# 11.5 Volume of Pyramids and Cones



Last Modified: Dec 25, 2014

## Learning Objectives

- Find the volume of a pyramid.
- Find the volume of a cone.

## Review Queue

1. Find the volume of a square prism with 8 inch base edges and a 12 inch height.
2. Find the volume of a cylinder with a *diameter* of 8 inches and a height of 12 inches.
3. In your answers from #1 and #2, which volume is bigger? Why do you think that is?
4. Find the surface area of a square pyramid with 10 inch base edges and a height of 12 inches.

**Know What?** The Khafre Pyramid is the second largest pyramid of the Ancient Egyptian Pyramids in Giza. It is a square pyramid with a base edge of 706 feet and an original height of 407.5 feet. What was the original volume of the Khafre Pyramid?

[Figure 1]

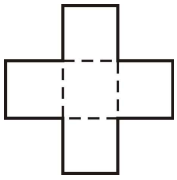
## Volume of a Pyramid

Recall that the volume of a prism is $Bh$, where $B$ is the area of the base. The volume of a pyramid is closely related to the volume of a prism with the same sized base.

**Investigation 11-1: Finding the Volume of a Pyramid**

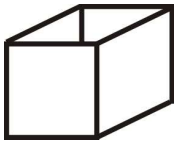
Tools needed: pencil, paper, scissors, tape, ruler, dry rice or sand.

Make an open net (omit one base) of a cube, with 2 inch sides.



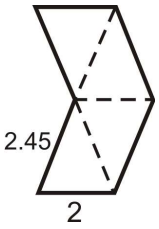
[Figure 2]

Cut out the net and tape up the sides to form an open cube.



[Figure 3]

Make an open net (no base) of a square pyramid, with lateral edges of 2.45 inches and base edges of 2 inches. This will make the overall height 2 inches.




[Figure 4]

Cut out the net and tape up the sides to form an open pyramid.

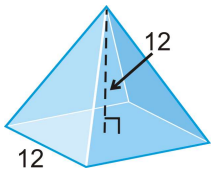
[Figure 5]

1. Fill the pyramid with dry rice. Then, dump the rice into the open cube. How many times do you have to repeat this to fill the cube?

**Volume of a Pyramid:** If $B$ is the area of the base and $h$ is the height, then the volume of a pyramid is $V = \frac{1}{3}Bh$.

The investigation showed us that you would need to repeat this process three times to fill the cube. This means that the pyramid is one-third the volume of a prism with the same base.

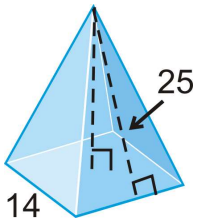
**Example 1:** Find the volume of the pyramid.




[Figure 6]

**Solution:** $V = \frac{1}{3}(12^2)12 = 576\ units^3$

**Example 2:** Find the volume of the pyramid.



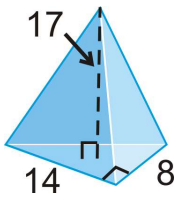

[Figure 7]

**Solution:** In this example, we are given the slant height. For volume, we need the height, so we need to use the Pythagorean Theorem to find it.

$$\begin{aligned} 7^2 + h^2 &= 25^2 \\ h^2 &= 576 \\ h &= 24 \end{aligned}$$

Using the height, the volume is $\frac{1}{3}(14^2)(24) = 1568\ units^3$ .

**Example 3:** Find the volume of the pyramid.




[Figure 8]

**Solution:** The base of this pyramid is a right triangle. So, the area of the base is $\frac{1}{2}(14)(8) = 56\ units^2$ .

$$V = \frac{1}{3}(56)(17) \approx 317.33\ units^3$$

**Example 4:** A rectangular pyramid has a base area of $56\ cm^2$ and a volume of $224\ cm^3$. What is the height of the pyramid?

**Solution:** The formula for the volume of a pyramid works for any pyramid, as long as you can find the area of the base.

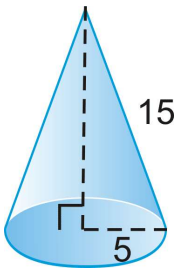
$$\begin{aligned} 224 &= 56h \\ 4 &= h \end{aligned}$$

## Volume of a Cone

The volume of cone has the same relationship with a cylinder as pyramid does with a prism. If the bases of a cone and a cylinder are the same, then the volume of a cone will be one-third the volume of the cylinder.

**Volume of a Cone:** If $r$ is the radius of a cone and $h$ is the height, then the volume is $V = \frac{1}{3}\pi r^2 h$.

**Example 5:** Find the volume of the cone.




[Figure 9]

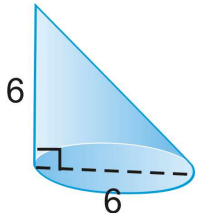
**Solution:** To find the volume, we need the height, so we have to use the Pythagorean Theorem.

$$\begin{aligned} 5^2 + h^2 &= 15^2 \\ h^2 &= 200 \\ h &= 10\sqrt{2} \end{aligned}$$

Now, we can find the volume.

$$V = \frac{1}{3}(5^2)\left(10\sqrt{2}\right)\pi \approx 370.24$$

**Example 6:** Find the volume of the cone.




[Figure 10]

**Solution:** Even though this doesn't look like the cone in Example 5, we can still find the volume in the same way. Use the *radius* in the formula.

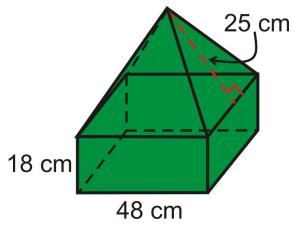
$$V = \frac{1}{3}\pi(3^2)(6) = 18\pi \approx 56.55$$

**Example 7:** The volume of a cone is $484\pi\ cm^3$ and the height is 12 cm. What is the radius?

**Solution:** Plug in what you know to the volume formula.

$$\begin{aligned} 484\pi &= \frac{1}{3}\pi r^2(12) \\ 121 &= r^2 \\ 11 &= r \end{aligned}$$

**Example 8:** Find the volume of the composite solid. All bases are squares.




[Figure 11]

**Solution:** This is a square prism with a square pyramid on top. Find the volume of each separately and then add them together to find the total volume. First, we need to find the height of the pyramid portion. The slant height is 25 and the edge is 48. Using have of the edge, we have a right triangle and we can use the Pythagorean Theorem.
$h = \sqrt{25^2 - 24^2} = 7$

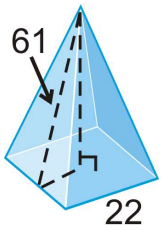
$$\begin{aligned} V_{prism} &= (48)(48)(18) = 41472\ cm^3 \\ V_{pyramid} &= \frac{1}{3}(48^2)(7) = 5376\ cm^3 \end{aligned}$$

The total volume is $41472 + 5376 = 46,848\ cm^3$ .

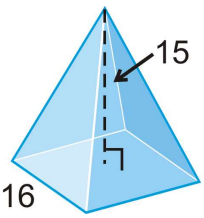
**Know What? Revisited** The original volume of the pyramid is $\frac{1}{3}(706^2)(407.5) \approx 67,704,223.33\ ft^3$ .

## Review Questions

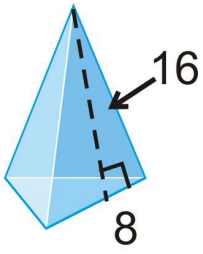
Find the volume of each regular pyramid and right cone. Round any decimal answers to the nearest hundredth. The bases of these pyramids are either squares or equilateral triangles.



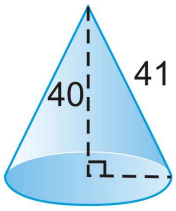
[Figure 12]



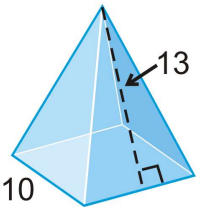
[Figure 13]



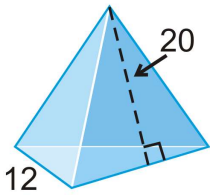
[Figure 14]



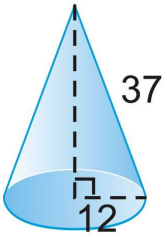
[Figure 15]



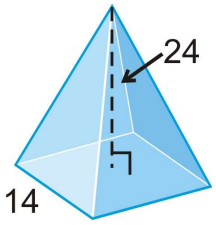
[Figure 16]



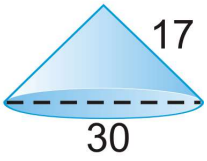
[Figure 17]



[Figure 18]

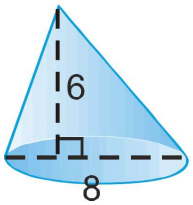


[Figure 19]

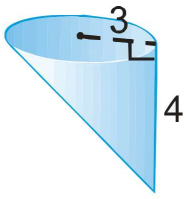


[Figure 20]

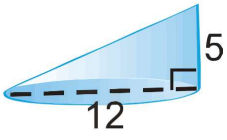
Find the volume of the following non-regular pyramids and cones. Round any decimal answers to the nearest hundredth.



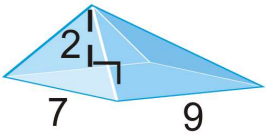
[Figure 21]



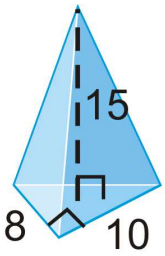
[Figure 22]



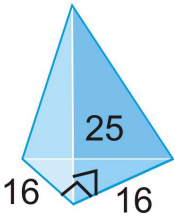
[Figure 23]



[Figure 24]

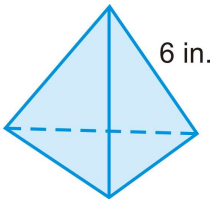


[Figure 25]



[Figure 26]

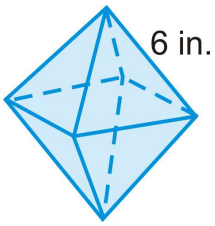
A ***regular tetrahedron*** has four equilateral triangles as its faces. Use the diagram to answer questions 16-19.




[Figure 27]

16. What is the area of the base of this regular tetrahedron?
17. What is the height of this figure? Be careful!
18. Find the volume. Leave your answer in simplest radical form.
19. ***Challenge*** If the sides are length $s$, what is the volume?

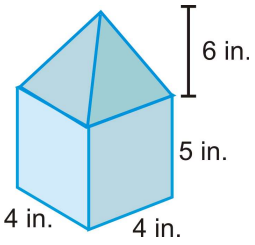
A ***regular octahedron*** has eight equilateral triangles as its faces. Use the diagram to answer questions 20-22.



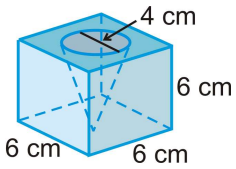

[Figure 28]

20. *Describe* how you would find the volume of this figure.
21. Find the volume. Leave your answer in simplest radical form.
22. ***Challenge*** If the sides are length $s$, what is the volume?
23. The volume of a square pyramid is 72 square inches and the base edge is 4 inches. What is the height?
24. If the volume of a cone is $30\pi\ cm^2$ and the radius is 5 cm, what is the height?
25. If the volume of a cone is $105\pi\ cm^2$ and the height is 35 cm, what is the radius?

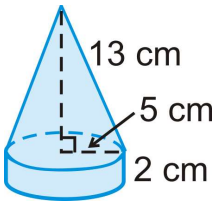
Find the volume of the composite solids. Round your answer to the nearest hundredth.




[Figure 29]



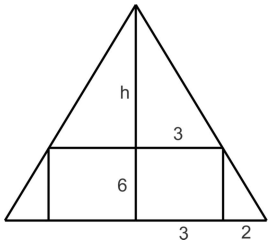

[Figure 30]




[Figure 31]

26. The ratio of the height to radius in a cone is 3:2. If the volume is $108\pi\ m^3$, find the height and radius of the cone.

A teepee is to be built such that there is a minimal cylindrical shaped central living space contained within the cone shape of diameter 6 ft and height 6 ft. If the radius of the entire teepee is 5 ft, find the total height of the teepee.




[Figure 32]

[Figure 33]

## Review Queue Answers

1. $(8^2)(12) = 768\ in^3$
2. $(4^2)(12)\pi = 192\pi \approx 603.19$
3. The volume of the square prism is greater because the square base is larger than a circle with the same diameter as the square's edge.
4. Find slant height, $l = 13$. $SA = 100 + \frac{1}{2}(40)(13) = 360\ in^2$