12.2 Translations and Vectors

FlexBooks® 2.0 > American HS Geometry > Translations and Vectors

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Learning Objectives

- Graph a point, line, or figure and translate it x and y units.
- Write a translation rule.
- Use vector notation.

Review Queue

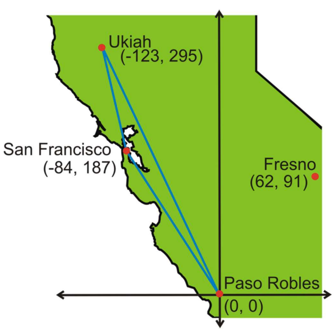
1. Find the equation of the line that contains (9, -1) and (5, 7).
2. What type of quadrilateral is formed by $A(1,-1), B(3,0), C(5,-5)$ and $D(-3,0)$?
3. Find the equation of the line parallel to #1 that passes through (4, -3).
4. Find the equation of the line perpendicular to #1 that passes through (4, -3).

Know What? Lucy currently lives in San Francisco, S , and her parents live in Paso Robles, P . She will be moving to Ukiah, U , in a few weeks. All measurements are in miles. Find:

a) The component form of $\overrightarrow{PS}, \overrightarrow{SU}$ and $\overrightarrow{PU}$.

b) Lucy's parents are considering moving to Fresno, F . Find the component form of $\overrightarrow{PF}$ and $\overrightarrow{UF}$.

c) Is Ukiah or Paso Robles closer to Fresno?



[Figure 1]

Transformations

Recall from Lesson 7.6, we learned about dilations, which is a type of transformation. Now, we are going to continue learning about other types of transformations. All of the transformations in this chapter are rigid transformations.

Transformation: An operation that moves, flips, or changes a figure to create a new figure.

Rigid Transformation: A transformation that preserves size and shape.

The rigid transformations are: translations, reflections, and rotations. The new figure created by a transformation is called the ***image***. The original figure is called the ***preimage***. Another word for a rigid transformation is an ***isometry***. Rigid transformations are also called congruence transformations.

Also in Lesson 7.6, we learned how to label an image. If the preimage is A, then the image would be labeled A', said “a prime.” If there is an image of A', that would be labeled A'', said “a double prime.”

Translations

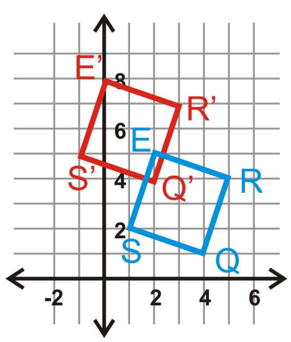
The first of the rigid transformations is a translation.

Translation: A transformation that moves every point in a figure the same distance in the same direction.

In the coordinate plane, we say that a translation moves a figure x units and y units.

Example 1: Graph square $S(1,2), Q(4,1), R(5,4)$ and $E(2,5)$. Find the image after the translation $(x,y) \rightarrow (x-2, y+3)$. Then, graph and label the image.

Solution: The translation notation tells us that we are going to move the square to the left 2 and up 3.

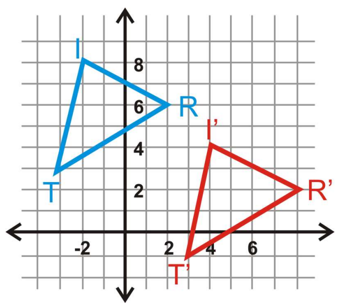


[Figure 2]

$$\begin{aligned}(x,y) &\rightarrow (x-2,y+3)\\ S(1,2) &\rightarrow S'(-1,5)\\ Q(4,1) &\rightarrow Q'(2,4)\\ R(5,4) &\rightarrow R'(3,7)\\ E(2,5) &\rightarrow E'(0,8)\end{aligned}$$

Example 2: Find the translation rule for $\triangle TRI$ to $\triangle T'R'I'$.

Solution: Look at the movement from T to T' . T is (-3, 3) and T' is (3, -1). The change in x is 6 units to the right and the change in y is 4 units down. Therefore, the translation rule is $(x,y) \rightarrow (x+6,y-4)$.



[Figure 3]

From both of these examples, we see that a translation preserves congruence. Therefore, ***a translation is an isometry***. We can show that each pair of figures is congruent by using the distance formula.

Example 3: Show $\triangle TRI \cong \triangle T'R'I'$ from Example 2.

Solution: Use the distance formula to find all the lengths of the sides of the two triangles.

$$\begin{aligned}&\underline{\triangle TRI}\\ &TR = \sqrt{(-3-2)^2+(3-6)^2} = \sqrt{34}\\ &RI = \sqrt{(2-(-2))^2+(6-8)^2} = \sqrt{20}\\ &TI = \sqrt{(-3-(-2))^2+(3-8)^2} = \sqrt{26}\end{aligned}$$

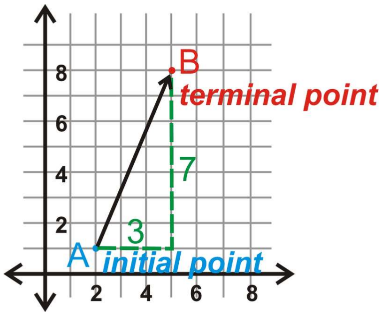
$$\begin{aligned}&\underline{\triangle T'R'I'}\\ &T'R' = \sqrt{(3-8)^2+(-1-2)^2} = \sqrt{34}\\ &R'I' = \sqrt{(8-4)^2+(2-4)^2} = \sqrt{20}\\ &T'I' = \sqrt{(3-4)^2+(-1-4)^2} = \sqrt{26}\end{aligned}$$

Vectors

Another way to write a translation rule is to use vectors.

Vector: A quantity that has direction and size.

In the graph below, the line from A to B, or the distance traveled, is the vector. This vector would be labeled $\overrightarrow{AB}$ because A is the ***initial point*** and B is the ***terminal point***. The terminal point always has the arrow pointing towards it and has the half-arrow over it in the label.

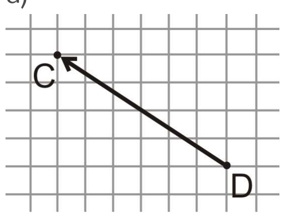


[Figure 4]

The ***component form*** of $\overrightarrow{AB}$ combines the horizontal distance traveled and the vertical distance traveled. We write the component form of $\overrightarrow{AB}$ as $\langle 3,7 \rangle$ because $\overrightarrow{AB}$ travels 3 units to the right and 7 units up. Notice the brackets are pointed, $\langle 3,7 \rangle$, not curved.

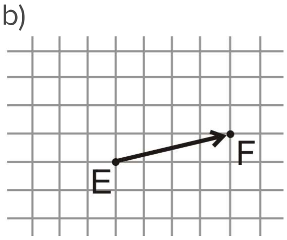
Example 4: Name the vector and write its component form.

a)



[Figure 5]

b)



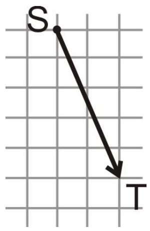
[Figure 6]

Solution:

a) The vector is $\overrightarrow{DC}$. From the initial point D to terminal point C, you would move 6 units to the left and 4 units up. The component form of $\overrightarrow{DC}$ is $\langle -6,4 \rangle$.

b) The vector is $\overrightarrow{EF}$. The component form of $\overrightarrow{EF}$ is $\langle 4,1 \rangle$.

Example 5: Draw the vector $\overrightarrow{ST}$ with component form $\langle 2,-5 \rangle$.

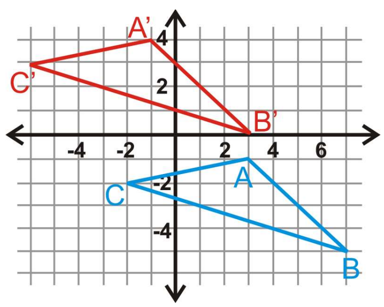


[Figure 7]

Solution: The graph above is the vector $\overrightarrow{ST}$. From the initial point S it moves down 5 units and to the right 2 units.

The positive and negative components of a vector always correlate with the positive and negative parts of the coordinate plane. We can also use vectors to translate an image.

Example 6: Triangle $\triangle ABC$ has coordinates $A(3,-1), B(7,-5)$ and $C(-2,-2)$. Translate $\triangle ABC$ using the vector $\langle -4,5 \rangle$. Determine the coordinates of $\triangle A'B'C'$.



[Figure 8]

Solution: It would be helpful to graph $\triangle ABC$. To translate $\triangle ABC$, add each component of the vector to each point to find $\triangle A'B'C'$.

$$A(3,-1) + \langle -4,5 \rangle = A'(-1,4)$$
$$B(7,-5) + \langle -4,5 \rangle = B'(3,0)$$
$$C(-2,-2) + \langle -4,5 \rangle = C'(-6,3)$$

Example 7: Write the translation rule for the vector translation from Example 6.

Solution: To write $\langle -4,5 \rangle$ as a translation rule, it would be $(x,y) \rightarrow (x-4, y+5)$.

Know What? Revisited

a) $\overrightarrow{PS} = \langle -84,187 \rangle, \overrightarrow{SU} = \langle -39,108 \rangle, \overrightarrow{PU} = \langle -123,295 \rangle$

b) $\overrightarrow{PF} = \langle 62,91 \rangle, \overrightarrow{UF} = \langle 185,-204 \rangle$

c) You can plug the vector components into the Pythagorean Theorem to find the distances. Paso Robles is closer to Fresno than Ukiah.

$$UF = \sqrt{185^2 + (-204)^2} \cong 275.4\, miles, PF = \sqrt{62^2 + 91^2} \cong 110.1\, miles$$

Review Questions

1. What is the difference between a vector and a ray?

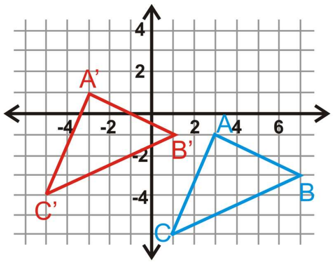
Use the translation $(x, y) \rightarrow (x+5, y-9)$ for questions 2-8.

2. What is the image of $A(-6, 3)$?
3. What is the image of $B(4, 8)$?
4. What is the preimage of $C'(5, -3)$?
5. What is the image of A' ?
6. What is the preimage of $D'(12, 7)$?
7. What is the image of A'' ?
8. Plot A, A', A'' , and A''' from the questions above. What do you notice? Write a conjecture.

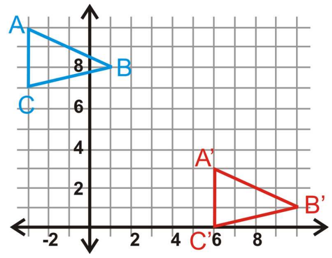
The vertices of $\triangle ABC$ are $A(-6, -7), B(-3, -10)$ and $C(-5, 2)$. Find the vertices of $\triangle A'B'C'$, given the translation rules below.

9. $(x, y) \rightarrow (x-2, y-7)$
10. $(x, y) \rightarrow (x+11, y+4)$
11. $(x, y) \rightarrow (x, y-3)$
12. $(x, y) \rightarrow (x-5, y+8)$

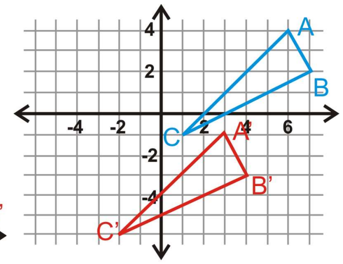
In questions 13-16, $\triangle A'B'C'$ is the image of $\triangle ABC$. Write the translation rule.



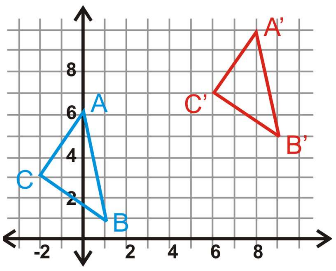
[Figure 9]



[Figure 10]



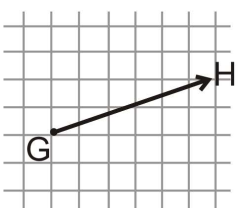
[Figure 11]



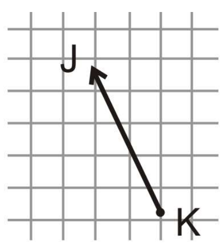
[Figure 12]

13. Verify that a translation is an isometry using the triangle from #15.
14. If $\triangle A'B'C'$ was the *preimage* and $\triangle ABC$ was the image, write the translation rule for #16.

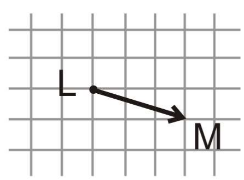
For questions 19-21, name each vector and find its component form.



[Figure 13]



[Figure 14]



[Figure 15]

For questions 22-24, plot and correctly label each vector.

22. $\overrightarrow{AB} = \langle 4, -3 \rangle$
23. $\overrightarrow{CD} = \langle -6, 8 \rangle$
24. $\overrightarrow{FE} = \langle -2, 0 \rangle$
25. The coordinates of $\triangle DEF$ are $D(4,-2), E(7,-4)$ and $F(5,3)$. Translate $\triangle DEF$ using the vector $\langle 5, 11 \rangle$ and find the coordinates of $\triangle D'E'F'$.
26. The coordinates of quadrilateral $QUAD$ are $Q(-6,1), U(-3,7), A(4,-2)$ and $D(1,-8)$. Translate $QUAD$ using the vector $\langle -3, -7 \rangle$ and find the coordinates of $Q'U'A'D'$.

For problems 27-29, write the translation rule as a translation vector.

27. $(x,y) \rightarrow (x-3,y+8)$
28. $(x,y) \rightarrow (x+9,y-12)$
29. $(x,y) \rightarrow (x,y-7)$

For problems 30-32, write the translation vector as a translation rule.

30. $\langle -7,2 \rangle$
31. $\langle 11,25 \rangle$
32. $\langle 15,-9 \rangle$

Review Queue Answers

1. $y = -2x + 17$
2. Kite
3. $y = -2x + 5$
4. $y = \frac{1}{2}x - 5$