10.1 Triangles and Parallelograms

FlexBooks® 2.0 > American HS Geometry > Triangles and Parallelograms

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Learning Objectives

- Understand the basic concepts of area.

- Use formulas to find the area of triangles and parallelograms.

Review Queue

1. Define perimeter and area, in your own words.

2. Solve the equations below. Simplify any radicals.

   a. $x^2 = 121$

   b. $4x^2 = 80$

   c. $x^2 - 6x + 8 = 0$

3. If a rectangle has sides 4 and 7, what is the perimeter?

Know What? Ed's parents are getting him a new bed. He has decided that he would like a king bed. Upon further research, Ed discovered there are two types of king beds, an Eastern (or standard) King and a California King. The Eastern King has $76'' \times 80''$ dimensions, while the California King is $72'' \times 84''$ (both dimensions are *width* $\times$ *length*). Which bed has a larger area to lie on? Which one has a larger perimeter? If Ed is 6'4", which bed makes more sense for him to buy?
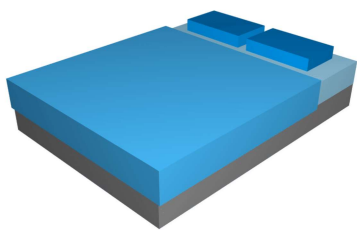
[Figure 1]

Areas and Perimeters of Squares and Rectangles

Perimeter: The distance around a shape. Or, the sum of all the edges of a two-dimensional figure.

The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write "units."

Example 1: Find the perimeter of the figure to the left.

Solution: First, notice there are no units, but the figure is on a grid. Here, we can use the grid as our units. Count around the figure to find the perimeter. We will start at the bottom left-hand corner and go around the figure clockwise.

$$5 + 1 + 1 + 1 + 5 + 1 + 3 + 1 + 1 + 1 + 1 + 2 + 4 + 7$$

The answer is 34 units.
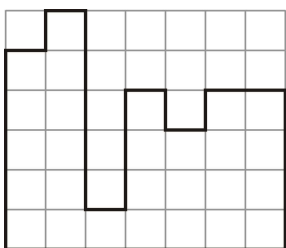
[Figure 2]

You are probably familiar with the area of squares and rectangles from a previous math class. Recall that you must always establish a unit of measure for area. Area is always measured in square units, square feet $\left(ft.^2\right)$, square inches $\left(in.^2\right)$, square centimeters $\left(cm.^2\right)$, etc. Make sure that the length and width are in the same units before applying any area formula. If no specific units are given, write "$units^2$."

Example 2: Find the area of the figure from Example 1.

Solution: If the figure is not a standard shape, you can count the number of squares within the figure. If we start on the left and count each column, we would have:

$$5 + 6 + 1 + 4 + 3 + 4 + 4 = 27\ units^2$$

Area of a Rectangle: The area of a rectangle is the product of its base (width) and height (length) $A = bh$.

Example 3: Find the area and perimeter of a rectangle with sides 4 cm by 9 cm.

9 cm

4 cm

[Figure 3]

Solution: The perimeter is $4 + 9 + 4 + 9 = 36\ cm$. The area is $A = 9 \cdot 4 = 36\ cm^2$.

In this example we see that a formula can be generated for the perimeter of a rectangle.

Perimeter of a Rectangle: $P = 2b + 2h$, where b is the base (or width) and h is the height (or length).

If a rectangle is a square, with sides of length s , the formulas are as follows:

Perimeter of a Square: $P_{square} = 2s + 2s = 4s$

Area of a Square: $A_{sqaure} = s \cdot s = s^2$

Example 4: The area of a square is $75\ in^2$. Find the perimeter.

Solution: To find the perimeter, we need to find the length of the sides.

$$A = s^2 = 75\ in^2$$
$$s = \sqrt{75} = 5\sqrt{3}\ in$$
$$\text{From this, } P = 4\left(5\sqrt{3}\right) = 20\sqrt{3}\ in.$$

Area Postulates

Congruent Areas Postulate: If two figures are congruent, they have the same area.

This postulate needs no proof because congruent figures have the same amount of space inside them. However, two figures with the same area are not necessarily congruent.

Example 5: Draw two different rectangles with an area of $36\ cm^2$.

Solution: Think of all the different factors of 36. These can all be dimensions of the different rectangles.
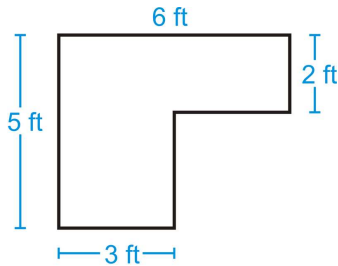
Other possibilities could be $6 \times 6, 2 \times 18$, and 1×36 .
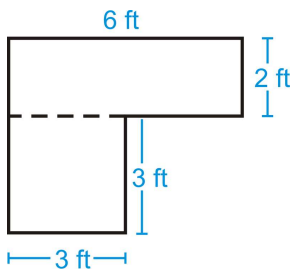
Area Addition Postulate: If a figure is composed of two or more parts that do not overlap each other, then the area of the figure is the sum of the areas of the parts.

Example 6: Find the area of the figure below. You may assume all sides are perpendicular.

[Figure 5]

Solution: Split the shape into two rectangles and find the area of each.
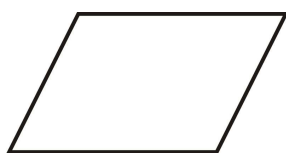
[Figure 6]

$$A_{top\,rectangle} = 6 \cdot 2 = 12 \ ft^2$$
$$A_{bottom\,square} = 3 \cdot 3 = 9 \ ft^2$$

The total area is $12 + 9 = 21 \ ft^2$.

Area of a Parallelogram

Recall that a parallelogram is a quadrilateral whose opposite sides are parallel.

[Figure 7]

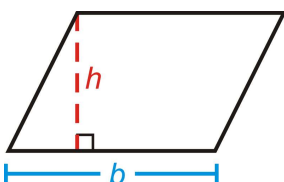To find the area of a parallelogram, make it into a rectangle.
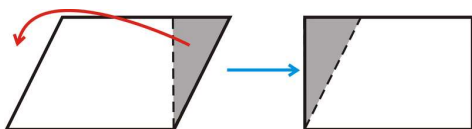
[Figure 8]

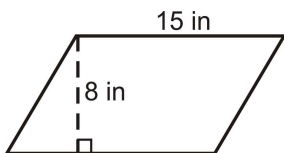From this, we see that the area of a parallelogram is the same as the area of a rectangle.

Area of a Parallelogram: The area of a parallelogram is $A = bh$.

Be careful! The height of a parallelogram is always perpendicular to the base. This means that the sides are **not** the height.

[Figure 9]

Example 7: Find the area of the parallelogram.

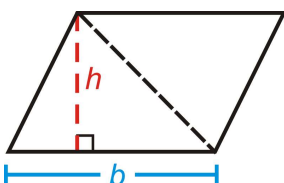[Figure 10]

Solution: $A = 15 \cdot 8 = 120 \ in^2$

Example 8: If the area of a parallelogram is $56 \ units^2$ and the base is 4 units, what is the height?

Solution: Plug in what we know to the area formula and solve for the height.

$$56 = 4h$$
$$14 = h$$

Area of a Triangle

[Figure 11]

If we take parallelogram and cut it in half, along a diagonal, we would have two congruent triangles. Therefore, the formula for the area of a triangle is the same as the formula for area of a parallelogram, but cut in half.

Area of a Triangle: $A = \dfrac{1}{2}bh$ or $A = \dfrac{bh}{2}$.

In the case that the triangle is a right triangle, then the height and base would be the legs of the right triangle. If the triangle is an obtuse triangle, the altitude, or height, could be outside of the triangle.

Example 9: Find the area and perimeter of the triangle.
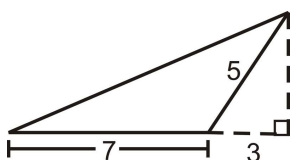
[Figure 12]

Solution: This is an obtuse triangle. First, to find the area, we need to find the height of the triangle. We are given the two sides of the small right triangle, where the hypotenuse is also the short side of the obtuse triangle. From these values, we see that the height is 4 because this is a 3-4-5 right triangle. The area is $A = \dfrac{1}{2}(4)(7) = 14 \ units^2$.

To find the perimeter, we would need to find the longest side of the obtuse triangle. If we used the dotted lines in the picture, we would see that the longest side is also the hypotenuse of the right triangle with legs 4 and 10. Use the Pythagorean Theorem.

$$
\begin{aligned}
4^2 + 10^2 &= c^2 \\
16 + 100 &= c^2 \\
c &= \sqrt{116} \approx 10.77 \qquad \text{The perimeter is } 7 + 5 + 10.77 = 22.77 \ units
\end{aligned}
$$

Example 10: Find the area of the figure below.

[Figure 13]

Solution: Divide the figure into a triangle and a rectangle with a small rectangle cut out of the lower right-hand corner.

[Figure 14]

$$A = A_{top\,triangle} + A_{rectangle} - A_{small\,triangle}$$
$$A = \left(\frac{1}{2} \cdot 6 \cdot 9\right) + (9 \cdot 15) + \left(\frac{1}{2} \cdot 3 \cdot 6\right)$$
$$A = 27 + 135 + 9$$
$$A = 171 \; units^2$$

Know What? Revisited The area of an Eastern King is $6080 \; in^2$ and the California King is $6048 \; in^2$. The perimeter of both beds is 312 in. Because Ed is 6'4", he should probably get the California King because it is 4 inches longer.

Review Questions

1. Find the area and perimeter of a square with sides of length 12 in.

2. Find the area and perimeter of a rectangle with height of 9 cm and base of 16 cm.

3. Find the area of a parallelogram with height of 20 m and base of 18 m.

4. Find the area and perimeter of a rectangle if the height is 8 and the base is 14.

5. Find the area and perimeter of a square if the sides are 18 ft.

6. If the area of a square is $81 \; ft^2$, find the perimeter.

7. If the perimeter of a square is 24 in, find the area.

8. Find the area of a triangle with base of length 28 cm and height of 15 cm.

9. What is the height of a triangle with area $144 \; m^2$ and a base of 24 m?

10. The perimeter of a rectangle is 32. Find two different dimensions that the rectangle could be.

11. Draw two different rectangles that haven an area of $90 \; mm^2$.

12. Write the converse of the Congruent Areas Postulate. Determine if it is a true statement. If not, write a counterexample. If it is true, explain why.

Use the triangle to answer the following questions.

[Figure 15]

13. Find the height of the triangle by using the geometric mean.

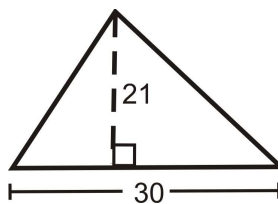14. Find the perimeter.

15. Find the area.

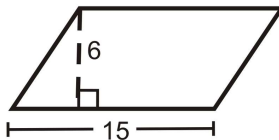Use the triangle to answer the following questions.
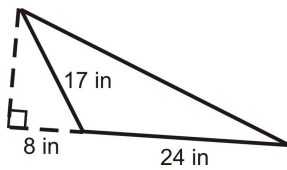
[Figure 16]

16. Find the height of the triangle.

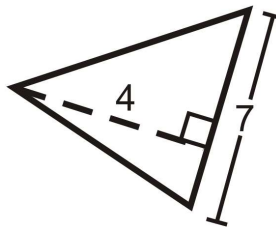17. Find the perimeter.

18. Find the area.

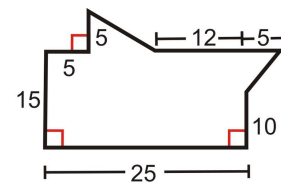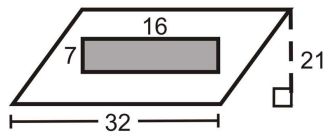Find the area of the following shapes.

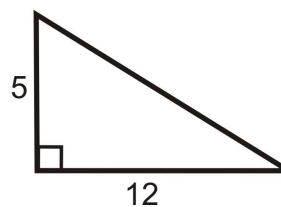[Figure 17]

[Figure 18]

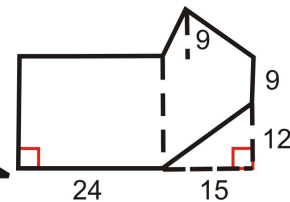[Figure 19]

[Figure 20]

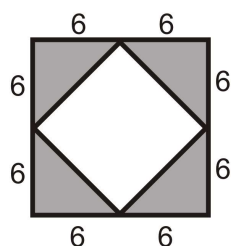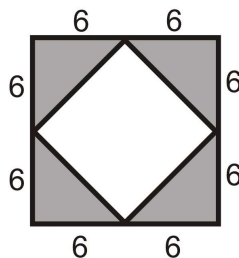Find the area of the unshaded region.

[Figure 21]

[Figure 22]

[Figure 23]

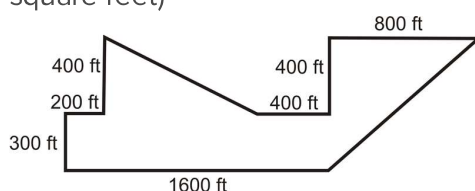Find the area of the shaded region.   Find the area of the unshaded region.

[Figure 24]   [Figure 25]

Lin bought a tract of land for a new apartment complex. The drawing below shows the measurements of the sides of the tract. Approximately how many acres of land did Lin buy? You may assume any angles that look like right angles are $90°$. (1 acre $\approx$ 40,000 square feet)

[Figure 26]

Challenge Problems

For problems 29 and 30 find the dimensions of the rectangles with the given information.

29. A rectangle with a perimeter of 20 units and an area of $24 \; units^2$.

30. A rectangle with a perimeter of 72 units and an area of $288 \; units^2$.

For problems 31 and 32 find the height and area of the equilateral triangle with the given perimeter.

31. Perimeter 18 units.

32. Perimeter 30 units.

33. Generalize your results from problems 31 and 32 into a formula to find the height and area of an equilateral triangle with side length x.

Linus has 100 ft of fencing to use in order to enclose a 1200 square foot rectangular pig pen. The pig pen is adjacent to the barn so he only needs to form three sides of the rectangular area as shown below. What dimensions should the pen be?
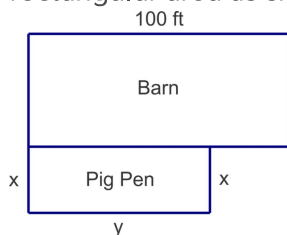
[Figure 27]

A rectangle with perimeter 138 units is divided into 8 congruent rectangles as shown in the diagram below. Find the perimeter and area of one of the 8 congruent rectangles.
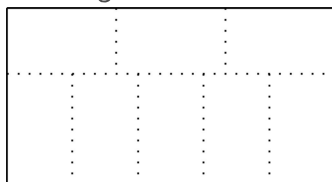
[Figure 28]

Review Queue Answers

1. *Possible* Answers

Perimeter: The distance around a shape.

Area: The space inside a shape.

2. (a) $x = \pm 11$

(b) $x = \pm 2\sqrt{5}$

(c) $x = 4, 2$

3. $4 + 4 + 7 + 7 = 22$