10.2 Trapezoids, Rhombi, and Kites

FlexBooks® 2.0 > American HS Geometry > Trapezoids, Rhombi, and Kites

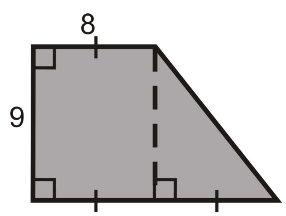
Last Modified: Dec 25, 2014

Learning Objectives

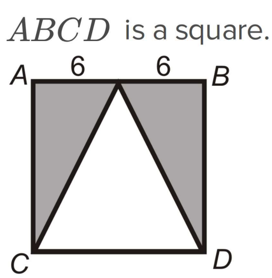
- Derive and use the area formulas for trapezoids, rhombi, and kites.

Review Queue

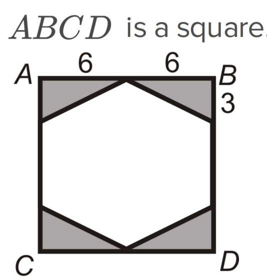
Find the area the *shaded* regions in the figures below.



[Figure 1]



[Figure 2]



[Figure 3]

1. Find the area of #1 using a different method.

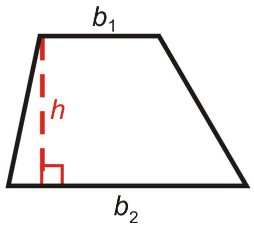
Know What? The Brazilian flag is to the right. The flag has dimensions of 20×14 (units vary depending on the size, so we will not use any here). The vertices of the yellow rhombus in the middle are 1.7 units from the midpoint of each side.

[Figure 4]

Find the total area of the flag and the area of the rhombus (including the circle). *Do not round your answers.*

Area of a Trapezoid

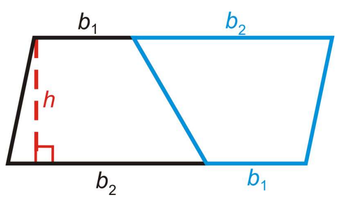
Recall that a trapezoid is a quadrilateral with one pair of parallel sides. The lengths of the parallel sides are the bases. The perpendicular distance between the parallel sides is the height, or altitude, of the trapezoid.



[Figure 5]

To find the area of the trapezoid, let's turn it into a parallelogram. To do this, make a copy of the trapezoid and then rotate the copy 180° .

Now, this is a parallelogram with height h and base $b_1 + b_2$. Let's find the area of this shape.



[Figure 6]

$$A = h(b_1 + b_2)$$

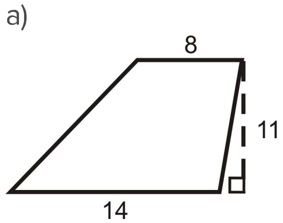
Because the area of this parallelogram is made up of two congruent trapezoids, the area of one trapezoid would be $A = \frac{1}{2}h(b_1 + b_2)$.

Area of a Trapezoid: The area of a trapezoid with height h and bases b_1 and b_2 is $A = \frac{1}{2}h(b_1 + b_2)$.

The formula for the area of a trapezoid could also be written as *the average of the bases time the height.*

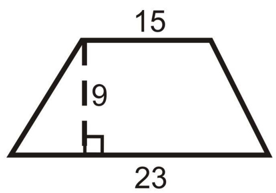
Example 1: Find the area of the trapezoids below.

a)



[Figure 7]

b)



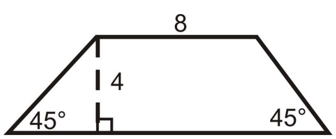
[Figure 8]

Solution:

a)
$$A = \frac{1}{2}(11)(14+8)$$
$$A = \frac{1}{2}(11)(22)$$
$$A = 121\ units^2$$

b)
$$A = \frac{1}{2}(9)(15+23)$$
$$A = \frac{1}{2}(9)(38)$$
$$A = 171\ units^2$$

Example 2: Find the perimeter and area of the trapezoid.



[Figure 9]

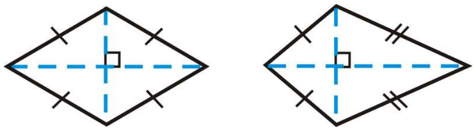
Solution: Even though we are not told the length of the second base, we can find it using special right triangles. Both triangles at the ends of this trapezoid are isosceles right triangles, so the hypotenuses are $4\sqrt{2}$ and the other legs are of length 4.

$$P = 8 + 4\sqrt{2} + 16 + 4\sqrt{2} \qquad A = \frac{1}{2}(4)(8+16)$$
$$P = 24 + 8\sqrt{2} \approx 35.3\ units \qquad A = 48\ units^2$$

Area of a Rhombus and Kite

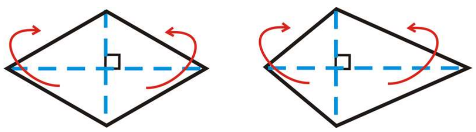
Recall that a rhombus is an equilateral quadrilateral and a kite has adjacent congruent sides.

Both of these quadrilaterals have perpendicular diagonals, which is how we are going to find their areas.



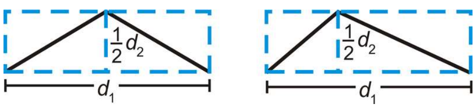
[Figure 10]

Notice that the diagonals divide each quadrilateral into 4 triangles. In the rhombus, all 4 triangles are congruent and in the kite there are two sets of congruent triangles. If we move the two triangles on the bottom of each quadrilateral so that they match up with the triangles above the horizontal diagonal, we would have two rectangles.



[Figure 11]

So, the height of these rectangles is half of one of the diagonals and the base is the length of the other diagonal.



[Figure 12]

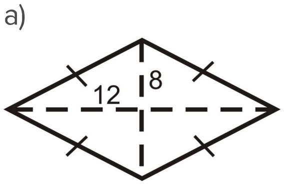
Area of a Rhombus: If the diagonals of a rhombus are d_1 and d_2 , then the area is $A = \frac{1}{2}d_1d_2$.

Area of a Kite: If the diagonals of a kite are d_1 and d_2 , then the area is $A = \frac{1}{2}d_1d_2$.

You could also say that the area of a kite and rhombus are *half the product of the diagonals.*

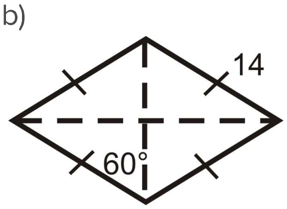
Example 3: Find the perimeter and area of the rhombi below.

a)



[Figure 13]

b)



[Figure 14]

Solution: In a rhombus, all four triangles created by the diagonals are congruent.

a) To find the perimeter, you must find the length of each side, which would be the hypotenuse of one of the four triangles. Use the Pythagorean Theorem.

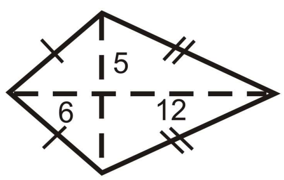
$$\begin{aligned} 12^2+8^2 &= side^2 \\ 144+64 &= side^2 \\ side &= \sqrt{208}=4\sqrt{13} \\ P &= 4(4\sqrt{13})=16\sqrt{13} \end{aligned} \qquad \begin{aligned} A &= \frac{1}{2}\cdot 16\cdot 24 \\ A &= 192 \end{aligned}$$

b) Here, each triangle is a 30-60-90 triangle with a hypotenuse of 14. From the special right triangle ratios the short leg is 7 and the long leg is $7\sqrt{3}$.

$$P=4\cdot 14=56 \qquad A=\frac{1}{2}\cdot 7\cdot 7\sqrt{3}=\frac{49\sqrt{3}}{2}\approx 42.44$$

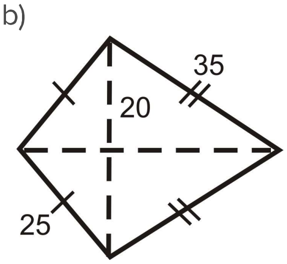
Example 4: Find the perimeter and area of the kites below.

a)



[Figure 15]

b)



[Figure 16]

Solution: In a kite, there are two pairs of congruent triangles. You will need to use the Pythagorean Theorem in both problems to find the length of sides or diagonals.

a)

$$\begin{aligned}&\text{Shorter sides of kite}\\&6^2+5^2=s_1^2\\&36+25=s_1^2\\&\quad s_1=\sqrt{61}\\&P=2(\sqrt{61})+2(13)=2\sqrt{61}+26\approx 41.6\\&A=\frac{1}{2}(10)(18)=90\end{aligned}$$

$$\begin{aligned}&\text{Longer sides of kite}\\&12^2+5^2=s_2^2\\&144+25=s_2^2\\&\quad s_2=\sqrt{169}=13\end{aligned}$$

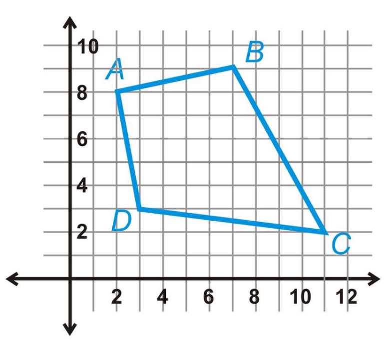
b)

$$\begin{aligned}&\text{Smaller diagonal portion}\\&20^2+d_s^2=25^2\\&\quad d_s^2=225\\&\quad d_s=15\\&P=2(25)+2(35)=120\\&A=\frac{1}{2}(15+5\sqrt{33})(40)\approx 874.5\end{aligned}$$

$$\begin{aligned}&\text{Larger diagonal portion}\\&20^2+d_l^2=35^2\\&\quad d_l^2=825\\&\quad d_l=5\sqrt{33}\end{aligned}$$

Example 5: The vertices of a quadrilateral are $A(2,8), B(7,9), C(11,2)$, and $D(3,3)$. Determine the type of quadrilateral and find its area.

Solution: For this problem, it might be helpful to plot the points. From the graph we can see this is probably a kite. Upon further review of the sides, $AB = AD$ and $BC = DC$ (you can do the distance formula to verify). Let's see if the diagonals are perpendicular by calculating their slopes.



[Figure 17]

$$\begin{aligned}m_{AC}&=\frac{2-8}{11-2}=-\frac{6}{9}=-\frac{2}{3}\\m_{BD}&=\frac{9-3}{7-3}=\frac{6}{4}=\frac{3}{2}\end{aligned}$$

Yes, the diagonals are perpendicular because the slopes are opposite signs and reciprocals. $ABCD$ is a kite. To find the area, we need to find the length of the diagonals. Use the distance formula.

$$\begin{aligned} d_1 &= \sqrt{(2-11)^2+(8-2)^2} \\ &= \sqrt{(-9)^2+6^2} \\ &= \sqrt{81+36} = \sqrt{117} = 3\sqrt{13} \end{aligned} \qquad \begin{aligned} d_2 &= \sqrt{(7-3)^2+(9-3)^2} \\ &= \sqrt{4^2+6^2} \\ &= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \end{aligned}$$

Now, plug these lengths into the area formula for a kite.

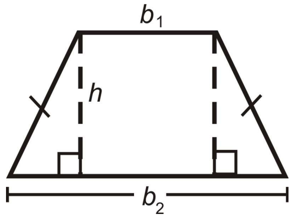
$$A = \frac{1}{2}(3\sqrt{13})(2\sqrt{13}) = 39\, units^2$$

Know What? Revisited The total area of the Brazilian flag is $A = 14 \cdot 20 = 280\, units^2$. To find the area of the rhombus, we need to find the length of the diagonals. One diagonal is $20-1.7-1.7 = 16.6\, units$ and the other is $14-1.7-1.7 = 10.6\, units$. The area is $A = \frac{1}{2}(16.6)(10.6) = 87.98\, units^2$.

Review Questions

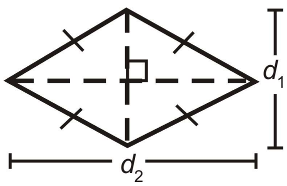
1. Do you think all rhombi and kites with the same diagonal lengths have the same area? *Explain* your answer.

Use the isosceles trapezoid to show that the area of this trapezoid can also be written as the sum of the area of the two triangles and the rectangle in the middle. Write the formula and then reduce it to equal $\frac{1}{2}h(b_1+b_2)$ or $\frac{h}{2}(b_1+b_2)$.



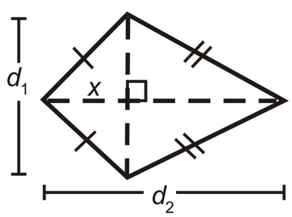
[Figure 18]

Use this picture of a rhombus to show that the area of a rhombus is equal to the sum of the areas of the four congruent triangles. Write a formula and reduce it to equal $\frac{1}{2}d_1d_2$.



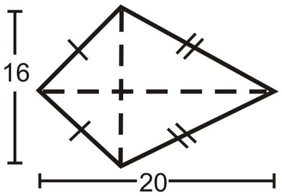
[Figure 19]

Use this picture of a kite to show that the area of a kite is equal to the sum of the areas of the two pairs of congruent triangles. Recall that d_1 is bisected by d_2 . Write a formula and reduce it to equal $\frac{1}{2}d_1d_2$.

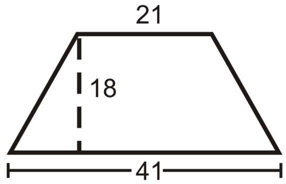


[Figure 20]

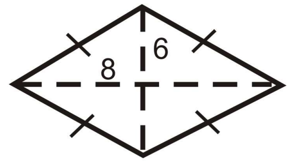
Find the area of the following shapes. *Leave answers in simplest radical form*.



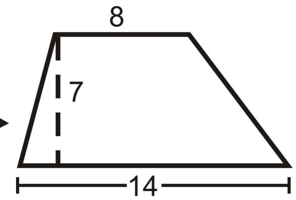
[Figure 21]



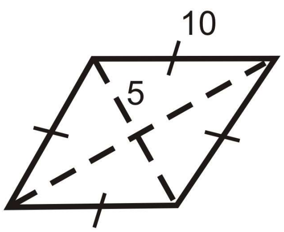
[Figure 22]



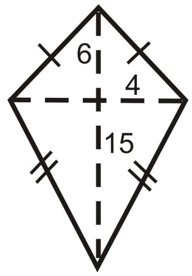
[Figure 23]



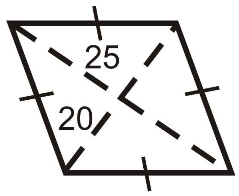
[Figure 24]



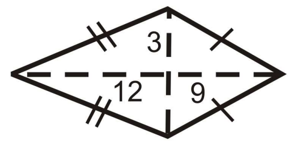
[Figure 25]



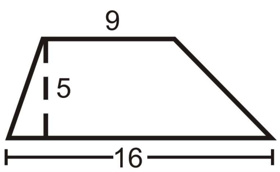
[Figure 26]



[Figure 27]

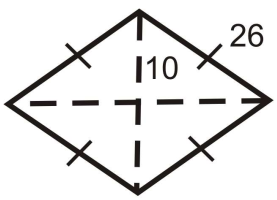


[Figure 28]

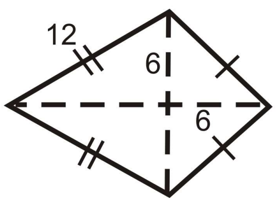


[Figure 29]

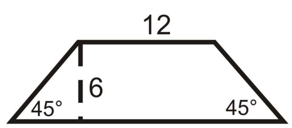
Find the area and perimeter of the following shapes. *Leave answers in simplest radical form.*



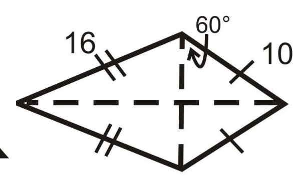
[Figure 30]



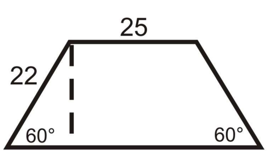
[Figure 31]



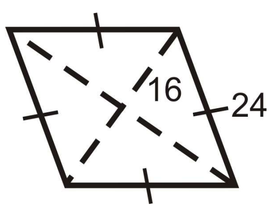
[Figure 32]



[Figure 33]



[Figure 34]

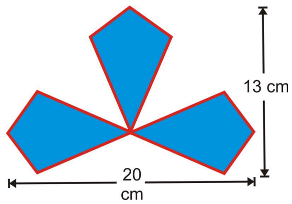


[Figure 35]

14. Quadrilateral $ABCD$ has vertices $A(-2,0), B(0,2), C(4,2)$, and $D(0,-2)$. Show that $ABCD$ is a trapezoid and find its area. *Leave your answer in simplest radical form*.
15. Quadrilateral $EFGH$ has vertices $E(2,-1), F(6,-4), G(2,-7)$, and $H(-2,-4)$. Show that $EFGH$ is a rhombus and find its area.
16. The area of a rhombus is $32\ units^2$. What are two possibilities for the lengths of the diagonals?

17. The area of a kite is $54 \; units^2$. What are two possibilities for the lengths of the diagonals?

    Sherry designed the logo for a new company. She used three congruent kites. What is the area of the entire logo?

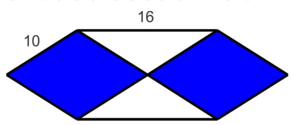


[Figure 36]

For problems 25-27, determine what kind of quadrilateral $ABCD$ is and find its area.

25. $A(-2,3),B(2,3),C(4,-3),D(-2,-1)$
26. $A(0,1),B(2,6),C(8,6),D(13,1)$
27. $A(-2,2),B(5,6),C(6,-2),D(-1,-6)$
28. Given that the lengths of the diagonals of a kite are in the ratio 4:7 and the area of the kite is 56 square units, find the lengths of the diagonals.
29. Given that the lengths of the diagonals of a rhombus are in the ratio 3:4 and the area of the rhombus is 54 square units, find the lengths of the diagonals.

    Sasha drew this plan for a wood inlay he is making. 10 is the length of the slanted side and 16 is the length of the horizontal line segment as shown in the diagram. Each shaded section is a rhombus. What is the total area of the shaded sections?



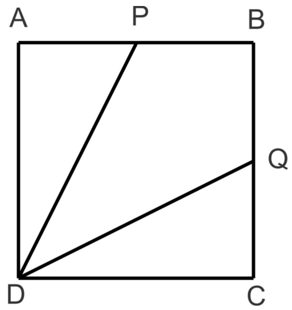
[Figure 37]

30. In the figure to the right, $ABCD$ is a square. $AP = PB = BQ$ and $DC = 20 \; ft$.

    a. What is the area of $PBQD$?

    b. What is the area of $ABCD$?

    c. What fractional part of the area of $ABCD$ is $PBQD$?



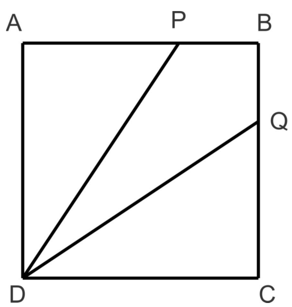
[Figure 38]

31. In the figure to the right, $ABCD$ is a square. $AP = 20\ ft$ and $PB = BQ = 10\ ft$.

a. What is the area of $PBQD$?

b. What is the area of $ABCD$?

c. What fractional part of the area of $ABCD$ is $PBQD$?



[Figure 39]

Review Queue Answers

1. $A = 9(8) + \left[\frac{1}{2}(9)(8)\right] = 72 + 36 = 108\ units^2$
2. $A = \frac{1}{2}(6)(12)2 = 72\ units^2$
3. $A = 4\left[\frac{1}{2}(6)(3)\right] = 36\ units^2$
4. $A = 9(16) - \left[\frac{1}{2}(9)(8)\right] = 144 - 36 = 108\ units^2$