8.5 Tangent, Sine and Cosine

FlexBooks® 2.0 > American HS Geometry > Tangent, Sine and Cosine

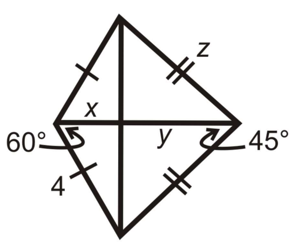
Last Modified: Oct 20, 2021

Learning Objectives

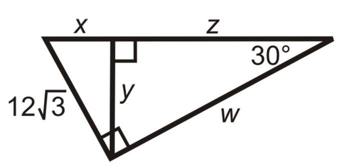
- Use the tangent, sine and cosine ratios in a right triangle.
- Understand these trigonometric ratios in special right triangles.
- Use a scientific calculator to find sine, cosine and tangent.
- Use trigonometric ratios in real-life situations.

Review Queue

1. The legs of an isosceles right triangle have length 14. What is the hypotenuse?
2. Do the lengths 8, 16, 20 make a right triangle? If not, is the triangle obtuse or acute?
3. In a 30-60-90 triangle, what do the 30, 60, and 90 refer to?
4. Find the measure of the missing lengths.

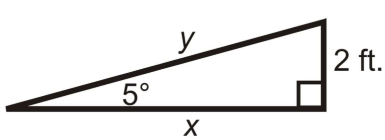
a. 

[Figure 1]

b. 

[Figure 2]

Know What? A restaurant needs to build a wheelchair ramp for its customers. The angle of elevation for a ramp is recommended to be 5° . If the vertical distance from the sidewalk to the front door is two feet, what is the horizontal distance that the ramp will take up (x) ? How long will the ramp be (y) ? Round your answers to the nearest hundredth.



[Figure 3]

What is Trigonometry?

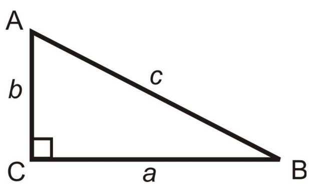
The word trigonometry comes from two words meaning *triangle* and *measure*. In this lesson we will define three trigonometric (or trig) functions. Once we have defined these functions, we will be able to solve problems like the **Know What?** above.

Trigonometry: The study of the relationships between the sides and angles of right triangles.

In trigonometry, sides are named in reference to a particular angle. The hypotenuse of a triangle is always the same, but the terms **adjacent** and **opposite** depend on which angle you are referencing. A side adjacent to an angle is the leg of the triangle that helps form the angle. A side opposite to an angle is the leg of the triangle that does not help form the angle. We never reference the right angle when referring to trig ratios.

$$\begin{array}{ll} a \text{ is } adjacent \text{ to } \angle B. & a \text{ is } opposite \ \angle A. \\ b \text{ is } adjacent \text{ to } \angle A. & b \text{ is } opposite \ \angle B. \end{array}$$

$$c \text{ is the } hypotenuse.$$



[Figure 4]

Sine, Cosine, and Tangent Ratios

The three basic trig ratios are called, sine, cosine and tangent. At this point, we will only take the sine, cosine and tangent of acute angles. However, you will learn that you can use these ratios with obtuse angles as well.

Sine Ratio: For an acute angle x in a right triangle, the $\sin x$ is equal to the ratio of the side opposite the angle over the hypotenuse of the triangle.

Using the triangle above, $\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$.

Cosine Ratio: For an acute angle x in a right triangle, the $\cos x$ is equal to the ratio of the side adjacent to the angle over the hypotenuse of the triangle.

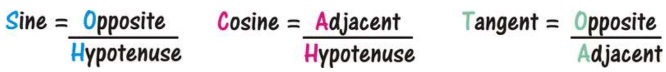
Using the triangle above, $\cos A = \frac{b}{c}$ and $\cos B = \frac{a}{c}$.

Tangent Ratio: For an acute angle x , in a right triangle, the $\tan x$ is equal to the ratio of the side opposite to the angle over the side adjacent to x .

Using the triangle above, $\tan A = \frac{a}{b}$ and $\tan B = \frac{b}{a}$.

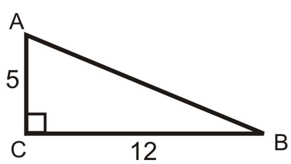
There are a few important things to note about the way we write these ratios. First, keep in mind that the abbreviations $\sin x, \cos x$, and $\tan x$ are all functions. Each ratio can be considered a function of the angle (see Chapter 10). Second, be careful when using the abbreviations that you still pronounce the full name of each function. When we write $\sin x$ it is still pronounced *sine,* with a long "i". When we write $\cos x$, we still say co-sine. And when we write $\tan x$, we still say tangent.

An easy way to remember ratios is to use the pneumonic SOH-CAH-TOA.



[Figure 5]

Example 1: Find the sine, cosine and tangent ratios of $\angle A$.



[Figure 6]

Solution: First, we need to use the Pythagorean Theorem to find the length of the hypotenuse.

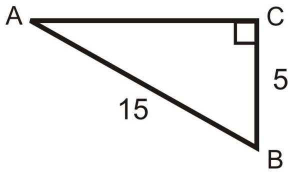
$$\begin{aligned} 5^2 + 12^2 &= h^2 \\ 13 &= h \end{aligned}$$

So, $\sin A = \frac{12}{13}, \cos A =$, and $\tan A = \frac{12}{5}$.

<u>A few important points:</u>

- Always reduce ratios when you can.
- Use the Pythagorean Theorem to find the missing side (if there is one).
- The tangent ratio can be bigger than 1 (the other two cannot).
- If two right triangles are similar, then their sine, cosine, and tangent ratios will be the same (because they will reduce to the same ratio).
- If there is a radical in the denominator, rationalize the denominator.

Example 2: Find the sine, cosine, and tangent of $\angle B$.



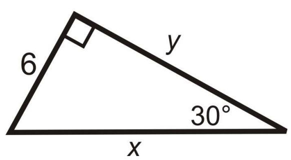
[Figure 7]

Solution: Find the length of the missing side.

$$\begin{aligned} AC^2+5^2 &= 15^2 \\ AC^2 &= 200 \\ AC &= 10\sqrt{2} \end{aligned}$$

Therefore, $\sin B = \frac{10\sqrt{2}}{15} = \frac{2\sqrt{2}}{3}, \cos B = \frac{5}{15} = \frac{1}{3}$, and $\tan B = \frac{10\sqrt{2}}{5} = 2\sqrt{2}$.

Example 3: Find the sine, cosine and tangent of 30° .



[Figure 8]

Solution: This is a special right triangle, a 30-60-90 triangle. So, if the short leg is 6, then the long leg is $6\sqrt{3}$ and the hypotenuse is 12.

$\sin 30^\circ = \frac{6}{12} = \frac{1}{2}, \cos 30^\circ = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$, and $\tan 30^\circ = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

In Example 3, we knew the angle measure of the angle we were taking the sine, cosine and tangent of. This means that ***the sine, cosine and tangent for an angle are fixed***.

Sine, Cosine, and Tangent with a Calculator

We now know that the trigonometric ratios are not dependent on the sides, but the ratios. Therefore, there is one fixed value for every angle, from 0° to 90° . Your scientific (or graphing) calculator knows the values of the sine, cosine and tangent of all of these angles. Depending on your calculator, you should have [SIN], [COS], and [TAN] buttons. Use these to find the sine, cosine, and tangent of any acute angle.

Example 4: Find the indicated trigonometric value, using your calculator.

a) $\sin 78^\circ$

b) $\cos 60^\circ$

c) $\tan 15^\circ$

Solution: Depending on your calculator, you enter the degree first, and then press the correct trig button or the other way around. For TI-83s and TI-84s you press the trig button first, followed by the angle. Also, make sure the mode of your calculator is in DEGREES.

a) $\sin 78^\circ = 0.9781$

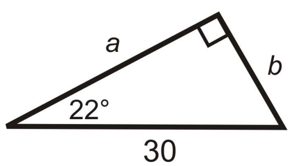
b) $\cos 60^\circ = 0.5$

c) $\tan 15^\circ = 0.2679$

Finding the Sides of a Triangle using Trig Ratios

One application of the trigonometric ratios is to use them to find the missing sides of a right triangle. All you need is one angle, other than the right angle, and one side. Let's go through a couple of examples.

Example 5: Find the value of each variable. Round your answer to the nearest hundredth.



[Figure 9]

Solution: We are given the hypotenuse, so we would need to use the sine to find b, because it is opposite 22° and cosine to find a, because it is adjacent to 22°.

$$\begin{aligned} \sin 22^\circ &= \frac{b}{30} & \cos 22^\circ &= \frac{a}{30} \\ 30 \cdot \sin 22^\circ &= b & 30 \cdot \cos 22^\circ &= a \\ b &\approx 11.24 & a &\approx 27.82 \end{aligned}$$

Example 6: Find the value of each variable. Round your answer to the nearest hundredth.

[Figure 10]

Solution: Here, we are given the adjacent leg to 42°. To find c, we need to use cosine and to find d we will use tangent.

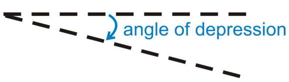
$$\cos 42^\circ = \frac{9}{c} \qquad \tan 42^\circ = \frac{d}{9}$$
$$c \cdot \cos 42^\circ = 9 \qquad 9 \cdot \tan 42^\circ = d$$
$$c = \frac{9}{\cos 42^\circ} \approx 12.11 \qquad d \approx 8.10$$

Notice in both of these examples, you should only use the information that you are given. For example, you should not use the found value of b to find a (in Example 5) because b is an ***approximation***. Use exact values to give the most accurate answers. However, in both examples you could have also used the complementary angle to the one given.

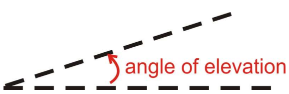
Angles of Depression and Elevation

Another practical application of the trigonometric functions is to find the measure of lengths that you cannot measure. Very frequently, angles of depression and elevation are used in these types of problems.

Angle of Depression: The angle measured from the horizon or horizontal line, down.



[Figure 11]



[Figure 12]

Angle of Elevation: The angle measure from the horizon or horizontal line, up.

Example 7: An inquisitive math student is standing 25 feet from the base of the Washington Monument. The angle of elevation from her horizontal line of sight is 87.4°. If her "eye height" is 5ft, how tall is the monument?

[Figure 13]

Solution: We can find the height of the monument by using the tangent ratio and then adding the eye height of the student.

$$\tan 87.4^\circ = \frac{h}{25}$$
$$h = 25 \cdot \tan 87.4^\circ = 550.54$$

Adding 5 ft, the total height of the Washington Monument is 555.54 ft.

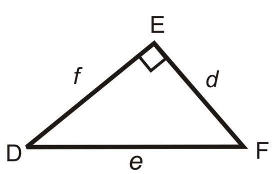
According to Wikipedia, the actual height of the monument is 555.427 ft.

Know What? Revisited To find the horizontal length and the actual length of the ramp, we need to use the tangent and sine.

$$\tan 5^\circ = \frac{2}{x} \qquad \sin 5^\circ = \frac{2}{y}$$
$$x = \frac{2}{\tan 5^\circ} = 22.86 \qquad y = \frac{2}{\sin 5^\circ} = 22.95$$

Review Questions

Use the diagram to fill in the blanks below.



[Figure 14]

1. $\tan D = \frac{?}{?}$
2. $\sin F = \frac{?}{?}$
3. $\tan F = \frac{?}{?}$
4. $\cos F = \frac{?}{?}$
5. $\sin D = \frac{?}{?}$
6. $\cos D = \frac{?}{?}$

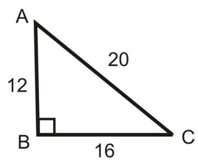
From questions 1-6, we can conclude the following. Fill in the blanks.

7. $\cos \_\_\_\_ = \sin F$ and $\sin \_\_\_\_ = \cos F$
8. The sine of an angle is ______________ to the cosine of its ________________.
9. $\tan D$ and $\tan F$ are _____________ of each other.

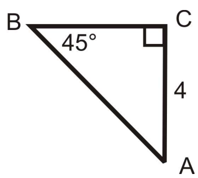
Use your calculator to find the value of each trig function below. Round to four decimal places.

10. $\sin 24^\circ$
11. $\cos 45^\circ$
12. $\tan 88^\circ$
13. $\sin 43^\circ$

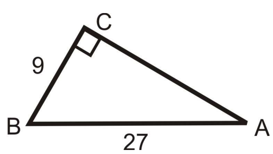
Find the sine, cosine and tangent of $\angle A$. Reduce all fractions and radicals.



14. [Figure 15]

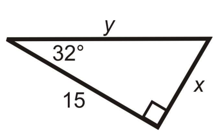


15. [Figure 16]

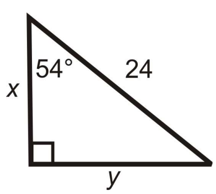


16. [Figure 17]

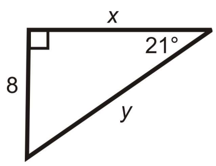
Find the length of the missing sides. Round your answers to the nearest hundredth.



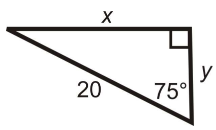
17. [Figure 18]



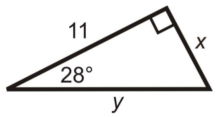
18. [Figure 19]



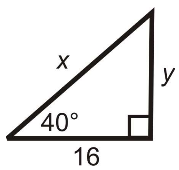
19. [Figure 20]



20. [Figure 21]

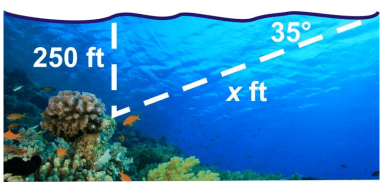


21. [Figure 22]



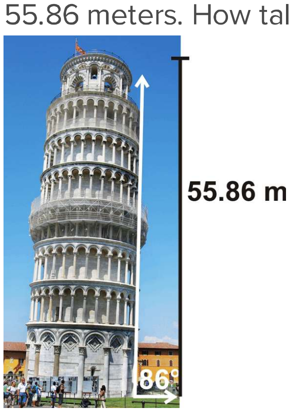
22. [Figure 23]

23. Kristin is swimming in the ocean and notices a coral reef below her. The angle of depression is 35° and the depth of the ocean, at that point, is 250 feet. How far away is she from the reef?



[Figure 24]

24. The Leaning Tower of Piza currently "leans" at a 4° angle and has a vertical height of 55.86 meters. How tall was the tower when it was originally built?



[Figure 25]

25. The angle of depression from the top of an apartment building to the base of a fountain in a nearby park is 72° . If the building is 78 ft tall, how far away is the fountain?

26. William spots a tree directly across the river from where he is standing. He then walks 20 ft upstream and determines that the angle between his previous position and the tree on

the other side of the river is 65° . How wide is the river?

27. Diego is flying his kite one afternoon and notices that he has let out the entire 120 ft of string. The angle his string makes with the ground is 52° . How high is his kite at this time?
28. A tree struck by lightning in a storm breaks and falls over to form a triangle with the ground. The tip of the tree makes a 36° angle with the ground 25 ft from the base of the tree. What was the height of the tree to the nearest foot?
29. Upon descent an airplane is 20,000 ft above the ground. The air traffic control tower is 200 ft tall. It is determined that the angle of elevation from the top of the tower to the plane is 15° . To the nearest mile, find the ground distance from the airplane to the tower.
30. ***Critical Thinking*** Why are the sine and cosine ratios always be less than 1?

Review Queue Answers

1. The hypotenuse is $14\sqrt{2}$.
2. No, $8^2+16^2<20^2,$ the triangle is obtuse.
3. $30^\circ, 60^\circ,$ and 90° refer to the angle measures in the special right triangle.
4. Answers:
   a. $x=2, y=2\sqrt{3}$
   b. $x=6\sqrt{3}, y=18, z=18\sqrt{3}, w=36$

8.6 Inverse Trigonometric Ratios

FlexBooks® 2.0 > American HS Geometry > Inverse Trigonometric Ratios

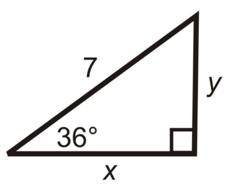
Last Modified: Dec 25, 2014

Learning Objectives

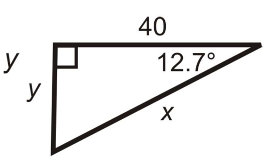
- Use the inverse trigonometric ratios to find an angle in a right triangle.
- Solve a right triangle.
- Apply inverse trigonometric ratios to real-life situation and special right triangles.

Review Queue

Find the lengths of the missing sides. Round your answer to the nearest hundredth.



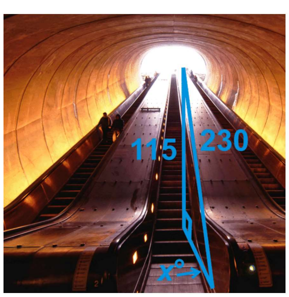
[Figure 1]



[Figure 2]

1. Draw an isosceles right triangle with legs of length 3. What is the hypotenuse?
2. Use the triangle from #3, to find the sine, cosine, and tangent of 45° .
3. Explain why $\tan 45^\circ = 1$.

Know What? The longest escalator in North America is at the Wheaton Metro Station in Maryland. It is 230 feet long and is 115 ft high. What is the angle of elevation, x° , of this escalator?



[Figure 3]

Inverse Trigonometric Ratios

The word ***inverse*** is probably familiar to you. In mathematics, once you learn how to do an operation, you also learn how to "undo" it. For example, you may remember that addition and subtraction are considered inverse operations. Multiplication and division are also inverse operations. In algebra you used inverse operations to solve equations and inequalities.

When we apply the word inverse to the trigonometric ratios, we can find the acute angle measures within a right triangle. Normally, if you are given an angle and a side of a right triangle, you can find the other two sides, using sine, cosine or tangent. With the inverse trig ratios, you can find the angle measure, given two sides.

Inverse Tangent: If you know the opposite side and adjacent side of an angle in a right triangle, you can use inverse tangent to find the measure of the angle.

Inverse tangent is also called arctangent and is labeled $\tan^{-1}$ or *arctan*. The “-1” indicates inverse.

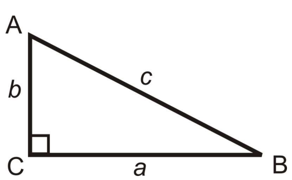
Inverse Sine: If you know the opposite side of an angle and the hypotenuse in a right triangle, you can use inverse sine to find the measure of the angle.

Inverse sine is also called arcsine and is labeled $\sin^{-1}$ or *arcsin*.

Inverse Cosine: If you know the adjacent side of an angle and the hypotenuse in a right triangle, you can use inverse cosine to find the measure of the angle.

Inverse cosine is also called arccosine and is labeled $\cos^{-1}$ or *arccos*.

Using the triangle below, the inverse trigonometric ratios look like this:



[Figure 4]

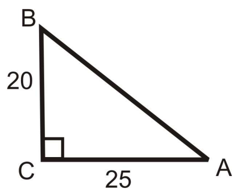
$$\begin{aligned} \tan^{-1}\left(\frac{b}{a}\right) &= m\angle B & \tan^{-1}\left(\frac{a}{b}\right) &= m\angle A \\ \sin^{-1}\left(\frac{b}{c}\right) &= m\angle B & \sin^{-1}\left(\frac{a}{c}\right) &= m\angle A \\ \cos^{-1}\left(\frac{a}{c}\right) &= m\angle B & \cos^{-1}\left(\frac{b}{c}\right) &= m\angle A \end{aligned}$$

In order to actually find the measure of the angles, you will need you use your calculator. On most scientific and graphing calculators, the buttons look like $[\text{SIN}^{-1}], [\text{COS}^{-1}]$, and $[\text{TAN}^{-1}]$. Typically, you might have to hit a shift or 2^{nd} button to access these functions.

For example, on the TI-83 and 84, $[2^{nd}][\text{SIN}]$ is $[\text{SIN}^{-1}]$. Again, make sure the mode is in degrees.

When you find the inverse of a trigonometric function, you put the word *arc* in front of it. So, the inverse of a tangent is called the arctangent (or arctan for short). Think of the arctangent as a tool you can use like any other inverse operation when solving a problem. If tangent tells you the ratio of the lengths of the sides opposite and adjacent to an angle, then tangent inverse tells you the measure of an angle with a given ratio.

Example 1: Use the sides of the triangle and your calculator to find the value of $\angle A$. Round your answer to the nearest tenth of a degree.

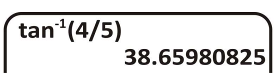


[Figure 5]

Solution: In reference to $\angle A$, we are given the *opposite* leg and the *adjacent* leg. This means we should use the *tangent* ratio.

$\tan A = \frac{20}{25} = \frac{4}{5}$, therefore $\tan^{-1}\left(\frac{4}{5}\right) = m\angle A$. Use your calculator.

If you are using a TI-83 or 84, the keystrokes would be: $[2^{nd}][\text{TAN}]\left(\frac{4}{5}\right)$ **[ENTER]** and the screen looks like:



[Figure 6]

So, $m\angle A = 38.7^\circ$

Example 2: $\angle A$ is an acute angle in a right triangle. Use your calculator to find $m\angle A$ to the nearest tenth of a degree.

a) $\sin A = 0.68$

b) $\cos A = 0.85$

c) $\tan A = 0.34$

Solution:

a) $m\angle A = \sin^{-1} 0.68 = 42.8^\circ$

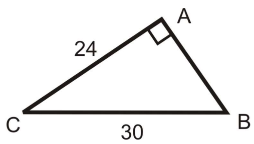
b) $m\angle A = \cos^{-1} 0.85 = 31.8^\circ$

c) $m\angle A = \tan^{-1} 0.34 = 18.8^\circ$

Solving Triangles

Now that we know how to use inverse trigonometric ratios to find the measure of the acute angles in a right triangle, we can solve right triangles. To solve a right triangle, you would need to find all sides and angles in a right triangle, using any method. When solving a right triangle, you could use sine, cosine or tangent, inverse sine, inverse cosine, or inverse tangent, or the Pythagorean Theorem. Remember when solving right triangles to only use the values that you are given.

Example 3: Solve the right triangle.



[Figure 7]

Solution: To solve this right triangle, we need to find $AB, m\angle C$ and $m\angle B$. Use AC and CB to give the most accurate answers.

$\underline{AB}$: Use the Pythagorean Theorem.

$$\begin{aligned} 24^2 + AB^2 &= 30^2 \\ 576 + AB^2 &= 900 \\ AB^2 &= 324 \\ AB &= \sqrt{324} = 18 \end{aligned}$$

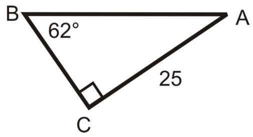
$m\angle B$: Use the inverse sine ratio.

$$\begin{aligned} \sin B &= \frac{24}{30} = \frac{4}{5} \\ \sin^{-1}\left(\frac{4}{5}\right) &= 53.1^\circ = m\angle B \end{aligned}$$

$m\angle C$: Use the inverse cosine ratio.

$$\begin{aligned} \cos C &= \frac{24}{30} = \frac{4}{5} \\ \cos^{-1}\left(\frac{4}{5}\right) &= 36.9^\circ = m\angle C \end{aligned}$$

Example 4: Solve the right triangle.



[Figure 8]

Solution: To solve this right triangle, we need to find AB, BC and $m\angle A$.

$\underline{AB}$: Use sine ratio.

$$\begin{aligned}\sin 62^\circ &= \frac{25}{AB}\\ AB &= \frac{25}{\sin 62^\circ}\\ AB &\approx 28.31\end{aligned}$$

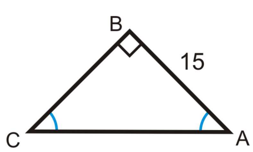
BC : Use tangent ratio.

$$\begin{aligned}\tan 62^\circ &= \frac{25}{BC}\\ BC &= \frac{25}{\tan 62^\circ}\\ BC &\approx 13.30\end{aligned}$$

$m\angle A$: Use Triangle Sum Theorem

$$\begin{aligned}62^\circ + 90^\circ + m\angle A &= 180^\circ\\ m\angle A &= 28^\circ\end{aligned}$$

Example 5: Solve the right triangle.



[Figure 9]

Solution: Even though, there are no angle measures given, we know that the two acute angles are congruent, making them both 45° . Therefore, this is a 45-45-90 triangle. You can use the trigonometric ratios or the special right triangle ratios.

Trigonometric Ratios

$$\tan 45^\circ = \frac{15}{BC} \qquad \sin 45^\circ = \frac{15}{AC}$$
$$BC = \frac{15}{\tan 45^\circ} = 15 \qquad AC = \frac{15}{\sin 45^\circ} \approx 21.21$$

45-45-90 Triangle Ratios

$$BC = AB = 15, AC = 15\sqrt{2} \approx 21.21$$

Real-Life Situations

Much like the trigonometric ratios, the inverse trig ratios can be used in several real-life situations. Here are a couple examples.

Example 6: A 25 foot tall flagpole casts a 42 feet shadow. What is the angle that the sun hits the flagpole?

[Figure 10]

Solution: First, draw a picture. The angle that the sun hits the flagpole is the acute angle at the top of the triangle, x° . From the picture, we can see that we need to use the inverse tangent ratio.

$$\tan x = \frac{42}{25}$$
$$\tan^{-1} \frac{42}{25} \approx 59.2^\circ = x$$

Example 7: Elise is standing on the top of a 50 foot building and spots her friend, Molly across the street. If Molly is 35 feet away from the base of the building, what is the angle of depression from Elise to Molly? Elise's eye height is 4.5 feet.

[Figure 11]

Solution: Because of parallel lines, the angle of depression is equal to the angle at Molly, or x° . We can use the inverse tangent ratio.

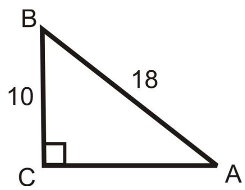
$$\tan^{-1}\left(\frac{54.5}{30}\right) = 61.2^\circ = x$$

Know What? Revisited To find the escalator's angle of elevation, we need to use the inverse sine ratio.

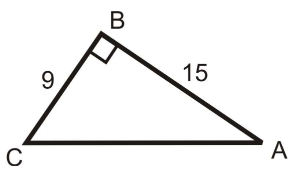
$$\sin^{-1}\left(\frac{115}{230}\right) = 30^\circ \qquad \text{The angle of elevation is } 30^\circ.$$

Review Questions

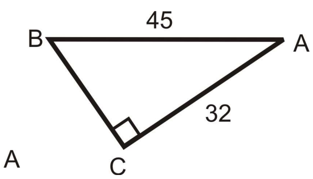
Use your calculator to find $m\angle A$ to the nearest tenth of a degree.



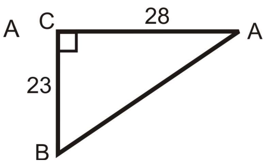
[Figure 12]



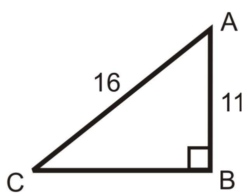
[Figure 13]



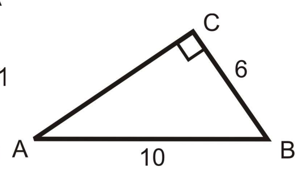
[Figure 14]



[Figure 15]



[Figure 16]

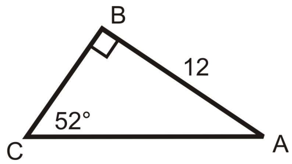


[Figure 17]

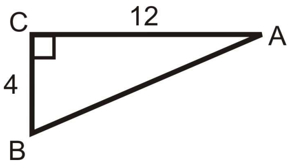
Let $\angle A$ be an acute angle in a right triangle. Find $m\angle A$ to the nearest tenth of a degree.

7. $\sin A = 0.5684$
8. $\cos A = 0.1234$
9. $\tan A = 2.78$

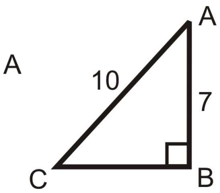
Solving the following right triangles. Find all missing sides and angles.



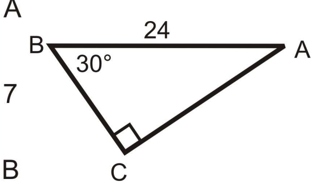
[Figure 18]



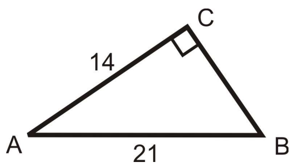
[Figure 19]



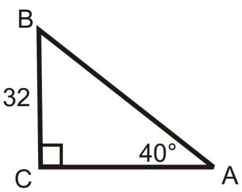
[Figure 20]



[Figure 21]



[Figure 22]



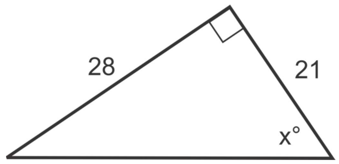
[Figure 23]

10. ***Writing*** Explain when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

Real-Life Situations Use what you know about right triangles to solve for the missing angle. If needed, draw a picture. Round all answers to the nearest tenth of a degree.

17. A 75 foot building casts an 82 foot shadow. What is the angle that the sun hits the building?
18. Over 2 miles (horizontal), a road rises 300 feet (vertical). What is the angle of elevation?
19. A boat is sailing and spots a shipwreck 650 feet below the water. A diver jumps from the boat and swims 935 feet to reach the wreck. What is the angle of depression from the boat to the shipwreck?
20. Elizabeth wants to know the angle at which the sun hits a tree in her backyard at 3 pm. She finds that the length of the tree's shadow is 24 ft at 3 pm. At the same time of day, her shadow is 6 ft 5 inches. If Elizabeth is 4 ft 8 inches tall, find the height of the tree and hence the angle at which the sunlight hits the tree.
21. Alayna is trying to determine the angle at which to aim her sprinkler nozzle to water the top of a 5 ft bush in her yard. Assuming the water takes a straight path and the sprinkler is on the ground 4 ft from the tree, at what angle of inclination should she set it?
22. ***Science Connection*** Would the answer to number 20 be the same every day of the year? What factors would influence this answer? How about the answer to number 21? What factors might influence the path of the water?

Tommy was solving the triangle below and made a mistake. What did he do wrong?



[Figure 24]

$$\tan^{-1}\left(\frac{21}{28}\right) \approx 36.9^\circ$$

23. Tommy then continued the problem and set up the equation: $\cos 36.9^\circ = \frac{21}{h}$. By solving this equation he found that the hypotenuse was 26.3 units. Did he use the correct trigonometric ratio here? Is his answer correct? Why or why not?
24. How could Tommy have found the hypotenuse in the triangle another way and avoided making his mistake?

Examining Patterns Below is a table that shows the sine, cosine, and tangent values for eight different angle measures. Answer the following questions.

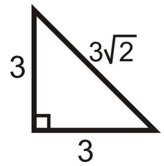
	10°	20°	30°	40°	50°	60°	70°	80°
Sine	0.1736	0.3420	0.5	0.6428	0.7660	0.8660	0.9397	0.9848
Cosine	0.9848	0.9397	0.8660	0.7660	0.6428	0.5	0.3420	0.1736
Tangent	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321	2.7475	5.6713

26. What value is equal to $\sin 40^\circ$?
27. What value is equal to $\cos 70^\circ$?
28. Describe what happens to the sine values as the angle measures increase.
29. Describe what happens to the cosine values as the angle measures increase.
30. What two numbers are the sine and cosine values between?
31. Find $\tan 85^\circ, \tan 89^\circ$, and $\tan 89.5^\circ$ using your calculator. Now, describe what happens to the tangent values as the angle measures increase.
32. Explain why all of the sine and cosine values are less than one. (hint: think about the sides in the triangle and the relationships between their lengths)

Review Queue Answers

1. $\sin 36^\circ = \frac{y}{7}$ $\quad$ $\cos 36^\circ = \frac{x}{7}$
   $y = 4.11$ $\quad$ $x = 5.66$

2. $$\cos 12.7^\circ = \frac{40}{x} \qquad \tan 12.7^\circ = \frac{y}{40}$$
$$x = 41.00 \qquad y = 9.01$$



[Figure 25]

3. $$\sin 45^\circ = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\cos 45^\circ = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\tan 45^\circ = \frac{3}{3} = 1$$

4. The tangent of 45° equals one because it is the ratio of the opposite side over the adjacent side. In an isosceles right triangle, or 45-45-90 triangle, the opposite and adjacent sides are the same, making the ratio always 1.