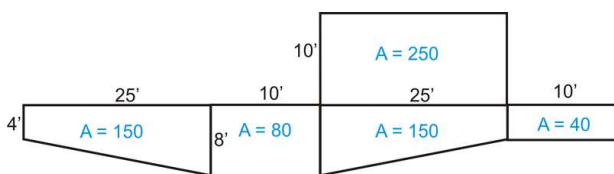
Solution: If the circumference of the base is 16π , then we can solve for the radius.

$$2\pi r = 16\pi$$
$$r = 8$$

Now, we can find the surface area.

$$SA = 2\pi(8)^2 + (16\pi)(21)$$
$$= 128\pi + 336\pi$$
$$= 464\pi$$

Know What? Revisited To the right is the net of the pool (minus the top). From this, we can see that your parents would need 670 square feet of siding. This means that the total cost would be $5583.33 for the siding.
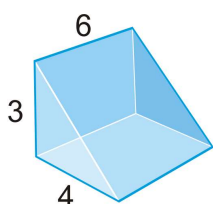
[Figure 16]

Review Questions

1. How many square feet are in a square yard?

2. How many square centimeters are in a square meter?

Use the right triangular prism to answer questions 3-6.

[Figure 17]

3. What shape are the bases of this prism? What are their areas?

4. What are the dimensions of each of the lateral faces? What are their areas?

5. Find the lateral surface area of the prism.

6. Find the total surface area of the prism.

7. *Writing* Describe the difference between lateral surface area and total surface area.

8. The lateral surface area of a cylinder is what shape? What is the area of this shape?
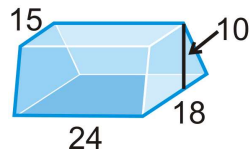
   Fuzzy dice are cubes with 4 inch sides.
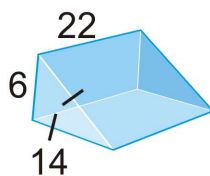

[Figure 18]

   a. What is the surface area of one die?

   b. Typically, the dice are sold in pairs. What is the surface area of two dice?

9. A right cylinder has a 7 cm radius and a height of 18 cm. Find the surface area.

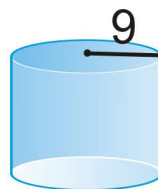Find the surface area of the following solids. Leave answers in terms of π.
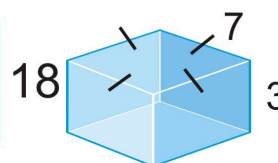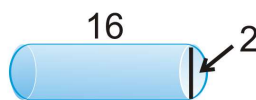
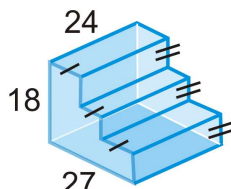bases are isosceles trapezoids

[Figure 19]

[Figure 20]

[Figure 21]

[Figure 22]

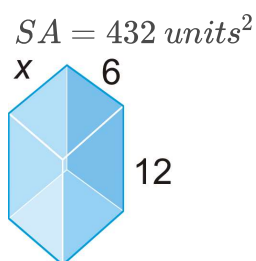[Figure 23]

[Figure 24]
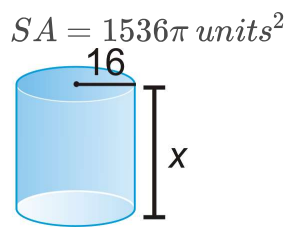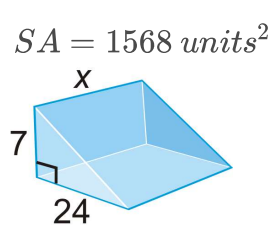
Algebra Connection Find the value of x, given the surface area.

$SA = 432 \ units^2$

[Figure 25]

$SA = 1536\pi \ units^2$

[Figure 26]

$SA = 1568 \ units^2$
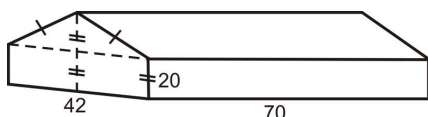
[Figure 27]

17. The area of the base of a cylinder is $25\pi \ in^2$ and the height is 6 in. Find the *lateral* surface area.

18. The circumference of the base of a cylinder is $80\pi \ cm$ and the height is 36 cm. Find the total surface area.

19. The lateral surface area of a cylinder is $30\pi \ m^2$. What is one possibility for height of the cylinder?

Use the diagram below for questions 23-27. The barn is shaped like a pentagonal prism with dimensions shown in feet.
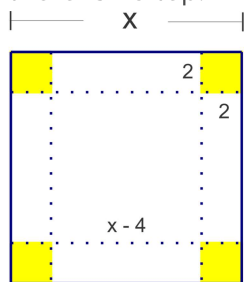
[Figure 28]

23. What is the area of the roof? (Both sides)

24. What is the floor area of the barn?

25. What is the area of the sides of the barn?

26. The farmer wants to paint the sides of the roof (excluding the roof). If a gallon of paint covers 250 square feet, how many gallons will he need?

27. A gallon of paint costs $15.50. How much will it cost for him to paint the sides of the barn?

28. Charlie started a business canning artichokes. His cans are 5 in tall and have diameter 4 in. If the label must cover the entire lateral surface of the can and the ends must overlap by at least one inch, what are the dimensions and area of the label?

    An open top box is made by cutting out 2 in by 2 in squares from the corners of a large square piece of cardboard. Using the picture as a guide, find an expression for the surface area of the box. If the surface area is $609 \ in^2$, find the length of x. Remember, there is no top.

[Figure 29]

29. Find an expression for the surface area of a cylinder in which the ratio of the height to the diameter is 2:1. If x is the diameter, use your expression to find x if the surface area is 160π.

Review Queue Answers

    a. 54

    b. 44

    c. $80\sqrt{3}$

1. $s = 6$

2. $s = 3\sqrt{5}$

3. $A = 60 + 30 + 20 = 110 \ cm^2$