# 11.4 Volume of Prisms and Cylinders

FlexBooks® 2.0 > American HS Geometry > Volume of Prisms and Cylinders

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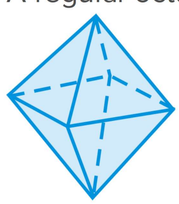
## Learning Objectives

- Find the volume of a prism.
- Find the volume of a cylinder.

## Review Queue

1. Define volume in your own words.
2. What is the surface area of a cube with 3 inch sides?
3. What is the surface area of a cube with $4\sqrt{2}$ inch sides?

   A regular octahedron has 8 congruent equilateral triangles as the faces.

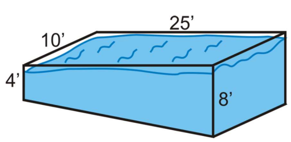


[Figure 1]

   a. If each edge is 4 cm, what is the surface area of the figure?

   b. If each edge is $s$, what is the surface area of the figure?

**Know What?** The pool is done and your family is ready to fill it with water. Recall that the shallow end is 4 ft. and the deep end is 8 ft. The pool is 10 ft. wide by 25 ft. long. How many gallons of water will it take to fill the pool? There are approximately 7.48 gallons in a cubic foot.




[Figure 2]

## Volume of a Rectangular Prism

**Volume:** The measure of how much space a three-dimensional figure occupies.

Another way to define volume would be how much a three-dimensional figure can hold, water or sand, for example. The basic unit of volume is the cubic unit: cubic centimeter $(cm^3)$, cubic inch $(in^3)$, cubic meter $(m^3)$, cubic foot $(ft^3)$, etc. Each basic cubic unit has a measure of one for each: length, width, and height.

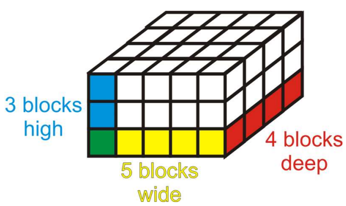
**Volume of a Cube Postulate:** The volume of a cube is the cube of the length of its side, or $s^3$.

What this postulate tells us is that every solid can be broken down into cubes, going along with our basic unit of measurement, the cubic unit. For example, if we wanted to find the volume of a cube with one inch sides, it would be $1^3 = 1\, in^3$. If we wanted to find the volume of a cube with 9 inch sides, it would be $9^3 = 729\, in^3$.

**Volume Congruence Postulate:** If two solids are congruent, then their volumes are congruent.

**Volume Addition Postulate:** The volume of a solid is the sum of the volumes of all of its non-overlapping parts.

**Example 1:** Find the volume of the right rectangular prism below.




[Figure 3]

**Solution:** A rectangular prism can be made from any square cubes. To find the volume, we would simply count the cubes. The bottom layer has 20 cubes, or 4 times 5, and there are 3 layers, or the same as the height. Therefore there are 60 cubes in this prism and the volume would be $60\, units^3$.

But, what if we didn't have cubes? Let's generalize this formula for any rectangular prism. Notice that each layer is the same as the area of the base. Then, we multiplied by the height. Here is our formula.

**Volume of a Rectangular Prism:** If a rectangular prism is $h$ units high, $w$ units wide, and $l$ units long, then its volume is $V = l \cdot w \cdot h$.

**Example 2:** A typical shoe box is 8 in by 14 in by 6 in. What is the volume of the box?

**Solution:** We can assume that a shoe box is a rectangular prism. Therefore, we can use the formula above.

$$V = (8)(14)(6) = 672\, in^2$$

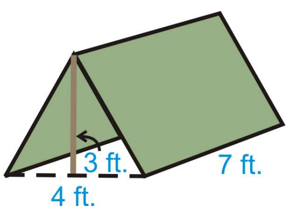
## Volume of any Prism

If we further analyze the formula for the volume of a rectangular prism, we would see that $l \cdot w$ is equal to the area of the base of the prism, a rectangle. If the bases are not rectangles, this would still be true, however we would have to rewrite the equation a little.

**Volume of a Prism:** If the area of the base of a prism is $B$ and the height is $h$, then the volume is $V = B \cdot h$.

Notice that "$B$" is not always going to be the same. So, to find the volume of a prism, you would first find the area of the base and then multiply it by the height.

**Example 3:** You have a small, triangular prism shaped tent. How much volume does it have, once it is set up?

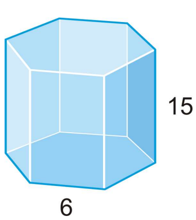



[Figure 4]

**Solution:** First, we need to find the area of the base. That is going to be $B = \frac{1}{2}(3)(4) = 6\ ft^2$. Multiplying this by 7 we would get the entire volume. The volume is $42\ ft^3$.

Even though the height in this problem does not look like a "height," it is, when referencing the formula. Usually, the height of a prism is going to be the last length you need to use.

**Example 4:** Find the volume of the regular hexagonal prism below.




[Figure 5]

**Solution:** Recall that a regular hexagon is divided up into six equilateral triangles. The height of one of those triangles would be the apothem. If each side is 6, then half of that is 3 and half of an equilateral triangle is a 30-60-90 triangle. Therefore, the apothem is going to be $3\sqrt{3}$. The area of the base is:

$$B = \frac{1}{2}(3\sqrt{3})(6)(6) = 54\sqrt{3}\,units^2$$

And the volume will be:

$$V = Bh = (54\sqrt{3})(15) = 810\sqrt{3}\,units^3$$

## Cavalieri's Principle

Recall that earlier in this section we talked about oblique prisms. These are prisms that lean to one side and the height is outside the prism. What would be the area of an oblique prism? To answer this question, we need to introduce Cavalieri's Principle. Consider to piles of books below.

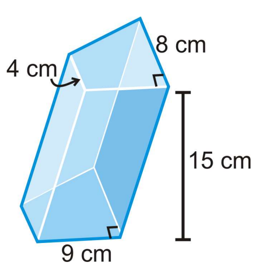
Both piles have 15 books, therefore they will have the same volume. However, one pile is leaning. Cavalieri's Principle says that this does not matter, as long as the heights are the same and every horizontal cross section has the same area as the base, the volumes are the same.

[Figure 6]

**Cavalieri's Principle:** If two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

Basically, if an oblique prism and a right prism have the same base area and height, then they will have the same volume.

**Example 5:** Find the area of the oblique prism below.




[Figure 7]

**Solution:** This is an oblique right trapezoidal prism. First, find the area of the trapezoid.

$$B = \frac{1}{2}(9)(8+4) = 9(6) = 54\ cm^2$$

Then, multiply this by the height.

$$V = 54(15) = 810\ cm^3$$

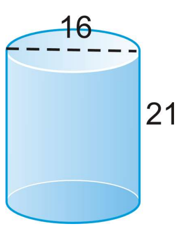
## Volume of a Cylinder

If we use the formula for the volume of a prism, $V = Bh$ , we can find the volume of a cylinder. In the case of a cylinder, the base, or $B$ , would be the area of a circle. Therefore, the volume of a cylinder would be $V = (\pi r^2)h$ , where $\pi r^2$ is the area of the base.

**Volume of a Cylinder:** If the height of a cylinder is $h$ and the radius is $r$ , then the volume would be $V = \pi r^2 h$ .

Also, like a prism, Cavalieri's Principle holds. So, the volumes of an oblique cylinder and a right cylinder have the same formula.

**Example 6:** Find the volume of the cylinder.

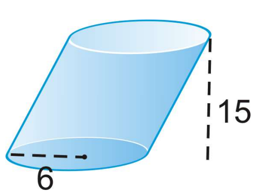



[Figure 8]

**Solution:** If the diameter is 16, then the radius is 8.

$$V = \pi 8^2(21) = 1344\pi\ units^3$$

**Example 7:** Find the volume of the cylinder.




[Figure 9]

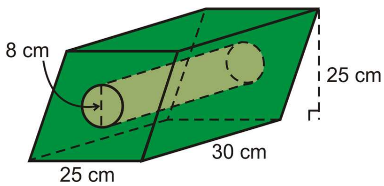
**Solution:** $V = \pi 6^2(15) = 540\pi\ units^3$

**Example 8:** If the volume of a cylinder is $484\pi\ in^3$ and the height is 4 in, what is the radius?

**Solution:** Plug in what you know to the volume formula and solve for $r$.

$$\begin{aligned} 484\pi &= \pi r^2(4) \\ 121 &= r^2 \\ 11 &= r \end{aligned}$$

**Example 9:** Find the volume of the solid below.




[Figure 10]

**Solution:** This solid is a parallelogram-based prism with a cylinder cut out of the middle. To find the volume, we need to find the volume of the prism and then subtract the volume of the cylinder.

$$\begin{aligned} V_{prism} &= (25 \cdot 25)30 = 18750\ cm^3 \\ V_{cylinder} &= \pi(4)^2(30) = 480\pi\ cm^3 \end{aligned}$$

The total volume is $18750 - 480\pi \approx 17242.04\ cm^3$.

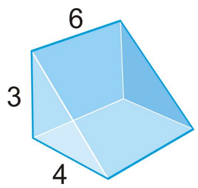
**Know What? Revisited** Even though it doesn't look like it, the trapezoid is considered the base of this prism. The area of the trapezoids are $\frac{1}{2}(4+8)25 = 150\ ft^2$. Multiply this by the height, 10 ft, and we have that the volume is $1500\ ft^3$. To determine the number of gallons that are needed, divide 1500 by 7.48. $\frac{1500}{7.48} \approx 200.53$ gallons are needed to fill the pool.

## Review Questions

1. Two cylinders have the same surface area. Do they have the same volume? How do you know?
2. How many one-inch cubes can fit into a box that is 8 inches wide, 10 inches long, and 12 inches tall? Is this the same as the volume of the box?
3. A cereal box in 2 inches wide, 10 inches long and 14 inches tall. How much cereal does the box hold?

4. A can of soda is 4 inches tall and has a diameter of 2 inches. How much soda does the can hold? Round your answer to the nearest hundredth.
5. A cube holds $216\ in^3$ . What is the length of each edge?
6. A cylinder has a volume of $486\pi\ ft.^3$ . If the height is 6 ft., what is the diameter?

Use the right triangular prism to answer questions 7 and 8.




[Figure 11]

7. What is the length of the third base edge?
8. Find the volume of the prism.

   Fuzzy dice are cubes with 4 inch sides.

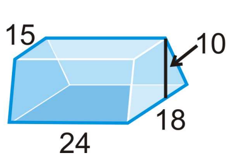
[Figure 12]

   a. What is the volume of one die?

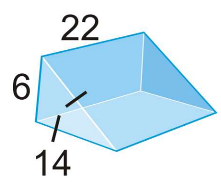
   b. What is the volume of both dice?

9. A right cylinder has a 7 cm radius and a height of 18 cm. Find the volume.

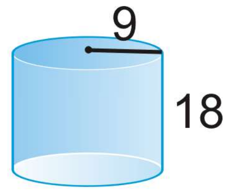
Find the volume of the following solids. Round your answers to the nearest hundredth.



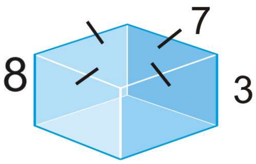
[Figure 13]



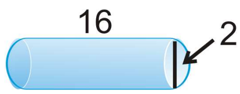
[Figure 14]



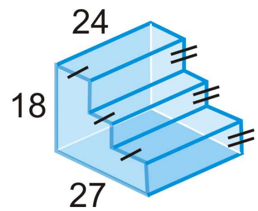
[Figure 15]



[Figure 16]

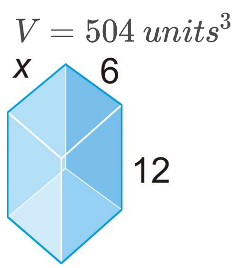


[Figure 17]

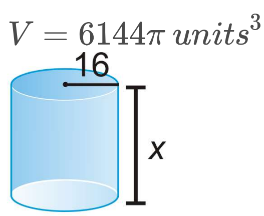


[Figure 18]

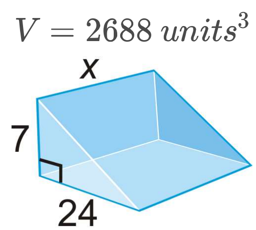
***Algebra Connection*** Find the value of $x$ , given the surface area.



[Figure 19]



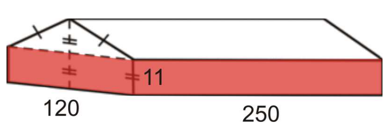
[Figure 20]



[Figure 21]

17. The area of the base of a cylinder is $49\pi\ in^2$ and the height is 6 in. Find the volume.
18. The circumference of the base of a cylinder is $80\pi\ cm$ and the height is 15 cm. Find the volume.
19. The lateral surface area of a cylinder is $30\pi\ m^2$ and the circumference is $10\pi\ m$ . What is the volume of the cylinder?

Use the diagram below for questions 23-25. The barn is shaped like a pentagonal prism with dimensions shown in feet.

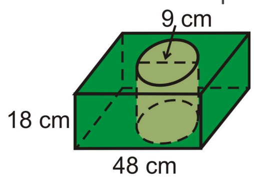


[Figure 22]

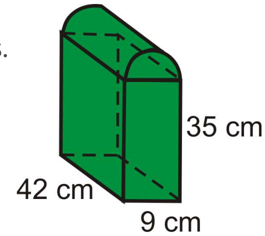
23. Find the volume of the red rectangular prism.
24. Find the volume of the triangular prism on top of the rectangular prism.
25. Find the total volume of the barn.

Find the volume of the composite solids below. Round your answers to the nearest hundredth.

The bases are squares.




[Figure 23]




[Figure 24]

26. The volume of a cylinder with height to radius ratio of 4:1 is $108\pi\ cm^3$ . Find the radius and height of the cylinder.
27. The length of a side of the base of a hexagonal prism is 8 cm and its volume is $1056\sqrt{3}\ cm^3$ . Find the height of the prism.
28. A cylinder fits tightly inside a rectangular prism with dimensions in the ratio 5:5:7 and volume $1400\ in^3$ . Find the volume of the space inside the prism that is not contained in the cylinder.

## Review Queue Answers

1. The amount a three-dimensional figure can hold.
2. $54\ in^2$
3. $192\ in^2$
4. Answers:
   a. $8\left(\frac{1}{4}\cdot 4^2\sqrt{3}\right) = 32\sqrt{3}\ cm^2$
   b. $8\left(\frac{1}{4}\cdot s^2\sqrt{3}\right) = 2s^2\sqrt{3}$